A Library Hierarchy for Implementing Scalable Parallel Search Algorithms

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Outline

• Motivation and background

• The Abstract Library for Parallel Search (ALPS)
  – Design and implementation
  – Preliminary computational results

• The Branch, Constrain, and Price Solver (BiCePS)
  – Design and implementation

• The BiCePS Linear Integer Solver (BLIS)
  – Design and implementation
  – Preliminary computational results

• Future work
Motivation

- ALPS is a C++ class library for implementing parallel tree search.
- ALPS is being developed in partnership with the COIN-OR Foundation, IBM, and NSF.
- A large number of frameworks and solvers already exist.
- What differentiates ALPS?
  - Intuitive interface and open source implementation.
  - Very general, base classes make minimal algorithmic assumptions.
  - Easy to specialize for particular problem classes.
  - Designed for maximum parallel scalability.
  - Explicitly supports data-intensive algorithms.
Tree Search Algorithms

- Tree search algorithms systematically search the nodes of a directed, acyclic graph for one or more goal nodes.
- This process is ostensibly easy to parallelize.
- However, the graph is not known a priori and is constructed as the algorithm progresses.
- A generic tree search algorithm consists of the following elements:
  - Processing method: Is goal achieved?
  - Search strategy: What should we work on next?
  - Fathoming rule: Can node can be fathomed?
  - Branching method: What are the successors?
- The algorithm consists of choosing a candidate node, processing it, and either fathoming or branching.
- During the course of the search, various information (knowledge) is generated and used to guide the search.
- Efficient knowledge sharing is the key to parallelization.
Parallel Computing Concepts

• The goal in parallelizing any algorithm is to minimize \textit{parallel overhead}.

• The main contributors to parallel overhead in tree search are
  
  – \textbf{Communication Overhead} (cost of sharing knowledge)
  
  – \textbf{Idle Time}
    
    * Handshaking/Synchronization (cost of sharing knowledge)
    
    * Task Starvation (cost of \textit{not} sharing knowledge)
    
    * Ramp Up Time (cost of sharing knowledge)
    
    * Ramp Down Time
  
  – \textbf{Performance of Redundant Work} (cost of \textit{not} sharing knowledge)

• Knowledge sharing is the main driver of efficiency.

• This breakdown highlights the tradeoff between centralized and decentralized knowledge storage and decision-making.
ALPS: Features

- Generality
  - ALPS only assumes that the graph to be searched is acyclic.
  - The implementation is based on a very general concept of knowledge.

- Scalability
  - Knowledge is shared asynchronously through pools and brokers.
  - Management overhead is reduced with the master-hub-worker paradigm.
  - Overhead is decreased using dynamic task granularity.
  - Static and dynamic load balancing techniques are employed.
  - Tasks are managed locally by a task scheduler.
ALPS: Knowledge Handling

• All knowledge to be shared is considered as AlpsKnowledge and has an associated encoded form.

• The encoded form is issued for identification, storage, and communication.

• AlpsKnowledge is managed by one or more AlpsKnowledgePools.

• The knowledge pools communicate through AlpsKnowledgeBrokers.
ALPS: Master-Hubs-Workers Paradigm
ALPS: How to Develop an Application

Mainly comprises two parts:

- Derive the required and auxiliary problem-specific classes
  - AlpsModel
  - AlpsTreeNode
  - AlpsNodeDesc
  - AlpsSolution

- Write the main function

Two examples have been developed:

- Knapsack solver
- ALPS Branch and Cut (ABC)
ALPS: Computational Results of Knapsack Solver

- Test Environment:

  **Machine:** Beowulf cluster with 48 dual-processor nodes  
  **Processor:** 1.0 GHz Pentium III  
  **Memory:** 512M on 44 nodes, 2G on 4 nodes  
  **Operating System:** Red Hat Linux 7.2  
  **Message Passing:** LAM/MPI

- Experiment Design:
  - Generate *ten hard knapsack instances* based on the rule proposed in Martello (’90).
  - Run three trials for each instance, and take the average.
  - Some default parameters:
    - Two hubs are used when using 16 processes or more.
    - Dynamic load balance is activated.
    - Hubs do not processing subproblems.
ALPS: Computational Results of Knapsack Solver

- The *ten instances* have similar behavior, so we present summary results.

