# Implementing Custom Applications with CHiPPS

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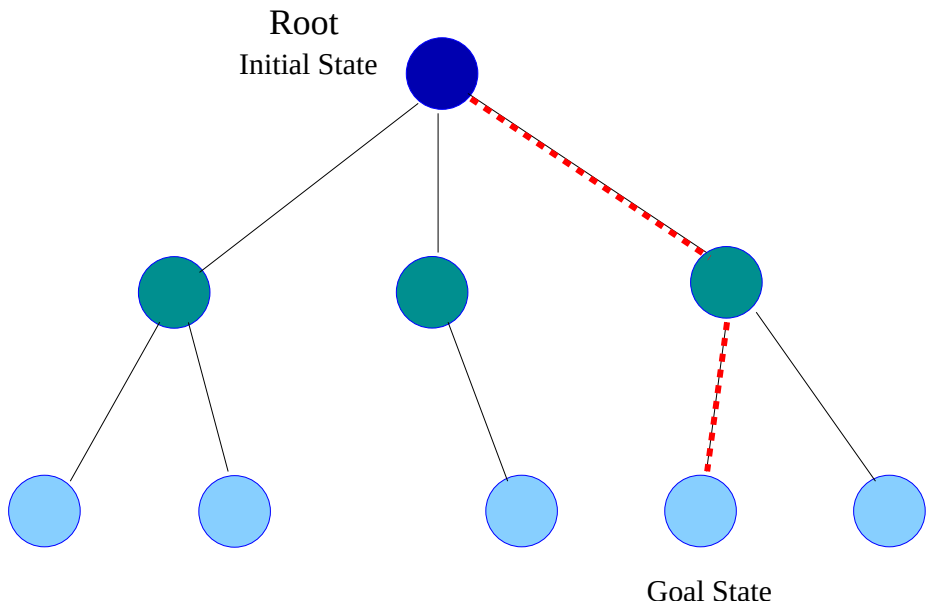
# Outline

# Tree Search Algorithms

- Tree search algorithms systematically search the nodes of an acyclic graph for certain *goal nodes*.




- Tree search algorithms have been applied in many areas such as
  - Constraint satisfaction,
  - Game search,
  - Artificial intelligence, and
  - Mathematical programming.

# Elements of Tree Search Algorithms

- A generic tree search algorithm consists of the following elements:

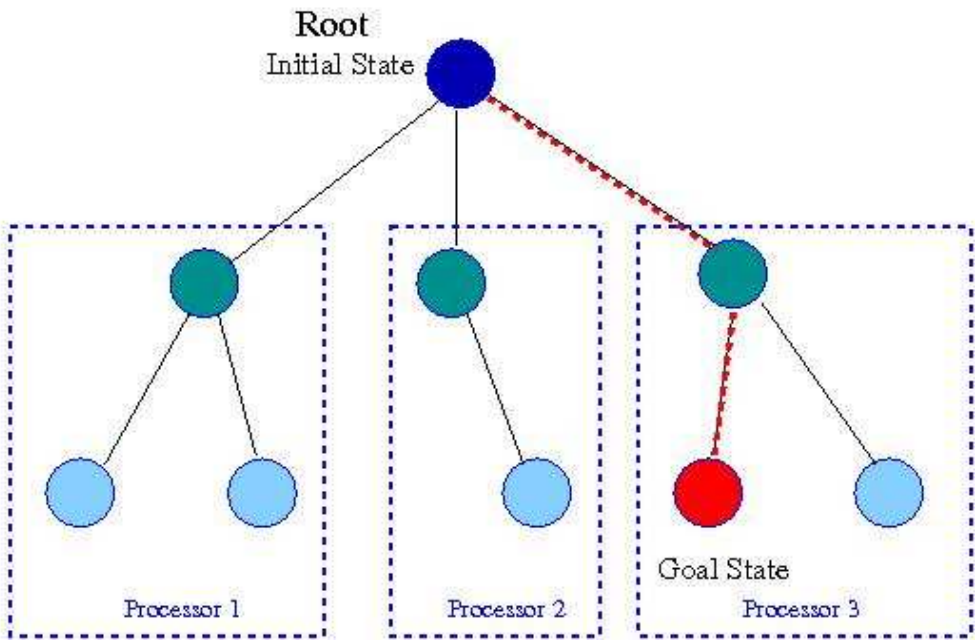
**Generic Tree Search Algorithm**

- Processing method: Is this a goal node?
- Fathoming rule: Can node can be fathomed?
- Branching method: What are the successors of this node?
- Search strategy: What should we work on next?

- The algorithm consists of choosing a candidate node, processing it, and either fathoming or branching.
- During the course of the search, various information (*knowledge*) is generated and can be used to guide the search.

# Parallelizing Tree Search Algorithms

- In general, the search tree can be very large.
- Fortunately, the generic algorithm appears very easy to parallelize.




- The appearance is deceiving, as the search graph is not generally known a priori and naïve parallelization strategies are not generally effective.

