Analysis of Parallel Algorithms

IE 496 Lecture 8

Reading for This Lecture

- Paper by Kumar and Gupta
- Paper by Gustafson
- Roosta, Chapter 5

Parallel Systems

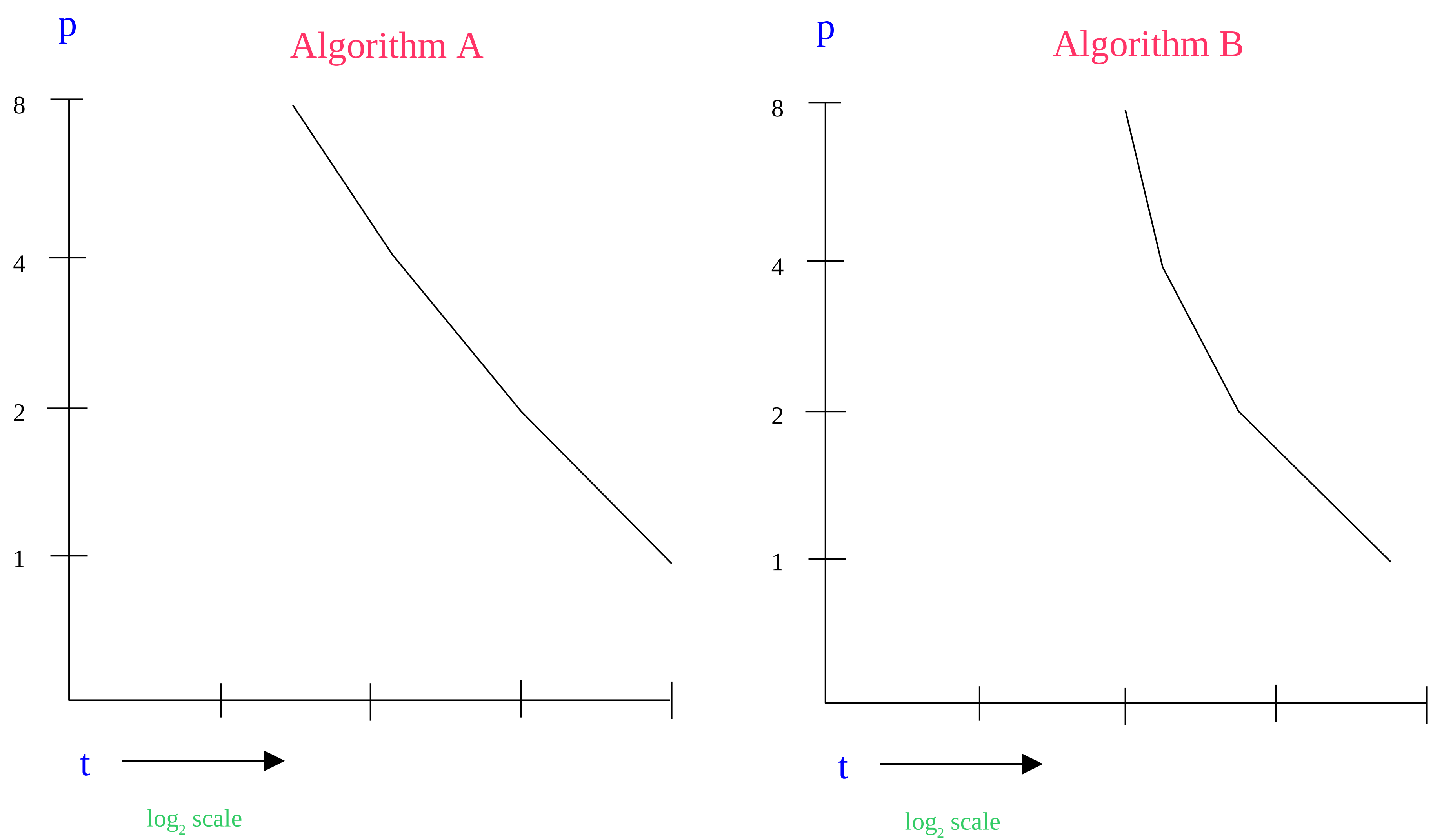
- A parallel system is a parallel algorithm plus a specified parallel architecture.
- Unlike sequential algorithms, parallel algorithms cannot be analyzed very well in isolation.
- One of our primary measures of goodness of a parallel system will be its scalability.
- Scalability is the ability of a parallel system to take advantage of increased computing resources (primarily more processors).

Empirical Analysis of Parallel Algorithms

- Modern parallel computing platforms are essentially all asynchronous.
- Threads/processes do not share a global clock.
- In practice, this means that the execution of parallel algorithms is non-deterministic.
- For analysis of all but the simplest parallel algorithms, we must depend primarily on empirical analysis.
- The realities ignored by our models of parallel computation are actually important in practice.

Scalability Example

Which is better?



