Parallel Integer Programming with ALPS

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Outline

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   - Parallel Computing
   - Motivation

2. The Abstract Library for Parallel Search
   - Design and Implementation
   - Preliminary Computational Results

3. The ALPS Optimization Libraries
   - The Branch, Constrain, and Price Software
   - The BiCePS Linear Integer Solver
   - Preliminary Computational Results

4. Conclusions
ALPS is a C++ class library for implementing parallel tree search. ALPS is an on-going open source software project originally developed in partnership with the COIN-OR Foundation, IBM, and NSF. ALPS is an attempt to further improve on existing solvers and frameworks in a number of ways.

What Differentiates ALPS?

- Intuitive interface and open source implementation.
- Very general, base classes make minimal algorithmic assumptions.
- Easy to specialize for particular problem classes.
- Designed for parallel scalability.
- Explicitly supports data-intensive algorithms.
- Operates effectively in both parallel and sequential environments.
Tree Search Algorithms

- Tree search algorithms systematically search the nodes of a directed, acyclic graph for one or more goal nodes.
- This process is ostensibly easy to parallelize, but the graph may not be known a priori.
- A generic tree search algorithm consists of the following elements:

  **Generic Tree Search Algorithm**

  - **Processing method**: Is goal achieved?
  - **Search strategy**: What should we work on next?
  - **Fathoming rule**: Can node can be fathomed?
  - **Branching method**: What are the successors?

- The algorithm consists of choosing a candidate node, processing it, and either fathoming or branching.
- During the course of the search, various information (knowledge) is generated and used to guide the search.
- Efficient knowledge sharing is the key to parallelization.
The goal in parallelizing any algorithm is to minimize parallel overhead.

### Components of Parallel Overhead in Tree Search

- **Communication Overhead** (cost of sharing knowledge)
- **Idle Time**
  - Handshaking/Synchronization (cost of sharing knowledge)
  - Task Starvation (cost of not sharing knowledge)
  - Ramp Up Time
  - Ramp Down Time
- **Performance of Redundant Work** (cost of not sharing knowledge)

Knowledge sharing is the main driver of efficiency.

This breakdown highlights the tradeoff between centralized and decentralized knowledge storage and decision-making.
## Illustrating Overhead and Scalability

Results solving MIPLIB instances with SYMPHONY 5.1

<table>
<thead>
<tr>
<th>Tree Size</th>
<th>Ramp Up</th>
<th>Ramp Down</th>
<th>Comm</th>
<th>Handshake</th>
<th>CPU</th>
<th>Wallclock</th>
<th>Eff</th>
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<td>220846</td>
<td>0.00</td>
<td>0.00</td>
<td>163.45</td>
<td>465.32</td>
<td>18280.70</td>
<td>19173.94</td>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0007</td>
<td>0.0021</td>
<td>0.0828</td>
<td>0.0868</td>
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<td>0.0000</td>
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<td>0.0000</td>
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<td>0.0007</td>
<td>0.0029</td>
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<td>0.0028</td>
<td>0.0006</td>
<td>0.0047</td>
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<td>0.0786</td>
<td>0.1196</td>
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</table>
ALPS: Feature Overview

Generality

- ALPS only assumes that the graph to be searched is acyclic.
- The implementation is based on a very general concept of knowledge.

Scalability

- Management overhead is reduced with the master-hub-worker paradigm.
- Knowledge is shared asynchronously through pools and brokers.
- Overhead is decreased using dynamic task granularity.
- Static and dynamic load balancing techniques are employed.
- Tasks are managed locally by a task scheduler.
ALPS: Master-Hub-Worker Paradigm
ALPS: Knowledge Sharing

- All knowledge to be shared is derived from the base class `AlpsKnowledge` and has an associated *encoded form*.
- The encoded form is used for *identification*, *storage*, and *communication*.
- `AlpsKnowledge` is maintained by one or more `AlpsKnowledgePools`.
- The knowledge pools communicate through `AlpsKnowledgeBrokers`.

![Diagram of ALPS Knowledge Sharing](image-url)
ALPS: Asynchronism

- A primary design goal for ALPS is to eliminate handshaking and synchronicity.
- Knowledge brokers can work completely asynchronously, as long as their local node pool is not empty.
- This asynchronism can result in an increase in the performance of redundant work.
- The task granularity also affects the degree of synchronism and is adjusted dynamically.
- To combat this, we need good load balancing.
Load balancing is a specific type of knowledge sharing in which tasks are distributed or redistributed.