# Previous Work

Previous tree search codes:

- Game tree search: ZUGZWANG and APHID
- Constraint programming: ECLiPSe, etc.
- Optimization:
  - Commercial: CPLEX, Lindo, Mosek, SAS/OR, Xpress, etc.
  - Serial: ABACUS, bc-opt, COIN/CBC, GLPK, MINTO, SCIP, etc.
  - Parallel: COIN/BCP, FATCOP, PARINO, PICO, SYMPHONY, Gurobi, XPress, etc.
- Parallelization of tree search have become an increasingly important and mainstream undertaking.
- *All tree search codes will have to be parallelized to stay competitive*.

# Quick Introduction to CHiPPS

- CHiPPS stands for COIN-OR High Performance Parallel Search.
- CHiPPS is a set of C++ class libraries for implementing tree search algorithms for both sequential and parallel environments.

## CHiPPS Components (Current)

ALPS (Abstract Library for Parallel Search)

- is the search-handling layer (parallel and sequential).
- provides various search strategies based on node priorities.

BiCePS (Branch, Constrain, and Price Software)

- is the data-handling layer for relaxation-based optimization.
- adds notion of variables and constraints.
- assumes iterative bounding process.

BLIS (BiCePS Linear Integer Solver)

- is a concretization of BiCePS.
- specific to models with linear constraints and objective function.

# ALPS: Design Goals

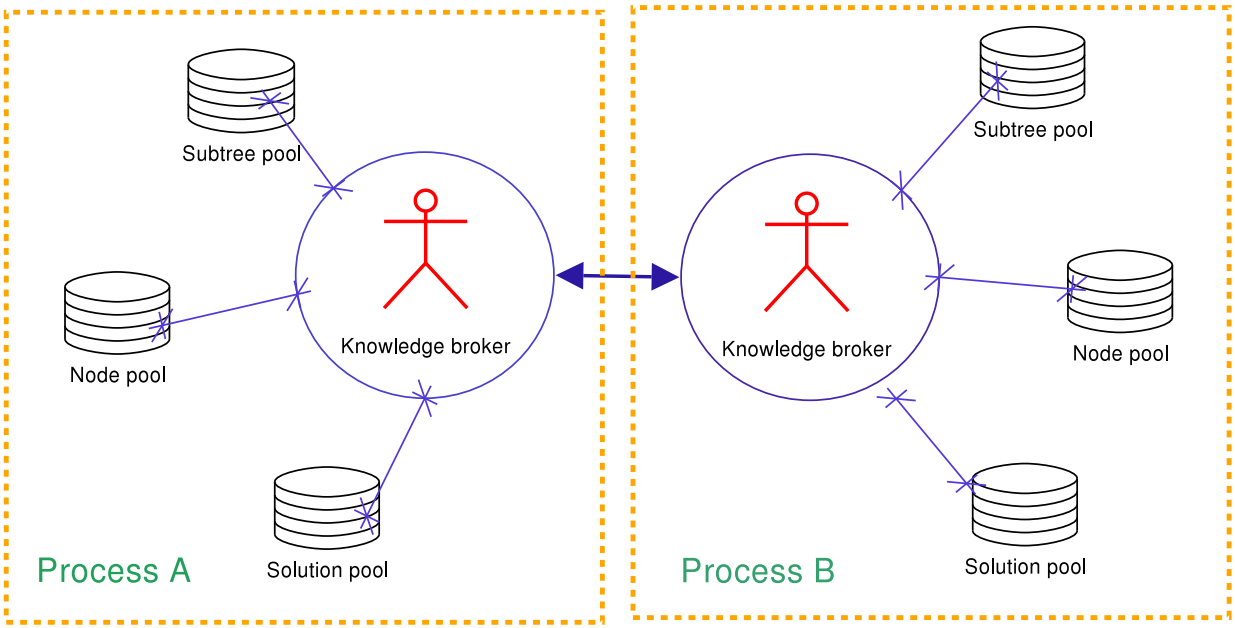
- Intuitive object-oriented class structure.
  - `AlpsModel`
  - `AlpsTreeNode`
  - `AlpsNodeDesc`
  - `AlpsSolution`
  - `AlpsParameterSet`
- Minimal algorithmic assumptions in the base class.
  - Support for a wide range of problem classes and algorithms.
  - Support for constraint programming.
- Easy for user to develop a custom solver.
- Design for *parallel scalability*, but operate effective in a sequential environment.
- Explicit support for *memory compression* techniques (packing/differencing) important for implementing optimization algorithms.

