Computational Experience with Parallel Integer Programming using the CHiPPS Framework

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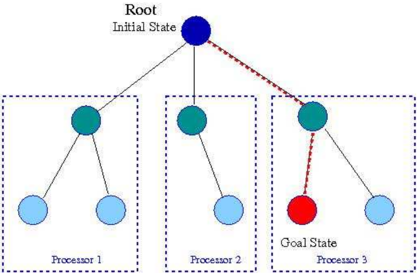
Outline

Quick Introduction to CHiPPS

- CHiPPS stands for COIN-OR High Performance Parallel Search.
- CHiPPS is a set of C++ class libraries for implementing tree search algorithms for both sequential and parallel environments.
- The generic algorithm appears very easy to parallelize.



- The appearance is deceiving, as the search graph is not generally known a priori and naïve parallelization strategies are not generally effective.

CHiPPS Components

ALPS (Abstract Library for Parallel Search)

- is the search-handling layer (parallel and sequential).
- provides various search strategies based on node priorities.

BiCePS (Branch, Constrain, and Price Software)

- is the data-handling layer for relaxation-based optimization.
- adds notion of variables and constraints.
- assumes iterative bounding process.

BLIS (BiCePS Linear Integer Solver)

- is a concretization of BiCePS.
- specific to models with linear constraints and objective function.

Previous Work

Previous tree search codes:

- Game tree search: ZUGZWANG and APHID
- Constraint programming: ECLiPSe, etc.
- Optimization:
  - Commercial: CPLEX, Lindo, Mosek, SAS/OR, Xpress, Gurobi, etc.
  - Serial: ABACUS, bc-opt, COIN/CBC, GLPK, MINTO, SCIP, etc.
  - Parallel: COIN/BCP, FATCOP, PARINO, PICO, SYMPHONY, CPLEX, Gurobi, XPress, etc.
- Parallelization of tree search have become an increasingly important and mainstream undertaking.
- *All tree search codes will have to be parallelized to stay competitive*.

ALPS: Design Goals

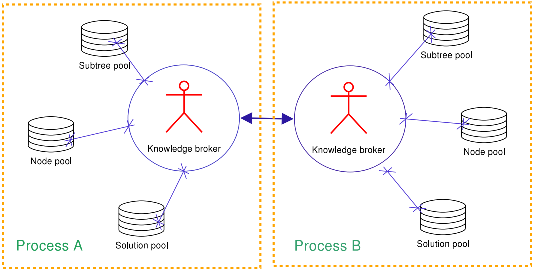
- Intuitive object-oriented class structure.
  - `AlpsModel`
  - `AlpsTreeNode`
  - `AlpsNodeDesc`
  - `AlpsSolution`
  - `AlpsParameterSet`
- Minimal algorithmic assumptions in the base class.
  - Support for a wide range of problem classes and algorithms.
  - Support for constraint programming.
- Easy for user to develop a custom solver.
- Design for *parallel scalability*, but operate effective in a sequential environment.
- Explicit support for *memory compression* techniques (packing/differencing) important for implementing optimization algorithms.

ALPS: Overview of Features

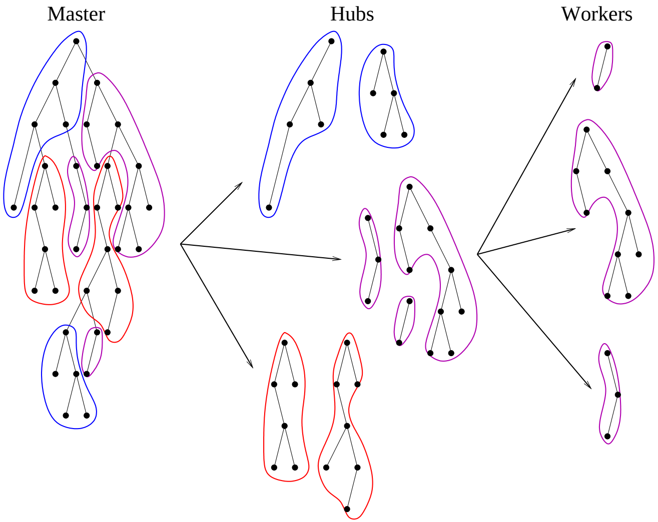
- The design is based on a very general concept of *knowledge*.
- Knowledge is shared asynchronously through *pools* and *brokers*.
- Management overhead is reduced with the *master-hub-worker* paradigm.
- Overhead is decreased using dynamic task granularity.
- Two static load balancing techniques are used.
- Three dynamic load balancing techniques are employed.
- Uses asynchronous messaging to the highest extent possible.
- A scheduler on each process manages tasks like
  - node processing,
  - load balancing,
  - update search states, and
  - termination checking, etc.