**Static load balancing**
- Determines the initial task distribution.
- In dynamic search algorithms, this can be difficult.
- This is the main source of ramp up overhead.

**Dynamic load balancing**
- Used periodically to redistribute the useful work.
- Work must be balanced both by *quantity* and *quality*.
- Donors and receivers are matched at both the hub and master level.
- A unique aspect of load balancing in ALPS is that subtrees are kept together in order to enable *differencing*.
ALPS: How to Develop an Application

- The first step is deriving a few classes to specify the algorithm.
  
  **User-written Classes**
  - AlpsModel
  - AlpsTreeNode
  - AlpsNodeDesc
  - AlpsSolution

- Once the classes have been implemented, the user writes a `main` function.

**Sample Applications**
- Knapsack problem
- ALPS branch and cut (ABC)
- Mixed-integer Stackelberg games (LBMZZSSPIC)
int main(int argc, char* argv[]) {
    UserModel model;

    #if defined(SERIAL)
    AlpsKnowledgeBrokerSerial broker(argc, argv, model);
    #elif defined(PARALLEL_MPI)
    AlpsKnowledgeBrokerMPI broker(argc, argv, model);
    #endif

    broker.registerClass("MODEL", new UserModel);
    broker.registerClass("SOLUTION", new UserSolution);
    broker.registerClass("NODE", new UserTreeNode);

    broker.search();
    broker.printResult();
    return 0;
}
ALPS: Preliminary Experiments

Test Environment

- **Machine:** Beowulf cluster with 48 dual-processor nodes
- **Processor:** 1.0 GHz Pentium III
- **Memory:** 512M on 44 nodes, 2G on 4 nodes
- **Operating System:** Red Hat Linux 7.2
- **Message Passing:** LAM/MPI

Experimental Design

- We generated *ten hard knapsack instances* based on the rule proposed by Martello ('90).
- We ran three trials for each instance, and take the average.
- Some default parameters:
  - Two hubs were used when for runs with 16 processes or more.
  - Dynamic load balancing was activated.
  - The hubs did not process subproblems.
The \textit{ten instances} have similar behavior, so we present summary results.

<table>
<thead>
<tr>
<th>P</th>
<th>Nodes</th>
<th>Time</th>
<th>Starvation</th>
<th>Ramp-up</th>
<th>Ramp-down</th>
<th>Eff</th>
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<td>8</td>
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<tr>
<td>32</td>
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<td>191.50</td>
<td>0.00</td>
<td>0.01</td>
<td>15.00</td>
<td>0.92</td>
</tr>
</tbody>
</table>

(NOTE: m is million, time is in seconds)

- Serial code could not solve several instance due to lack of memory.
- These indicate reasonable scalability.
- Starvation and ramp-up overhead is negligible.
- Ramp-down overhead has room to decrease.
Library Hierarchy for Optimization

**ALPS** (Abstract Library for Parallel Search)
- is the search-handling layer.
- prioritizes nodes based on quality.

**BiCePS** (Branch, Constrain, and Price Software)
- is the data-handling layer for relaxation-based optimization.
- adds notion of variables and constraints.
- uses an iterative bounding procedure.

**BLIS** (BiCePS Linear Integer Solver)
- is a concretization of BiCePS.
- specific to models with linear constraints and objective function.
BiCePS: Data-intensive Applications

- In applications such as *branch, cut, and price* (BCP), the amount of information needed to describe each search tree node is very large.
- This can make memory an issue and also increase communication overhead.
- We can think of each node as being described by a list of *objects*, i.e., *constraints* and *variables*.
- All objects have a domain and can be treated *symmetrically*.
- These objects can be generated throughout the search process.
- In BCP, the set of objects may not change much from parent to children.
- We can therefore store the description of an entire subtree very compactly using *differencing*. 
BiCePS: Objects

- BiCePS assumes an iterative bounding scheme.
- Each iteration, objects are generated and either stored in pools or used to augment the current relaxation.
- The number of objects can be huge, so duplicate and weak objects can be removed based on their hash keys and their effectiveness.
- Periodically, invalid and ineffective objects are purged.
- Effectively sharing objects is a challenge.
Problem $P$

$$\begin{align*}
\text{min} & \quad c^T x \\
\text{s.t.} & \quad Ax \leq b \\
& \quad x_i \in \mathbb{Z} \quad \forall i \in I
\end{align*}$$

where $(A, b) \in \mathbb{R}^{m \times (n+1)}, c \in \mathbb{R}^n$.