# ALPS: Overview of Features

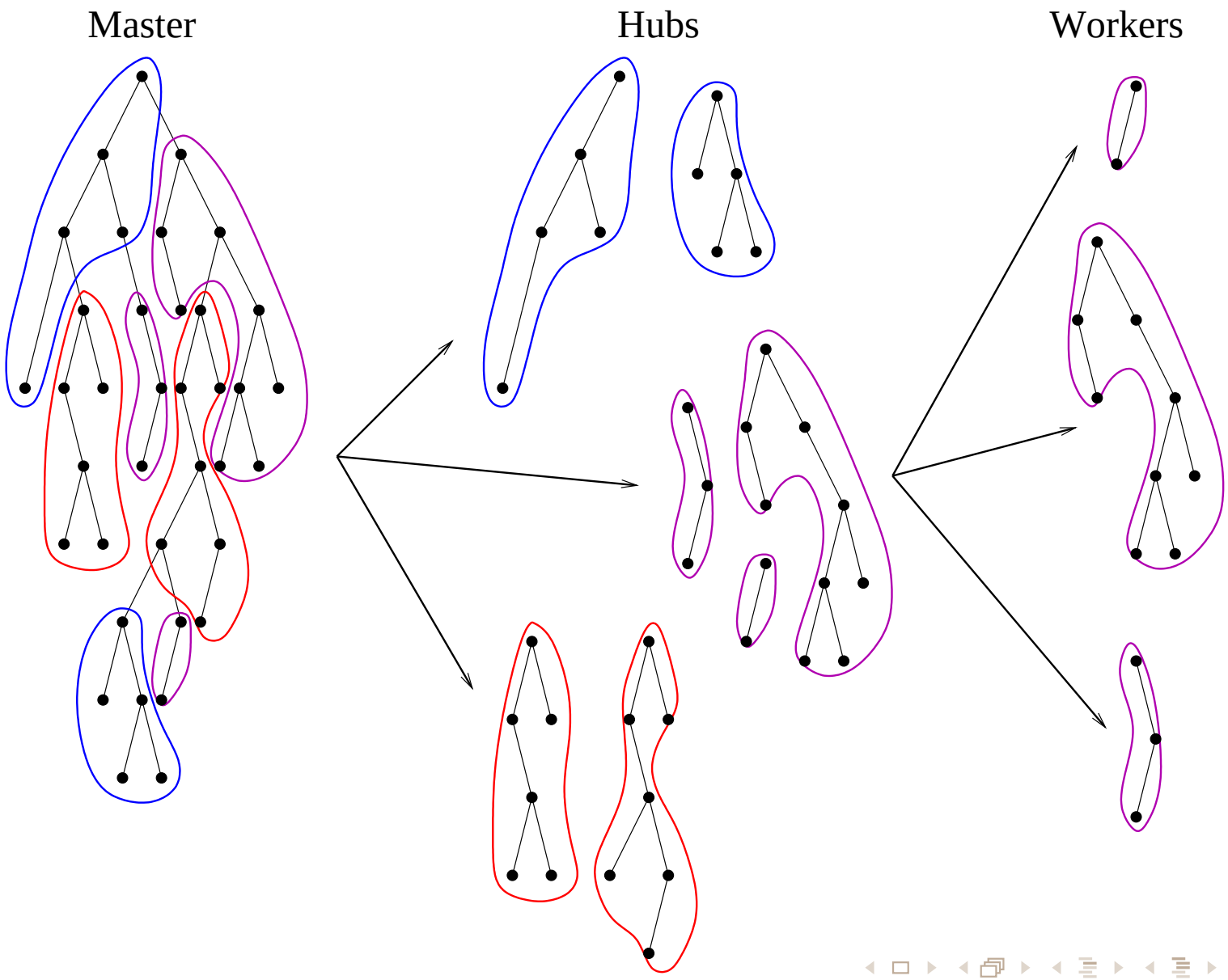
- The design is based on a very general concept of *knowledge*.
- Knowledge is shared asynchronously through *pools* and *brokers*.
- Management overhead is reduced with the *master-hub-worker* paradigm.
- Overhead is decreased using dynamic task granularity.
- Two static load balancing techniques are used.
- Three dynamic load balancing techniques are employed.
- Uses asynchronous messaging to the highest extent possible.
- A scheduler on each process manages tasks like
  - node processing,
  - load balancing,
  - update search states, and
  - termination checking, etc.