Knowledge Sharing

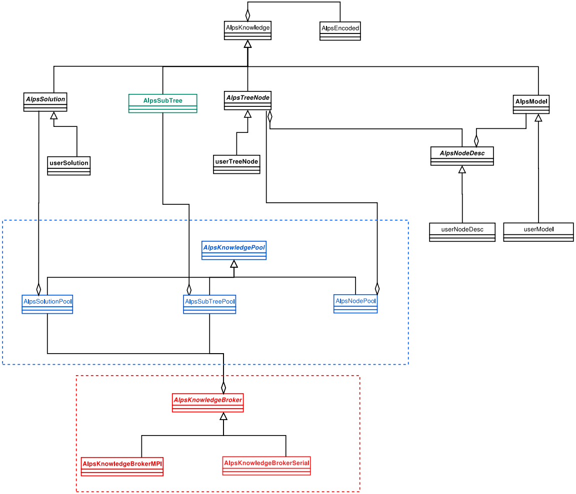
- All knowledge to be shared is derived from a single base class and has an associated *encoded form*.
- Encoded form is used for identification, storage, and communication.
- Knowledge is maintained by one or more *knowledge pools*.
- The knowledge pools communicate through *knowledge brokers*.



Master-Hub-Worker Paradigm



Alps Class Hierarchy



Using ALPS: A Knapsack Solver

The formulation of the binary knapsack problem is

$$\max\{\sum_{i=1}^{m} p_i x_i : \sum_{i=1}^{m} s_i x_i \leq c, x_i \in \{0,1\}, i = 1, 2, \ldots, m\}, \tag{1}$$

We derive the following classes:

- `KnapModel` (from `AlpsModel`) : Stores the data used to describe the knapsack problem and implements `readInstance()`
- `KnapTreeNode` (from `AlpsTreeNode`) : Implements `process()` (bound) and `branch()`
- `KnapNodeDesc` (from `AlpsNodeDesc`) : Stores information about which variables/items have been fixed by branching and which are still free.
- `KnapSolution` (from `AlpsSolution`) Stores a solution (which items are in the knapsack).

BiCePS: Support for Relaxation-based Optimization

- Adds notion of *modeling objects* (variables and constraints).
- Models are built from sets of such objects.
- Bounding is an iterative process that produces new objects.
- A differencing scheme is used to store the difference between the descriptions of a child node and its parent.

```
struct BcpsObjectListMod
{
    int  numRemove;
    int* posRemove;
    int  numAdd;
    BcpsObject **objects;
    BcpsFieldListMod<double> lbHard;
    BcpsFieldListMod<double> ubHard;
    BcpsFieldListMod<double> lbSoft;
    BcpsFieldListMod<double> ubSoft;
};
```

```
template<class T>
struct BcpsFieldListMod
{
    bool relative;
    int  numModify;
    int  *posModify;
    T    *entries;
};
```

BLIS: A Generic Solver for MILP

MILP

$$\min \quad c^T x \tag{2}$$
$$s.t. \quad Ax \leq b \tag{3}$$
$$x_i \in \mathbb{Z} \quad \forall\, i \in I \tag{4}$$

where $(A, b) \in \mathbb{R}^{m \times (n+1)}, c \in \mathbb{R}^n$.

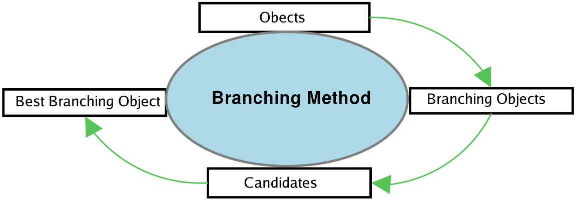
Basic Algorithmic Components

- Bounding method.
- Branching scheme.
- Object generators.
- Heuristics.

