Software Tools for Implementing Branch, Cut, and Price Algorithms

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Agenda

- Overview of COIN-OR
- Overview of branch, cut, and price
- Overview of COIN-OR branch, cut, and price toolbox
  - BCP
  - OSI
  - CGL
  - CLP
  - VOL
- Using the toolbox
  - Getting started
  - Developing an application
- Examples
Overview of COIN-OR

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- ⇒ Overview of COIN-OR ⇐
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- Overview of COIN-OR branch, cut, and price toolbox
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  - OSI
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What is COIN-OR?

- **The COIN-OR Project**
  - A *consortium* of researchers in both industry and academia dedicated to improving the state of computational research in OR.
  - An *initiative* promoting the development and use of interoperable, open-source software for operations research.

- **The COIN-OR Repository**
  - A *library* of interoperable software tools for building optimization codes, as well as a few stand alone packages.
  - A *venue for peer review* of OR software tools.
  - A *development platform* for open source projects, including a CVS repository.
Our Agenda

- **Accelerate the pace of research** in computational OR.
  - Reuse instead of reinvent.
  - Reduce development time and increase robustness.
  - Increase interoperability (standards and interfaces).

- **Provide for software what the open literature provides for theory.**
  - Peer review of software.
  - Free distribution of ideas.
  - Adherence to the principles of good scientific research.

- **Define standards and interfaces** that allow software components to interoperate.

- **Increase synergy** between various development projects.

- **Provide robust, open-source tools for practitioners.**
Open Source Development

- *Open source development* is a coding paradigm in which development is done in a cooperative and distributed fashion.

- Strictly speaking, an open source license must satisfy the requirements of the *Open Source Definition*.

- A license cannot call itself “open source” until it is approved by the *Open Source Initiative*.

- Basic properties of an open source license
  - Access to source code.
  - The right to redistribute.
  - The right to modify.

- The license may require that modifications also be kept open.
Components of the COIN-OR Library

- Branch, cut, price toolbox
  - OSI: Open Solver Interface
  - CGL: Cut Generator Library
  - BCP: Branch, Cut, and Price Library
  - VOL: Volume Algorithm
  - CLP: COIN-OR LP Solver

- Stand-alone components
  - IPOPT: Interior Point Optimization (Nonlinear)
  - DFO: Derivative Free Optimization
  - OTS: Open Tabu Search
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Introduction to Branch, Cut, and Price

• Consider problem $P$:

$$\begin{align*}
\min & \quad c^T x \\
\text{s.t.} & \quad Ax \leq b \\
& \quad x_i \in \mathbb{Z} \quad \forall i \in I
\end{align*}$$

where $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^{m}$, $c \in \mathbb{R}^{n}$.

• Let $P = \text{conv}\{x \in \mathbb{R}^{n} : Ax \leq b, x_i \in \mathbb{Z} \forall i \in I\}$.

• Basic Algorithmic Approach
  
  – Use LP relaxations to produce lower bounds.
  – Branch using hyperplanes.


**Branch, Cut, and Price**

- **Weyl-Minkowski**
  
  - \( \exists \bar{A} \in \mathbb{R}^{\bar{m} \times n}, \bar{b} \in \mathbb{R}^{\bar{m}} \text{ s.t. } \mathcal{P} = \{ x \in \mathbb{R}^n : \bar{A}x \leq \bar{b} \} \)
  
  - We want the solution to \( \min\{ c^T x : \bar{A}x \leq \bar{b} \} \).
  
  - Solving this LP isn’t practical (or necessary).

- **BCP Approach**
  
  - Form LP relaxations using submatrices of \( \bar{A} \).
  
  - The submatrices are defined by sets \( V \subseteq \{1, \ldots, n\} \) and \( C \subseteq \{1, \ldots, \bar{m}\} \).
  
  - Forming/managing these relaxations efficiently is one of the primary challenges of BCP.
The Challenge of BCP

- The efficiency of BCP depends heavily on the size (number of rows and columns) and tightness of the LP relaxations.

- Tradeoff
  - Small LP relaxations ⇒ faster LP solution.
  - Big LP relaxations ⇒ better bounds.

- The goal is to keep relaxations small while not sacrificing bound quality.

- We must be able to easily move constraints and variables in and out of the active set.

- This means dynamic generation and deletion.
An Object-oriented Approach

• The rows/columns of a static LP are called *constraints* and *variables*.