# ALPS: Knowledge Sharing

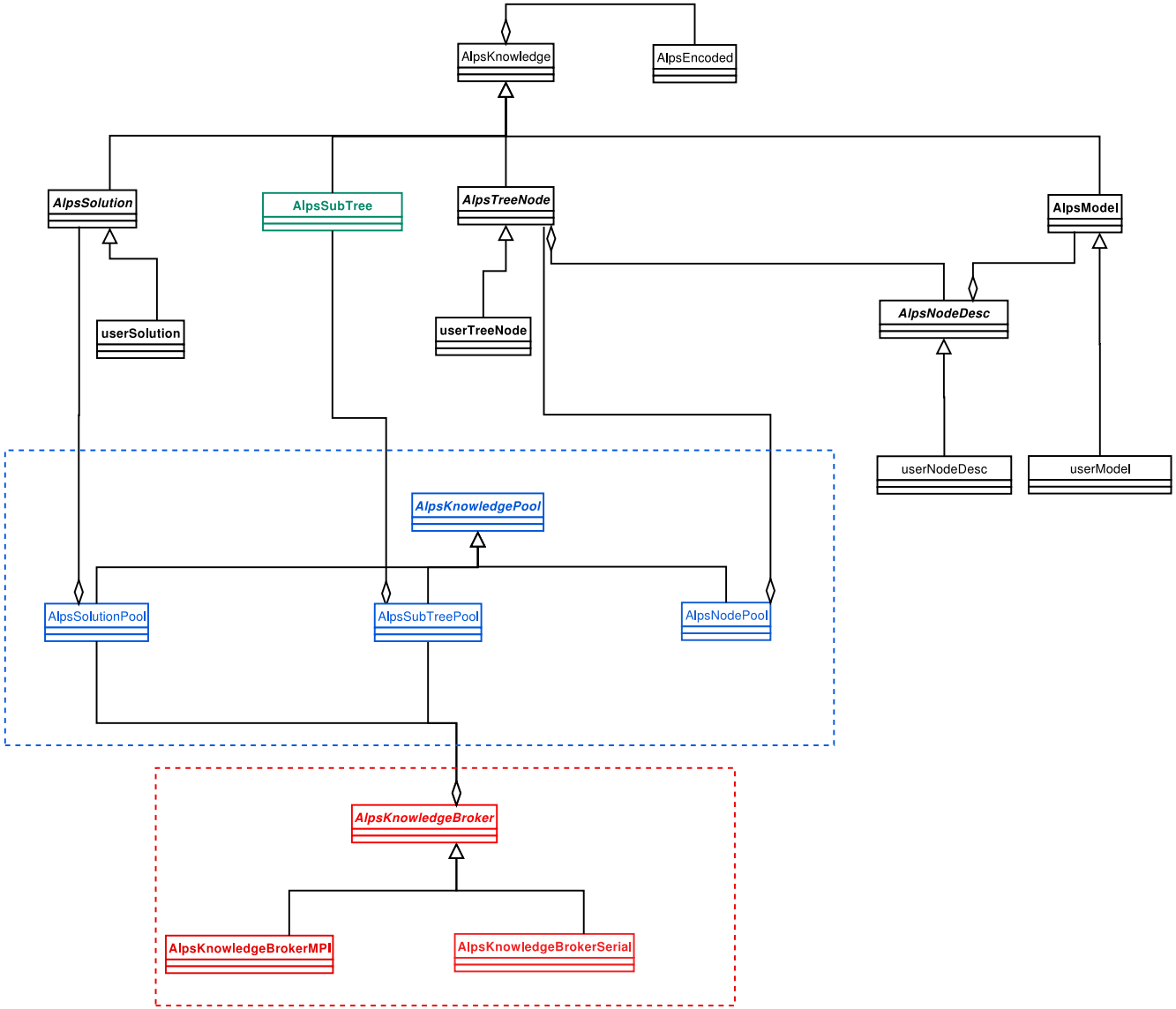
- All knowledge to be shared is derived from a single base class and has an associated *encoded form*.
- Encoded form is used for identification, storage, and communication.
- Knowledge is maintained by one or more *knowledge pools*.
- The knowledge pools communicate through *knowledge brokers*.



# ALPS: Master-Hub-Worker Paradigm



# ALPS: Class Hierarchy



# BiCePS: Support for Relaxation-based Optimization

- Adds notion of *modeling objects* (variables and constraints).
- Models are built from sets of such objects.
- Bounding is an iterative process that produces new objects.
- A differencing scheme is used to store the difference between the descriptions of a child node and its parent.

```
struct BcpsObjectListMod
{
    int  numRemove;
    int* posRemove;
    int  numAdd;
    BcpsObject **objects;
    BcpsFieldListMod<double> lbHard;
    BcpsFieldListMod<double> ubHard;
    BcpsFieldListMod<double> lbSoft;
    BcpsFieldListMod<double> ubSoft;
};
```

```
template<class T>
struct BcpsFieldListMod
{
    bool relative;
    int  numModify;
    int  *posModify;
    T    *entries;
};
```

# BLIS: A Generic Solver for MILP

## MILP

$$\begin{aligned} min \quad & c^T x && (1)\\ s.t. \quad & Ax \leq b && (2)\\ & x_i \in \mathbb{Z} \quad \forall\, i \in I && (3) \end{aligned}$$

where $(A, b) \in \mathbb{R}^{m \times (n+1)}$, $c \in \mathbb{R}^n$.