BLIS: Branching Scheme

BLIS Branching scheme comprise three components:

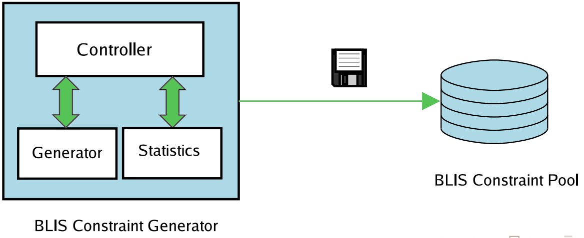
- Object: has feasible region and can be branched on.
- Branching Object:
  - is created from objects that do not lie in they feasible regions or objects that will be beneficial to the search if branching on them.
  - contains instructions for how to conduct branching.
- Branching method:
  - specifies how to create a set of candidate branching objects.
  - has the method to compare objects and choose the best one.



BLIS: Constraint Generators

BLIS constraint generator:

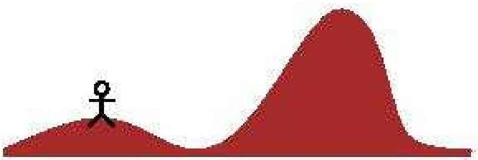
- provides an interface between BLIS and the algorithms in COIN/Cgl.
- provides a base class for deriving specific generators.
- has the ability to specify rules to control generator:
  - where to call: root, leaf?
  - how many to generate?
  - when to activate or disable?
- contains the statistics to guide generating.



BLIS: Heuristics

BLIS primal heuristic:

- defines the functionality to search for solutions.
- has the ability to specify rules to control heuristics.
  - where to call: before root, after bounding, at solution?
  - how often to call?
  - when to activate or disable?
- collects statistics to guide the heuristic.
- provides a base class for deriving specific heuristics.



Traditional Measures of Performance

- *Parallel System*: Parallel algorithm + parallel architecture.
- *Scalability*: How well a parallel system takes advantage of increased computing resources.

Terms

- Sequential runtime: T_s
- Parallel runtime: T_p
- Parallel overhead: $T_o = NT_p - T_s$
- Speedup: $S = T_s/T_p$
- Efficiency: $E = S/N$

- Standard analysis considers change in efficiency on a fixed test set as number of processors is increased.
- *Isoefficiency analysis* considers the increase in problem size to maintain a fixed efficiency as number of processors is increased.

Parallel Overhead

- The amount of *parallel overhead* determines the scalability.

Major Components of Parallel Overhead in Tree Search

- Communication Overhead (cost of sharing knowledge)
- Idle Time
  - Handshaking/Synchronization (cost of sharing knowledge)
  - Task Starvation (cost of *not* sharing knowledge)
  - Ramp Up Time
  - Ramp Down Time
- Performance of Redundant Work (cost of *not* sharing knowledge)

- Knowledge sharing is the main driver of efficiency.
- This breakdown highlights the tradeoff between centralized and decentralized knowledge storage and decision-making.

Challenges in Measuring Performance

- Traditional measures are not appropriate.
  - The interesting problems are the ones that take too long to solve sequentially.
  - Need to account for the possibility of failure.
- It's exceedingly difficult to construct a test set
  - Scalability varies substantially by instance.
  - Hard to know what test problems are appropriate.
  - A fixed test set will almost surely fail to measure what you want.
- Results are highly dependent on architecture
  - Difficult to make comparisons
  - Difficult to tune parameters
- Hard to get enough time on large-scale platforms for tuning and testing.
- Results are non-deterministic!
  - Determinism can be a false sense of security.
  - Lack of determinism requires more extensive testing.

Computational Results: Platforms

Clemson Cluster

Machine:	Beowulf cluster with 52 nodes
Node:	dual core PPC, speed 1654 MHz
Memory:	4G RAM each node
Operating System:	Linux
Message Passing:	MPICH

SDSC Blue Gene System

Machine:	IBM Blue Gene with 3,072 compute nodes
Node:	dual processor, speed 700 MHz
Memory:	512 MB RAM each node
Operating System:	Linux
Message Passing:	MPICH

KNAP Scalability for Moderately Difficult Instances

- Tested the moderately difficult instances at Clemson.
- The default algorithm was used except that
  - the static load balancing scheme was the two-level root initialization,
  - the number of nodes generated by the master was 3000, and
  - the size of a unit work was 300 nodes.