• What do these terms mean in a dynamic context?

• Conceptual Definitions

  – **Constraint**: A mapping that generates coefficients for the *realization* of an inequality for the current set of active variables.
  
  – **Variable**: A mapping that generates coefficients corresponding to a variable for the current set of active constraints.
  
  – **Subproblem**: Defined by a subset of the global set of variables and constraints.

• To construct a subproblem, an initial *core relaxation* is needed.

• From the core, we can build up other relaxations using the mappings.
Generating the Objects

- We will generically call the constraints and variables *objects*.

- We need to define methods for generating these objects.

- For constraints, such a method is a mapping

  \[ g^c(x) : \mathbb{R}^n \rightarrow 2\{1, \ldots, m\} \]

  where \( x \) is a primal solution vector.

- For variables, we have

  \[ g^v(y) : \mathbb{R}^m \rightarrow 2\{1, \ldots, n\} \]

  where \( y \) is a dual solution vector.
Object Representation

- In practice, we may not know the cardinality of the object set.
- We may not easily be able to assign indices to the objects.
- Instead, we must define abstract representations of these objects.
- **Example**: Subtour elimination constraints.
Example: Traveling Salesman Problem

Feasible solutions are those incidence vectors satisfying:

\[
\sum_{j=1}^{n} x_{ij} = 2 \quad \forall i \in V \\
\sum_{i \in S} x_{ij} \geq 2 \quad \forall S \subset V, \ |S| > 1.
\]

- The variables correspond to the edges of a graph (easy to index).
- The number of facets (constraints) is astronomical.
- The core
  - The \( k \) shortest edges adjacent to each node.
  - The degree constraints.
- Generate subtour elimination constraints and other variables dynamically.
Frameworks for BCP

- **Concept**: Provide a *framework* in which the user has only to define constraints, variables, and a core.
  - Branch and bound $\Rightarrow$ core only
  - Branch and cut $\Rightarrow$ core plus constraints
  - Branch and price $\Rightarrow$ core plus variables
  - Branch, cut, and price $\Rightarrow$ the whole caboodle

- **Existing BCP frameworks**
  - SYMPHONY (parallel, C)
  - COIN/BCP (parallel, C++)
  - ABACUS (sequential, C++)

- **Tools Needed**
  - Framework
  - LP Solver
  - Cut Generator
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The COIN-OR Branch, Cut, and Price Toolbox

Branch, cut, price toolbox

- **BCP**: Tool for implementing BCP algorithms
- **OSI**: Tool for interfacing to third-party solvers (particularly LP solvers)
- **CGL**: Tool for generating valid inequalities (within BCP)
- **VOL**: Fast approximate LP solver (with OSI interface)
- **CLP**: COIN-OR LP Solver (with OSI interface)
The COIN/BCP library is divided into modules, of which there are four basic types:

- **Master** Maintains problem instance data, spawns other processes, performs I/O, fault tolerance.
- **Tree Manager** Controls overall execution by tracking growth of the tree and dispatching subproblems to the LP solvers.
- **Node Processor** Perform processing and branching operations.
- **Object Generator** Generate objects.

The division into separate modules makes the code highly configurable and parallelizable.
COIN/BCP Overview: Node Processor

Handling of Constraints

• Cuts are generated by the cut generators and/or by the node processor itself.

• Violated cuts are received and processed by the LP modules.

• Each LP module maintains a small local cut pool.

• A limited number of cuts are added to the LP relaxations each iteration to prevent “saturation.”

• Ineffective (non-core) cuts are aggressively removed.
COIN/BCP Overview: Node Processor

Handling of Variables

- Reduced cost/logical fixing are used to remove (non-core) variables.
- Variable generation may be needed for very large problems.
- Exact generation must take place before fathoming!
- Two-phase algorithm
  - BCP is run to completion on the core variables before generating new ones.
  - Using the upper bound and cuts from the first phase, all variables are priced out in the root node and are then propagated down into the leaves as required.
  - The tree is trimmed by aggregating children back into their parent.
  - Afterwards, each leaf is processed again.
OSI Overview

Uniform interface to LP solvers, including:

- **CLP** (COIN-OR LP Solver)
- **CPLEX** (ILOG)
- **DyLP** (BonsaiG LP Solver)
- **GLPK** (GNU LP Kit)
- **OSL** (IBM)
- **SoPlex** (Konrad-Zuse-Zentrum für Informationstechnik Berlin)
- **Volume** (COIN-OR)
- **XPRESS** (Dash Optimization)
CLP Overview

- Open source (Common Public License).
- Distributed with COIN-OR branch-cut-price toolbox.
- Reasonably robust and fast.
VOL Overview

- Generalized subgradient optimization algorithm.
- Compatible with branch, cut, and price:
  - provides approximate primal and dual solutions,
  - provides a valid lower bound (feasible dual solution), and
  - provides a method for warm starting.
CGL Overview

- Collection of cut generating methods integrated with OSI.
- Intended to provide robust implementations, including computational tricks not usually published.
- Currently includes:
  - Simple rounding cut
  - Gomory cut
  - Knapsack cover cut
  - Rudimentary lift-and-project cut (covering and packing)
  - Odd hole cut
  - Probing cut
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Getting Started with COIN-OR

- The source can be obtained from www.coin-or.org as “tarball” or using CVS.

- Platforms/Requirements
  - Linux, gcc 2.95.3/2.96RH/3.2
  - Windows, Visual C++, CygWin make (untested)
  - Sun Solaris, gcc 2.95.3/3.2 or SunWorkshop C++

- Editing the Makefiles
  - Makefile.location
  - Makefile.<operating system>

- Make the necessary libraries. They'll be installed in ${CoinDir}/lib.
  - Change to appropriate directory and type make.
Developing an Application

- The user API is implemented via a C++ class hierarchy.
- To develop an application, the user must derive the appropriate classes and override the appropriate methods.
- Classes for customizing the behavior of the modules
  - BCP_tm_user
  - BCP_lp_user
  - BCP_cg_user
  - BCP_vg_user
- Classes for defining user objects
  - BCP_cut
  - BCP_var
  - BCP_solution
- Allowing COIN/BCP to create instances of the user classes.
  - The user must derive the class USER_initialize.
  - The function BCP_user_init() returns an instance of the derived initializer class.
Objects in COIN/BCP

• Most application-specific methods are related to handling of objects.

• Since representation is independent of the current LP, the user must define methods to add objects to a given subproblem.

• For parallel execution, the objects need to be packed into (and unpacked from) a buffer.

• Object Types
  
  – **Core objects** are objects that are active in every subproblem (BCP_xxx_core).
  
  – **Indexed objects** are extra objects that can be uniquely identified by an index (such as the edges of a graph) (BCP_xxx_indexed).
  
  – **Algorithmic objects** are extra objects that have an abstract representation (BCP_xxx_algo).
**Forming the LP Relaxations in COIN/BCP**