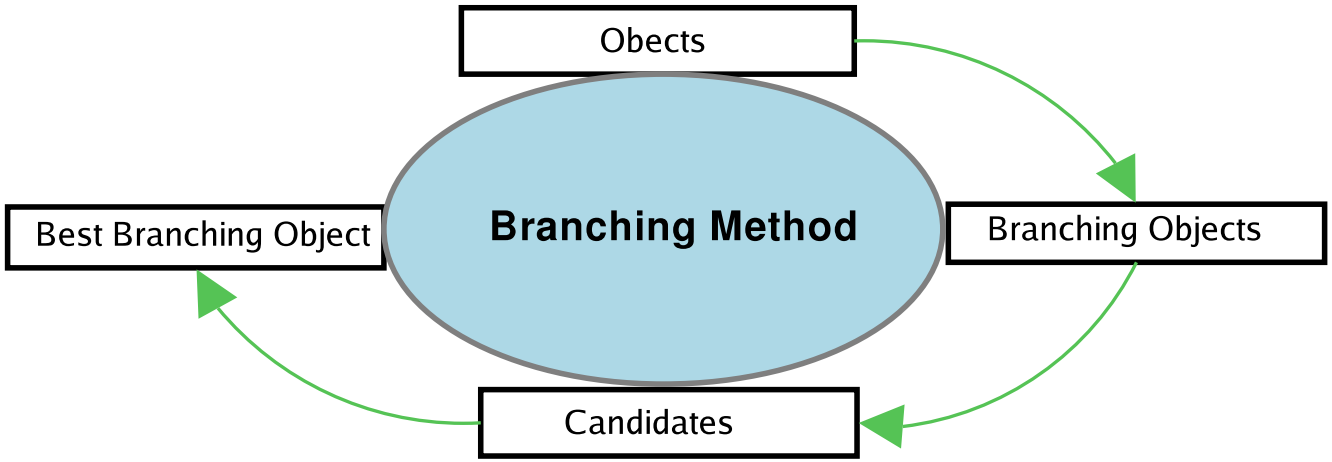
## Basic Algorithmic Components

- Bounding method.
- Branching scheme.
- Object generators.
- Heuristics.

# BLIS: Branching Scheme

BLIS Branching scheme comprise three components:

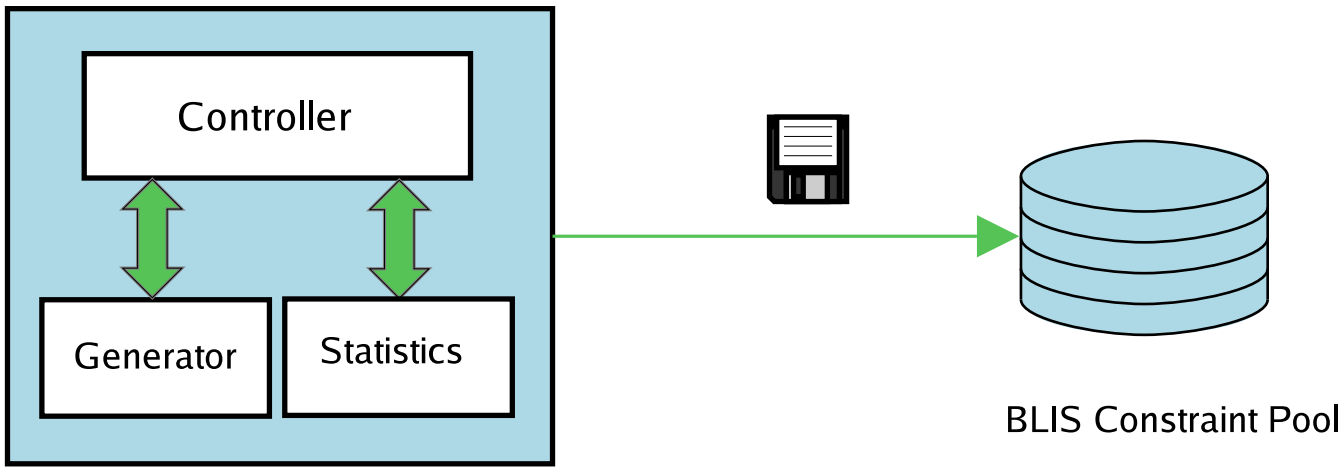
- Object: has feasible region and can be branched on.
- Branching Object:
  - is created from objects that do not lie in they feasible regions or objects that will be beneficial to the search if branching on them.
  - contains instructions for how to conduct branching.
- Branching method:
  - specifies how to create a set of candidate branching objects.
  - has the method to compare objects and choose the best one.



# BLIS: Constraint Generators

BLIS constraint generator:

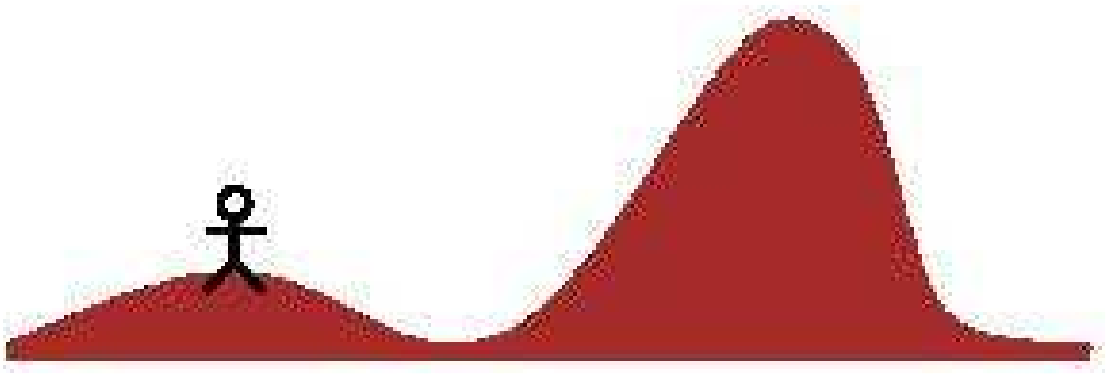
- provides an interface between BLIS and the algorithms in COIN/Cgl.
- provides a base class for deriving specific generators.
- has the ability to specify rules to control generator:
  - where to call: root, leaf?
  - how many to generate?
  - when to activate or disable?
- contains the statistics to guide generating.