P	Node	Ramp-up	Idle	Ramp-down	Wallclock	Eff
4	193057493	0.28%	0.02%	0.01%	586.90	1.00
8	192831731	0.58%	0.08%	0.09%	245.42	1.20
16	192255612	1.20%	0.26%	0.37%	113.43	1.29
32	191967386	2.34%	0.71%	1.47%	56.39	1.30
64	190343944	4.37%	2.27%	5.49%	30.44	1.21

- Super-linear speedup observed.
- Ramp-up, ramp-down, and idle time overhead remains low.

KNAP Scalability for Very Difficult Instances

- Tested difficult instances on SDSC Blue Gene,
- The default algorithm was used except that
  - the static load balancing scheme was the two-level root initialization,
  - nodes generated by the master varies from 10K to 30K.
  - nodes generated by hub varies from 10K to 20K.
  - the size of a unit of work was 3K nodes; and
  - multiple hubs were used.

P	Node	Ramp-up	Idle	Ramp-down	Wallclock	Eff
64	14733745123	0.69%	4.78%	2.65%	6296.49	1.00
128	14776745744	1.37%	6.57%	5.26%	3290.56	0.95
256	14039728320	2.50%	7.14%	9.97%	1672.85	0.94
512	13533948496	7.38%	4.30%	14.83%	877.54	0.90
1024	13596979694	8.33%	3.41%	16.14%	469.78	0.84
2048	14045428590	9.59%	3.54%	22.00%	256.22	0.77

- Note the inevitable increase in ramp-up and ramp-down.
- KNAP scales well even when using several thousand processors.

Does Differencing Make a Difference?

A Simple Test

- 38 MILP instances from Lehigh/CORAL and MIPLIB3.
- Solved to optimality by using BLIS in 10 minutes.
- PC, 2.8 GHz Pentium, 2G RAM, Linux, COIN/Clp.

	No	Yes	Geometric
Total Time	2016 seconds	1907 seconds	1.0
Total Peak Memory	1412 MB	286 MB	4.3

Problem	Time(No)	Memory(No)	Time(Yes)	Memory(Yes)
dcmulti	4.20 s	17.4 MB	4.19 s	1.4 MB
dsbmip	33.28 s	34.1 MB	33.16 s	2.4 MB
egout	0.18 s	0.3 MB	0.18 s	0.2 MB
enigma	6.41 s	13.1 MB	6.16 s	2.3 MB
fiber	6.60 s	17.6 MB	6.56 s	2.1 MB
...	...	...	...	...

BLIS Scalability for Moderately Difficult Instances

- Selected 18 MILP instances from Lehigh/CORAL, MIPLIB 3.0, MIPLIB 2003, BCOL, and markshare.
- Tested on the Clemson cluster.

Instance	Nodes	Ramp -up	Idle	Ramp -down	Comm Overhead	Wallclock	Eff
1 P Per Node	11809956	– –	– –	– –	– –	33820.53 0.00286	1.00
4P Per Node	11069710	0.03% 0.03%	4.62% 4.66%	0.02% 0.00%	16.33% 16.34%	10698.69 0.00386	0.79
8P Per Node	11547210	0.11% 0.10%	4.53% 4.52%	0.41% 0.53%	16.95% 16.95%	5428.47 0.00376	0.78
16P Per Node	12082266	0.33% 0.27%	5.61% 5.66%	1.60% 1.62%	17.46% 17.45%	2803.84 0.00371	0.75
32P Per Node	12411902	1.15% 1.22%	8.69% 8.78%	2.95% 2.93%	21.21% 21.07%	1591.22 0.00410	0.66
64P Per Node	14616292	1.33% 1.38%	11.40% 11.46%	6.70% 6.72%	34.57% 34.44%	1155.31 0.00506	0.46

Impact of Problem Properties

- Instance `input150_1` is a knapsack instance. When using 128 processors, BLIS achieved super-linear speedup mainly to the decrease of the tree size
- Instance `fc_30_50_2` is a fixed-charge network flow instance. It exhibits very significant increases in the size of its search tree.
- Instance `pk1` is a small integer program with 86 variables and 45 constraints. It is relatively easy to solve.