The current LP relaxation looks like this:

```
<table>
<thead>
<tr>
<th>core vars</th>
<th>extra vars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>core cuts</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>extra cuts</th>
<th></th>
</tr>
</thead>
</table>

Reason for this split: efficiency.
COIN/BCP Methods: Initialization

Solver Initialization (Tree Manager)

- create_root()
- initialize_core()
- xx_init()
- pack_module_data()
- BCP_user_init()
**COIN/BCP Methods: Steady State**

**Tree Manager**
- `unpack_feasible_solution()`
- `init_new_phase()`
- `compare_tree_nodes()`

**Cut Generator**
- `unpack_module_data()`
- `unpack_primal_solution()`
- `generate_cuts()`

**LP Solver**
- `unpack_module_data()`
- `initialize_search_tree_node()`

**Variable Generator**
- `unpack_module_data()`
- `unpack_dual_solution()`
- `generate_vars()`

See the solver loop figure.
COIN/BCP Methods: Node Processing Loop

- **modify_LP_parameters()**
  - **test_feasibility()**
  - **generate_heuristic_solution()**
  - **pack_{primal/dual}_solution()**
  - **unpack_{var/cut}_algo()**
  - **generate_{vars/cuts}_in_lp()**
  - **compare_{vars/cuts}()**
  - **vars_to_cols() / cuts_to_rows()**
  - **logical_fixing()**
  - **select_branching_candidates()**
  - **compare_branching_candidates()**
  - **set_actions_for_children()**
  - **purge_slack_pool()**

- **Possible fathoming**
- **pack_feasible_solution()**
- **Send primal and dual solutions to CG and VG**
- **Generating and comparing cuts and variables**
- **Strong branching functions**
## COIN/BCP Methods: Node Processing Loop

<table>
<thead>
<tr>
<th>Var generation</th>
<th>B&amp;B</th>
<th>Cut generation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>initialize_new_search_tree_node</td>
<td></td>
</tr>
<tr>
<td></td>
<td>modify_lp_parameters</td>
<td>pack_primal_solution</td>
</tr>
<tr>
<td></td>
<td>solve the LP relaxation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>display_lp_solution</td>
<td></td>
</tr>
<tr>
<td></td>
<td>test_feasibility</td>
<td></td>
</tr>
<tr>
<td></td>
<td>reduced cost fixing</td>
<td></td>
</tr>
<tr>
<td></td>
<td>logical_fixing</td>
<td></td>
</tr>
<tr>
<td></td>
<td>generate_heuristic_solution</td>
<td></td>
</tr>
<tr>
<td></td>
<td>select_branching_candidates</td>
<td></td>
</tr>
<tr>
<td></td>
<td>If branching is decided on</td>
<td></td>
</tr>
<tr>
<td></td>
<td>compare_branching_candidates</td>
<td></td>
</tr>
<tr>
<td></td>
<td>set_actions_for_children</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Otherwise</td>
<td></td>
</tr>
<tr>
<td></td>
<td>vars are added</td>
<td></td>
</tr>
<tr>
<td></td>
<td>generate_cuts_in_lp</td>
<td></td>
</tr>
<tr>
<td></td>
<td>cuts are added</td>
<td></td>
</tr>
<tr>
<td>compute_lower_bound</td>
<td></td>
<td></td>
</tr>
<tr>
<td>pack_dual_solution</td>
<td></td>
<td></td>
</tr>
<tr>
<td>generate_vars_in_lp</td>
<td></td>
<td></td>
</tr>
<tr>
<td>vars are added</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Misc methods in the TM (no particular order)**

- **unpack_feasible_solution.** Default: Unpack a generic solution. Write if own solution type used.
- **display_feasible_solution.** Default: display a generic solution. Write if own solution type used.
- **(un)pack_warmstart.** Default: handles all warmstarts defined in Osi. Unlikely to write.
- **(un)pack_var_algo.** Write if algorithmic vars are generated.
- **(un)pack_cut_algo.** Write if algorithmic cuts are generated.
- **compare_tree_nodes.** Defaults: Breadth/Depth/Best First Search. Unlikely to write (only if none of the defaults are satisfactory).
- **init_new_phase.** Unlikely to write (only if multiphase is used).
Misc methods in the LP (no particular order)

- **pack_feasible_solution.** Invoked whenever a feasible soln is found. Default: Pack a generic solution. Write if own solution type used.

- **(un)pack_warmstart.** Default: handles all warmstarts defined in Osi. Unlikely to write.

- **(un)pack_var_algo.** Write if algorithmic vars are generated.

- **(un)pack_cut_algo.** Write if algorithmic cuts are generated.

- **cuts_to_rows and vars_to_cols.** Write if any sort of cut/var generation is done.

- **compare_cuts and compare_vars.** Write if any sort of cut/var generation is done.
Parameters and using the finished code

- Create a parameter file
- Run your code with the parameter file name as an argument (command line switches will be added).
- BCP_ for BCP’s parameters
- Defined and documented in BCP_tm_par, BCP_lp_par, etc.
- Helper class for creating your parameters.
- Output controlled by verbosity parameters.
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Example: Uncapacitated Facility Location

User classes and methods

- **UFL_init**
  - tm_init()
  - lp_init()

- **UFL_lp**
  - unpack_module_data()
  - pack_cut_algo()
  - unpack_cut_algo()
  - generate_cuts_in_lp()
  - cuts_to_rows()

- **UFL_tm**
  - read_data()
  - initialize_core()
  - pack_module_data()

- **UFL_cut**
Example: Generic MIP solver

• Implement branch and cut to solve an IP by
  – reading in from an MPS file,
  – designating all vars as core vars,
  – selecting some of the constraints as core constraints
  – making the rest extra (indexed) constraints, and
  – interfacing to CGL to generate cuts.

• Classes and methods are similar to the previous example.
Final advice

Use the source, Luke...

...and feel free to ask questions either by email or on the discussion list.