# BLIS: Heuristics

BLIS primal heuristic:

- defines the functionality to search for solutions.
- has the ability to specify rules to control heuristics.
  - where to call: before root, after bounding, at solution?
  - how often to call?
  - when to activate or disable?
- collects statistics to guide the heuristic.
- provides a base class for deriving specific heuristics.



# ALPS Applications

- Relatively simple tree search algorithms can be implemented directly from ALPS.
- The user must derive a few classes to specify the algorithm and model.
  - `AlpsModel`: Implement `readInstance()` and store problem data.
  - `AlpsTreeNode`: Implement `process()` and `branch()`.
  - `AlpsNodeDesc`: Store branching information.
  - `AlpsSolution`: Strore solution data.
  - `AlpsParameterSet`: Define parameters (optional)
- To use the parallel mode, the user must also define `pack()` and `unpack()` methods for each knowledge type (easy with provided utilities).
- After deriving classes, use the resulting libray to implement an application.

# Using ALPS: A Knapsack Solver

The formulation of the binary knapsack problem is

$$\max\{\sum_{i=1}^{m} p_i x_i : \sum_{i=1}^{m} s_i x_i \leq c, x_i \in \{0,1\}, i = 1,2,\ldots,m\}, \tag{4}$$

We derive the following classes:

- `KnapModel` (from `AlpsModel`) : Stores the data used to describe the knapsack problem and implements `readInstance()`
- `KnapTreeNode` (from `AlpsTreeNode`) : Implements `process()` (bound) and `branch()`
- `KnapNodeDesc` (from `AlpsNodeDesc`) : Stores information about which variables/items have been fixed by branching and which are still free.
- `KnapSolution` (from `AlpsSolution`) Stores a solution (which items are in the knapsack).

# Using ALPS: Knapsack Solver Main Function

Then, supply the main function.

```
int main(int argc, char* argv[])
{
    KnapModel model;

#if defined(SERIAL)
    AlpsKnowledgeBrokerSerial broker(argc, argv, model);
#elif defined(PARALLEL_MPI)
    AlpsKnowledgeBrokerMPI broker(argc, argv, model);
#endif
    broker.registerClass(AlpsKnowledgeTypeModel, new KnapModel);
    broker.registerClass(AlpsKnowledgeTypeSolution,
                         new KnapSolution(&model));
    broker.registerClass(AlpsKnowledgeTypeNode,
                         new KnapTreeNode(&model));
    AlpsTreeNode* root = new KnapTreeNode(&model);
    broker.search(root);
    broker.printResult();
    return 0;
}
```

# Computational Experiments with KNAP

- Randomly generated 26 *difficult* Knapsack instances based on the rule proposed by Martello ('90).
- Tested on the SDSC Blue Gene System (Linux, MPICH, 700MHz Dual Processor, 512 MB RAM).

Table: Scalability for solving Difficult Knapsack Instances

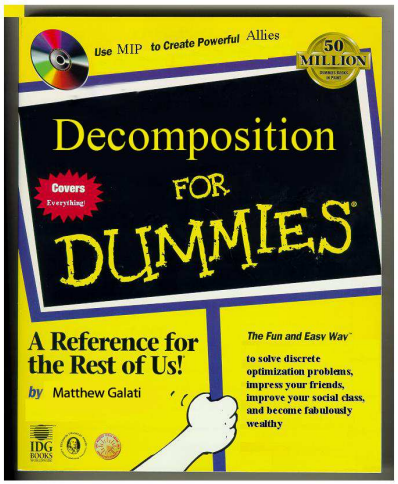
| P | Node | Ramp-up | Idle | Ramp-down | Wallclock | Eff |
|---|---|---|---|---|---|---|
| 64 | 14733745123 | 0.69% | 4.78% | 2.65% | 6296.49 | 1.00 |
| 128 | 14776745744 | 1.37% | 6.57% | 5.26% | 3290.56 | 0.95 |
| 256 | 14039728320 | 2.50% | 7.14% | 9.97% | 1672.85 | 0.94 |
| 512 | 13533948496 | 7.38% | 4.30% | 14.83% | 877.54 | 0.90 |
| 1024 | 13596979694 | 13.33% | 3.41% | 16.14% | 469.78 | 0.84 |
| 2048 | 14045428590 | 9.59% | 3.54% | 22.00% | 256.22 | 0.77 |

# ALPS Application: DECOMP

## DECOMP Framework

DECOMP is a software framework that provides a virtual sandbox for testing and comparing various decomposition-based bounding methods.