Terms and Notations

Sequential Runtime	T_1
Sequential Fraction	s
Parallel Fraction	$p = 1 - s$
Parallel Runtime	T_N
Cost	$C_N = NT_N$
Parallel Overhead	$T_o = C_N - T_1$
Speedup	$S_N = T_1 / T_N$
Efficiency	$E = S_N / N$

Definitions and Assumptions

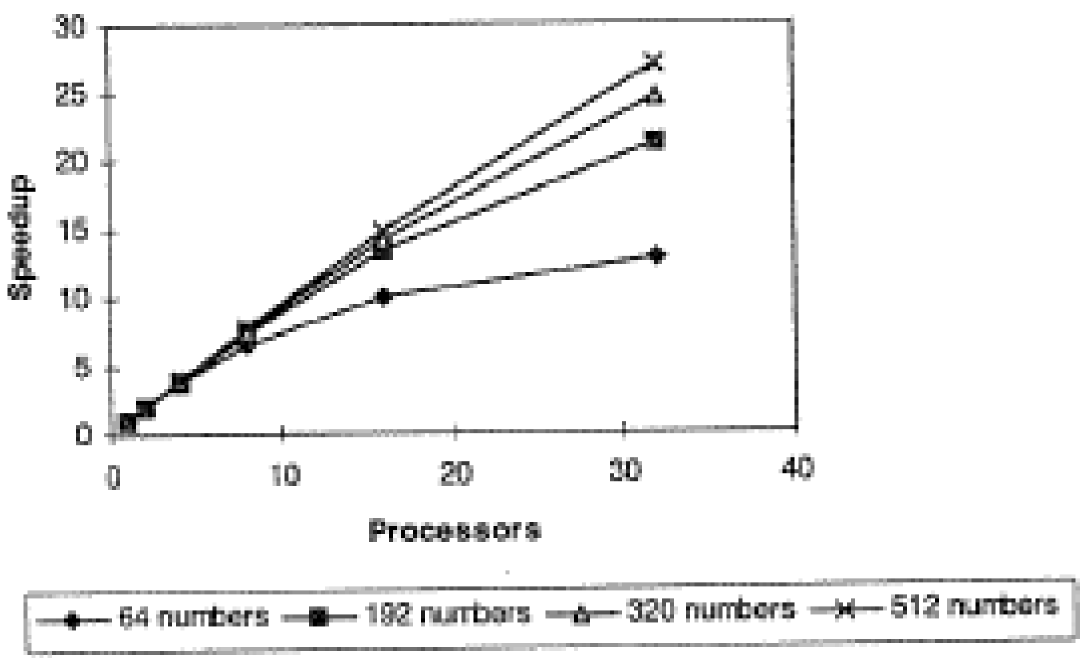
- The sequential running time is usually taken to be the running time of the best sequential algorithm.
- The sequential fraction is the part of the algorithm that is inherently sequential (reading in the data, splitting, etc.)
- The parallel overhead includes all additional work that is done due to parallelization.
  - communication
  - nonessential work
  - idle time

Cost, Speedup, and Efficiency

- These three concepts are closely related.
- A parallel system is cost optimal if $C_N \in O(T_1)$.
- A parallel system is said to exhibit linear speedup if $S \in O(N)$.
- Hence, linear speedup $\Leftrightarrow$ cost optimal $\Leftrightarrow$ $E = 1$
- If $E > 1$, this is called super-linear speedup.
- Superlinear speedup can arise in arise, though it it is not possible *in principle*.

Example: Parallel Semi-group

- With n data elements and p processors, we first combine n/p elements sequentially locally.
- Then combine local results.
- Parallel running time is $n/p + 2 \log p$.



Factors Affecting Speedup

- Sequential Fraction
- Parallel Overhead
  - Unecessary/duplicate work
  - Communication overhead/idle time
  - Time to split/combine
- Task Granularity
- Degree of Concurrency
- Sychronization/Data Dependency
- Work Distribution
- Ramp-up and Ramp-down Time

Amdahl's Law

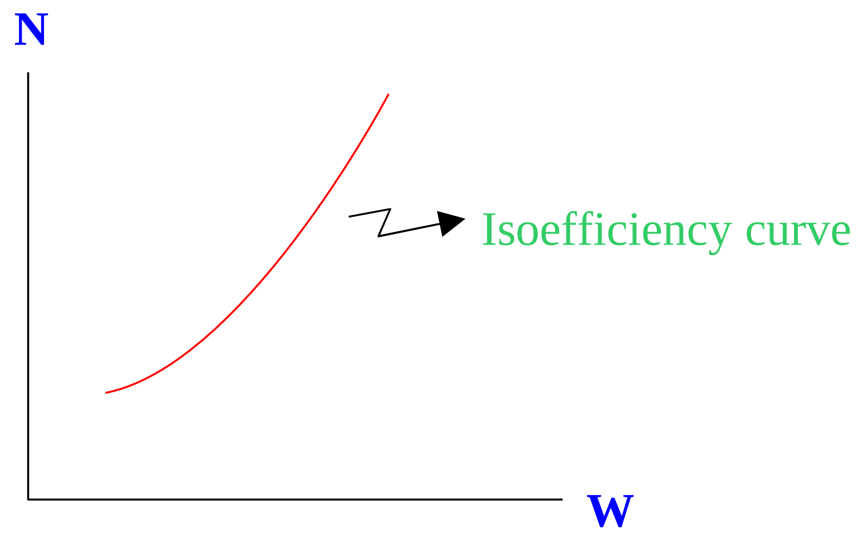
- Speedup is bounded by

$$(s + p)/(s + p/N) = 1/(s + p/N) = N/(sN + p)$$

- This means more processors $\Rightarrow$ less efficient!
- How do we combat this?
- Typically, larger problem size $\Rightarrow$ more efficient.
- This can be used to "overcome" Amdahl's Law.

The Isoefficiency Function

- The isoefficiency function *f(N)* of a parallel system represents the rate at which the problem size must be increased in order to maintain a fixed efficiency



- This function is a measure of scalability that can be analyzed using asymptotic analysis.

Gustafson's Viewpoint

- Gustafson noted that typically the serial fraction does not increase with problem size.
- This view leads to an alternative bound on speedup called scaled speedup.

$$(s + pN)/(s + p) = s + pN = N + (1-N)s$$

- This may be a more realistic viewpoint.