Instance	P	Node	Ramp-up	Idle	Ramp-down	Wallclock	Eff
input150_1	64	75723835	0.44%	3.38%	1.45%	1257.82	1.00
	128	64257131	1.18%	6.90%	2.88%	559.80	1.12
	256	84342537	1.62%	5.53%	7.02%	380.95	0.83
	512	71779511	3.81%	10.26%	10.57%	179.48	0.88
fc_30_50_2	64	3494056	0.15%	31.46%	9.18%	564.20	1.00
	128	3733703	0.22%	33.25%	21.71%	399.60	0.71
	256	6523893	0.23%	29.99%	28.99%	390.12	0.36
	512	13358819	0.27%	23.54%	29.00%	337.85	0.21
pk1	64	2329865	3.97%	12.00%	5.86%	103.55	1.00
	128	2336213	11.66%	12.38%	10.47%	61.31	0.84
	256	2605461	11.55%	13.93%	20.19%	41.04	0.63
	512	3805593	19.14%	9.07%	26.71%	36.43	0.36

BLIS Scalability for Very Difficult Instances

- Tests on Clemson's palmetto cluster (60 on the Top 500 list, 11/2008, Linux, MPICH, 8-core 2.33GHz Xeon/Opteron mix, 12-16GB RAM).
- Tests use one processor per node.

Raw Computational Results

Name	256	128	64	1
mcf2	926	1373	2091	43059
neos-1126860	2184	1830	2540	39856
neos-1122047	1676	1125	1532	NS
neos-1413153	4230	3500	2990	20980
neos-1456979		78.06%	NS	NS
neos-1461051	396	1082	536	NS
neos-1599274		1500	8108	9075
neos-548047		137.29%	376.48%	482%
neos-570431	1034	1255	1308	21873
neos-611838	712	956	886	8005
neos-612143	565	1716	1315	4837
neos-693347		1.28%	1.70%	NS
neos-912015	538	438	275	10674
neos-933364		6.67%	6.79%	11.80%
neos-933815		6.54%	8.77%	32.85%
neos-934184		6.67%	6.76%	9.15%
neos18		30.78%	30.78%	79344

Speedups

Name	256	128	64
mcf2	46.5	31.36	20.59
neos-1126860	18.25	21.78	15.69
neos-1413153	4.96	5.99	7.02
neos-1599274		6.05	1.12
neos-570431	21.15	17.43	16.72
neos-611838	11.24	8.37	9.03
neos-612143	8.56	2.82	3.68
neos-912015	19.84	24.37	38.81

Efficiency

Name	256	128	64
mcf2	0.18	0.25	0.32
neos-1126860	0.07	0.17	0.25
neos-1413153	0.02	0.05	0.11
neos-1599274		0.05	0.02
neos-570431	0.08	0.14	0.26
neos-611838	0.04	0.07	0.14
neos-612143	0.03	0.02	0.06
neos-912015	0.08	0.19	0.61

Shameless Promotion

In October, 2007, the VRP/TSP solver won the Open Contest of Parallel Programming at the 19th International Symposium on Computer Architecture and High Performance Computing.

ALPS

- Our methods implemented in ALPS seem effective in improving scalability.
- The framework is useful for implementing serial or parallel tree search applications.
- The KNAP application achieves very good scalability.
- There is still much room for improvement
  - load balancing,
  - fault tolerance,
  - hybrid architectures,
  - grid enable.

BLIS

- The performance of BLIS in serial mode is favorable when compared to state of the art non-commercial solvers.
- The scalability for solving generic MILPs is highly dependent on properties of individual instances.
- Based on BLIS, applications like VRP/TSP can be implemented in a straightforward way.
- Much work is still needed
  - Callable library API
  - Support for column generation
  - Enhanced heuristics
  - Additional capabilities

Obtaining CHiPPS

The CHiPPS framework is available for download at

`https://projects.coin-or.org/CHiPPS`

Thank You!

Questions?