- It's difficult to compare variants of decomposition-based algorithms.
- The method for separation/optimization over $\mathcal{P}'$ is the primary custom component of any of these algorithms.
- DECOMP abstracts the common, generic elements of these methods.
  - Key: The user defines methods in the space of the compact formulation.
  - The framework takes care of reformulation and implementation for all variants.



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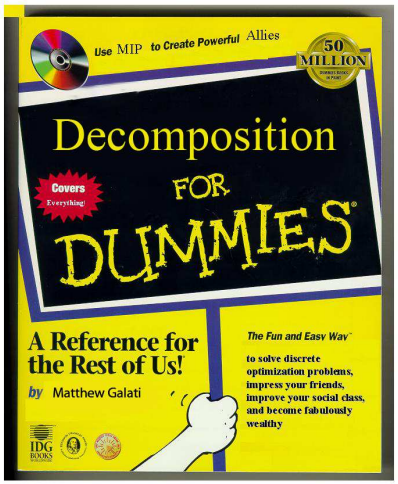

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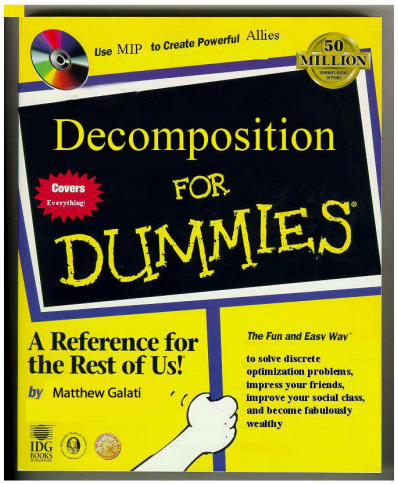

# DECOMP: Bounding Methods

- Decomposition methods
  - Cutting Plane Method (CPM) dynamically builds an *outer* approximation of the feasible region of a given relaxation. $\mathcal{Q}''$.
  - Dantzig-Wolfe Method (DW) builds an *inner* approximation of the feasible region of a given relaxation.
  - Lagrangian Method (LD) obtains a bound by solving a penalized version of the relaxation.
- These three methods can be seen as equivalent.
- The user has only to define the relaxation to get implementations of all three, plus hybrids.
- DECOMP can be inserted as a bounding method to drive a branch-and-bound built in ALPS.

# DECOMP: Implementation Overview

- The DECOMP framework, written in C++, is accessed through two user interfaces:
  - Applications Interface: `DecompApp`
  - Algorithms Interface: `DecompAlgo`
- DECOMP provides the bounding method for branch and bound.
- ALPS (Abstract Library for Parallel Search) provides the framework for parallel tree search.
  - `AlpsDecompModel : public AlpsModel`
    - a wrapper class that calls (data access) methods from `DecompApp`
  - `AlpsDecompTreeNode : public AlpsTreeNode`
    - a wrapper class that calls (algorithmic) methods from `DecompAlgo`

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# BLIS Applications: VRP Formulation

$$min \sum_{e \in E} c_e x_e$$

$$\sum_{e=\{0,j\} \in E} x_e = 2k, \tag{5}$$

$$\sum_{e=\{i,j\} \in E} x_e = 2 \ \ \forall i \in N, \tag{6}$$

$$\sum_{\substack{e=\{i,j\} \in E \\ i \in S, j \notin S}} x_e \geq 2b(S) \ \ \forall S \subset N, \ |S| > 1, \tag{7}$$

$$0 \leq x_e \leq 1 \ \ \forall e = \{i,j\} \in E, \ i,j \neq 0, \tag{8}$$

$$0 \leq x_e \leq 2 \ \ \forall e = \{i,j\} \in E, \tag{9}$$

$$x_e \in \mathbb{Z} \ \ \forall e \in E. \tag{10}$$

# BLIS Applications: VRP Implementation

- First, derive a few subclasses to specify the algorithm and model
  - `VrpModel` (from `BlisModel`),
  - `VrpSolution` (from `BlisSolution`),
  - `VrpCutGenerator` (from `BlisConGenerator`),
  - `VrpHeurTSP` (from `BlisHeuristic`),
  - `VrpVariable` (from `BlisVariable` ), and
  - `VrpParameterSet` (from `AlpsParameterSet`).
- The cut generator in this case is for the Generalized Subtour Elimination Constraints.
- Note that we derive our own variable class so that we can store a decription of the variables, i.e., the locations the edge connects.
- We now also have our own heuristic.
- Then, we write a `main` function as before.