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(NOTE: m is million, time is in seconds)

- Serial code has difficult to solve several instance due to memory fraction.
- These indicate reasonable scalability.
- Starvation and ramp-up overhead is tiny.
- Ramp-down overhead has room to decrease.
Library Hierarchy for Optimization Engine

**ALPS** (Abstract Library for parallel search)

- is the search handling layer.
- prioritizes nodes based on *quality*.

**BiCePS** (Branch, Constrain, and Price Software)

- is the data handling layer for relaxation-based optimization.
- adds notion of *variables* and *constraints*.
- uses an iterative bounding procedure.

**BLIS** (BiCePS Linear Integer Solver)

- is a concretization of BiCePS.
- specific to models with *linear* constraints and objective function.
BiCePS: Data-intensive Applications

• In applications such as *branch, cut, and price* (BCP), the amount of information needed to describe each search tree node is very large.

• This can make memory an issue and also increase communication overhead.

• We can think of each node as being described by a list of *objects*, i.e., *constraints* and *variables*.

• All objects have a domain and can be treated *symmetrically*.

• These objects can be generated throughout the search process.

• In BCP, the set of objects may not change much from parent to children.

• We can therefore store the description of an entire subtree very compactly using *differencing*.
BiCePS: Objects

- BiCePS assumes an iterative bounding scheme.
- Each iteration, objects and can be stored in object pools.
- The number of objects can be huge, duplicate and weak objects can be removed based on their hash keys and effectiveness.
- Periodically, invalid and ineffective objects are purged.
- Effectively sharing objects between processes is a challenge.
Consider problem $P$:

\[
\begin{align*}
\text{min} & \quad c^T x \\
\text{s.t.} & \quad Ax \leq b \\
x_i & \in \mathbb{Z} \quad \forall i \in I
\end{align*}
\]

where $(A, b) \in \mathbb{R}^{m \times (n+1)}$, $c \in \mathbb{R}^n$.

Basic Algorithmic Elements:

- bounding method.
- branching scheme.
- object generators.
- heuristics.
BLIS: Branching scheme

BLIS Branching scheme comprise three components:

- **Object**: has feasible region and can be branched on.
- **Branching Object**:
  - is created from an infeasible object.
  - contains instructions for how to conduct branching.
- **Branching strategy**:
  - specifies how to create a set of candidate branching objects.
  - has the method to compare objects and choose the best one.
BLIS: Constraint generators

BLIS constraint generator:

- provides an interface between BLIS and the algorithms in COIN/Cgl.
- has the ability to specify rules to control generator:
  - where to call: root, leaf?
  - how many to generate?
  - when to activate or disable?
- contains the statistics to guide generating.
BLIS: Heuristics

BLIS heuristic:

- Defines the functionality to search for solutions.
- Has the ability to specify rules to control heuristics.
  - where to call: after bounding, at solution?
  - how often to call?
  - when to activate or disable?
- Collects statistics to guide searching.
- Provides a base class for deriving various heuristics.
BLIS: Preliminary Computational Results

Test Machine: PC, 2.8 GHz Pentium, 2.0G RAM, Linux

Test instances: Select 33 instances from Lehigh/CORAL and Miplib3, which both solvers can solve in 10 minutes.

Solver settings:

- BLIS
  - Branching strategy: Pseudocost branching.
  - Cuts generators: Gomory, Knapsack, Flow Cover, MIR, Probing, and Clique.
  - Heuristics: Rounding.

- COIN/Cbc
  - Branching strategy: Strong branching.
  - Cut generators: Gomory, Knapsack, Flow Cover, MIR, Probing, and Clique.
  - Heuristics: Rounding and Local search.
### BLIS: Preliminary Computational Results

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BLIS: Preliminary Computational Results
Future work

• Improve ALPS:
  – Reduce ramp-up time when node processing time is long.
  – More effectively adjust parameters based on problem structures and searching progress.

• Complete the development of BiCePS and BLIS.
  – Finish parallel parts of the code.
  – Find answers to important research questions:
    ∗ How to share objects?
    ∗ How to efficiently avoid duplicated knowledge?
    ∗ How to deal with locally valid knowledge?
  – Add more customization features akin to COIN/BCP:
    ∗ Branch and price.
    ∗ Branch, cut, and price.