IGERT Experience In Switzerland
Logistics at Galenica Pharmaceutical

Matthew V Galati
Gérard Hêche

Department of Industrial and Systems Engineering
Lehigh University, Bethlehem, PA, USA
September 27, 2002
Outline

École Polytechnique Fédérale de Lausanne, Suisse (EPFL) - Travel

Galenica Pharmaceutical - Logistics

Vehicle Routing Problem with Time Windows (VRPTW)

Branch Cut and Price (BCP)

BCP for VRPTW
  - Formulation
  - Pricing Problem
  - Cutting Planes
  - Branching

Implementation of BCP for VRPTW

(Preliminary) Computational Results

Current Research
School of Basic Sciences - Institute of Mathematics
- