Written Problems

1. Given a graph $G = (V, E)$, a $k$-vertex coloring is an assignment of $k$ colors (represented by the integers 1 to $k$) to the vertices of $G$ in such a way that no two vertices adjacent in $G$ have the same color. The vertex coloring problem is that of finding the minimum $k$ such that there exists a $k$-vertex coloring of $G$.

   Design a branch and bound algorithm for solving the vertex coloring problem. Your algorithm specification should include

   - Specification of the state space
   - Methods of upper and lower bounding
   - A method of checking feasibility
   - A branching rule
   - A search strategy

   You should discuss what data structures would be necessary to implement your algorithm, including the graph representation, representation of the search nodes, and data structures for upper and lower bounding.

2. Show that the set of all 1-trees on a graph $G$ is a matroid. There are two different methods of showing this. Use either one.