# Computational Results with VRP

- Select 16 VRP instances from public sources (Ralphs,'03)
- Tested on a Clemson Cluster (Linux, MPICH, 1654 MHz Dual Core, 4G RAM).

| P | Nodes | Ramp-up | Idle | Ramp-down | Wallclock | Eff |
|---|---|---|---|---|---|---|
| 1 | 40250 | – | – | – | 19543.46 | 1.00 |
| 4 | 36200 | 7.06% | 7.96% | 0.39% | 5402.95 | 0.90 |
| 8 | 52709 | 9.88% | 6.15% | 1.29% | 4389.62 | 0.56 |
| 16 | 70865 | 14.16% | 8.81% | 3.76% | 3332.52 | 0.37 |
| 32 | 96160 | 15.85% | 10.75% | 16.91% | 3092.20 | 0.20 |
| 64 | 163545 | 18.19% | 10.65% | 19.02% | 2767.83 | 0.11 |

In October, 2007, the VRP/TSP solver won the Open Contest of Parallel Programming at the 19th International Symposium on Computer Architecture and High Performance Computing.

# BLIS Applications: MiBS

The Mixed Integer Bilevel Solver (MibS) has been developed to solve bilevel integer programming problems.

## COIN-OR Components Used

- The COIN High Performance Parallel Search Framework (CHiPPS) to manage the branch and bound.
- The COIN Branch and Cut (CBC) framework for solving MILPs.
- The COIN LP Solver (CLP) framework for solving LPs.
- The Cut Generation Library (CGL) for generating valid inequalities.
- The Open Solver Interface (OSI) for interfacing with CBC and CLP.

# BLIS Applications: MiBS Implementation

- As before, the main effort is in deriving a few classes to specify the algorithm.
  - `MibSModel`
  - `MibSSolution`
  - `MibSCutGenerator`
  - `MibSVariable`
  - `MibSParams`
- Once the classes have been implemented, the user writes a `main` function as before.
- Note that all the power of the generic elements of the MIP solver are available.

# Computational Results with MiBS: Knapsack Interdiction

- Below are results for knapsack interdiction problems.
- These use a special class of cuts that applies only to interdiction problems.
- There is also a heuristic.
- Instances come from the MCDM library.

| | Maximum Infeasibility | | | Strong Branching | | |
|---|---|---|---|---|---|---|
| $2n$ | Avg Nodes | Avg Depth | Avg CPU (s) | Avg Nodes | Avg Depth | Avg CPU (s) |
| 20 | 359.30 | 8.65 | 9.32 | 358.30 | 8.65 | 11.07 |
| 22 | 658.40 | 9.85 | 18.50 | 658.20 | 9.85 | 18.92 |
| 24 | 1414.80 | 10.85 | 46.03 | 1410.80 | 10.75 | 46.46 |
| 26 | 2725.00 | 12.05 | 97.55 | 2723.50 | 12.05 | 100.17 |
| 28 | 5326.40 | 12.90 | 214.97 | 5328.60 | 12.95 | 220.26 |
| 30 | 10625.00 | 14.05 | 482.70 | 10638.00 | 14.10 | 538.32 |

# Computational Results with MiBS: Assignment Interdiction

The setup here is similar, but we have assignment interdiction instead of knapsack.

| Instance | Nodes | Depth | CPU (s) | Instance | Nodes | Depth | CPU (s) |
|---|---|---|---|---|---|---|---|
| 2AP05-1 | 6203 | 33 | 290.25 | 2AP05-14 | 3173 | 28 | 261.08 |
| 2AP05-2 | 3881 | 32 | 384.97 | 2AP05-15 | 2509 | 32 | 127.05 |
| 2AP05-3 | 3909 | 32 | 205.93 | 2AP05-16 | 1699 | 29 | 44.61 |
| 2AP05-4 | 2441 | 36 | 102.66 | 2AP05-17 | 5417 | 29 | 201.34 |
| 2AP05-5 | 3505 | 33 | 119.18 | 2AP05-18 | 5785 | 32 | 176.67 |
| 2AP05-6 | 2031 | 35 | 80.31 | 2AP05-19 | 2259 | 32 | 79.70 |
| 2AP05-7 | 2957 | 29 | 153.02 | 2AP05-20 | 2585 | 31 | 77.35 |
| 2AP05-8 | 3549 | 32 | 224.77 | 2AP05-21 | 6039 | 33 | 161.44 |
| 2AP05-9 | 2271 | 33 | 111.13 | 2AP05-22 | 2479 | 29 | 48.06 |
| 2AP05-10 | 3299 | 31 | 211.07 | 2AP05-23 | 1519 | 25 | 49.40 |
| 2AP05-11 | 707 | 33 | 35.13 | 2AP05-24 | 15 | 5 | 1.32 |
| 2AP05-12 | 407 | 18 | 29.51 | 2AP05-25 | 3857 | 31 | 115.97 |
| 2AP05-13 | 391 | 18 | 23.80 | | | | |

The CHiPPS framework is available at

`https://projects.coin-or.org/CHiPPS`

# Questions? & Thank You!