

IE 495 Lecture 4

September 7, 2000

Reading for this lecture

- Primary
 - Miller and Boxer, Chapters 2 and 3
 - Aho, Hopcroft, and Ullman, Sections 2.5-2.9

Induction and Recursion

Mathematical Induction

- Induction is a technique for proving statements about consecutive integers.
- Principle of Mathematical Induction
 - Let $P(n)$ be a predicate that we want to prove TRUE for all positive n .
 - Method:
 1. Prove $P(1)$.
 2. Prove $P(k) \Rightarrow P(k+1) \forall k \geq 1$.
- Example: Prove $\sum_{i=1}^n i = \frac{n(n+1)}{2}$
- What does induction have to do with programming?

Recursion

- **Definition (Mathematics):** An expression, each term of which is determined by application of a formula to the preceding terms.
- In CS, a function that calls itself is called *recursive*.
- Recursion allows us to process large data sets based on our knowledge of smaller ones.
- The correctness and complexity of recursive algorithms can be proven by induction.
- Example: The *factorial* function.

Analyzing Recursive Algorithms

- For recursive algorithms, running times can also be defined recursively.

- Example:

$$T(0) = 1$$

$$T(n) = T(n-1) + 1$$

- What is $T(n)$?
- In general, we have something like $T(n) = aT(n/b - c) + f(n)$.

Example: Binary Search

- This example is in Miller and Boxer, p.37

Divide and Conquer

- A common approach is to divide the problem into smaller parts and solve each part independently.
- In this case, our recurrence relation looks like

$$T(n) = S(n) + aT(n/b) + C(n)$$

- $S(n)$ = time to split
- $C(n)$ = time to combine

Example: Merge Sort

- This example is in Miller and Boxer, p. 41

Balancing

- When splitting, it usually makes sense to split in such a way that the two resulting subproblems are of approximately equal size.
- If not, the recurrence relation may not hold, and efficiency will decrease.
- However, balancing is is not always easy to do.
- Improper balancing can cause problems for parallel algorithms (more on this later).

Example: Multiplying n -bit numbers

- This example is in AHU, p. 62.

The Master Theorem

- The Master Theorem can help solve some common recurrence relations.
- See Miller and Boxer, Chapter 3.

Implications for Parallelism

- Recursive algorithms have natural parallel formulations.
 - Split the problem until there is one part per processor.
 - Perform a sequential algorithm on each part in parallel.
 - Combine the results.
- Must be cognizant of the overhead involved in splitting and combining (this is also true in sequential algorithms).