Basic Algorithmic Elements

- Bounding method.
- Branching scheme.
- Object generators.
- Heuristics.
BLIS: Branching scheme

BLIS Branching scheme comprise three components:

- **Object**: has feasible region and can be branched on.
- **Branching Object**:
  - is created from an infeasible object.
  - contains instructions for how to conduct branching.
- **Branching strategy**:
  - specifies how to create a set of candidate branching objects.
  - has the method to compare objects and choose the best one.
BLIS: Constraint generators

BLIS constraint generator:
- provides an interface between BLIS and the algorithms in COIN/Cgl.
- has the ability to specify rules to control generator:
  - where to call: root, leaf?
  - how many to generate?
  - when to activate or disable?
- contains the statistics to guide generating.
BLIS: Heuristics

BLIS primal heuristic:
- Defines the functionality to search for solutions.
- Has the ability to specify rules to control heuristics.
  - where to call: after bounding, at solution?
  - how often to call?
  - when to activate or disable?
- Collects statistics to guide searching.
- Provides a base class for deriving various heuristics.
BLIS: Preliminary Experiments

To benchmark, we compare BLIS with COIN/Cbc.

**Test Bed**
- **Test Machine**: PC, 2.8 GHz Pentium, 2.0G RAM, Linux
- **Test instances**: Selected 33 instances from Lehigh/CORAL and MIPLIB 3, which both solvers can solve in 10 minutes.

**Solver settings**
- **BLIS**
  - Branching strategy: Pseudocost branching.
  - Cuts generators: Gomory, Knapsack, Flow Cover, MIR, Probing, and Clique.
  - Heuristics: Rounding.
- **COIN/Cbc**
  - Branching strategy: Strong branching.
  - Cut generators: Gomory, Knapsack, Flow Cover, MIR, Probing, and Clique.
  - Heuristics: Rounding and Local search.
### BLIS: Computational Results

<table>
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<th>Problem</th>
<th>Row</th>
<th>Column</th>
<th>Nonzero</th>
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<th>Time-CBC</th>
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<td><strong>1642.61</strong></td>
<td><strong>1238.32</strong></td>
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</table>
BLIS: Comparing with Cbc

Performance Profile

CBC
BLIS

0.2
0.4
0.6
0.8
1

0
1
2
4
8
16
32
What’s Available

- ALPS itself is available for download at
  https://projects.coin-or.org/Alps

- There is a user’s manual, but it is slightly out of date.
- There are several sample applications that come with ALPS.
- The base library will build and run on most platforms with the GNU autotools.
- MPI is required for parallel execution.
- BiCePS and BLIS will be publicly available in a week or two.
Case Study: Mixed-Integer Stackelberg Games

- In this research project, we are studying *mixed-integer Stackelberg games*.
- This effort required implementation of a solver for bilevel programs.
- We were able to implement such a solver using ALPS and a variety of other COIN-OR projects.

**COIN-OR Components Used**

- The *Abstract Library for Parallel Search* (ALPS) to perform the branch and bound.
- The *COIN Branch and Cut* (CBC) framework for solving the MILPs.
- The *COIN LP Solver* (CLP) framework for solving the LPs arising in the branch and cut.
- The *Cut Generation Library* (CGL) for generating cutting planes within CBC.
- The *Open Solver Interface* (OSI) for interfacing with CBC and CLP.
Future work

Improve ALPS

- Reduce ramp-up/down time when node processing times are long.
- More effectively adjust parameters dynamically based on problem structure and search progress.

Complete the development of BiCePS and BLIS

- Finish parallel parts of the code.
- Find answers to important research questions:
  - How to share objects?
  - How to efficiently avoid duplicated knowledge?
  - How to deal with locally valid knowledge?
- Add more customization features akin to COIN/BCP:
  - Branch and price.
  - Branch, cut, and price.
- Large-scale testing!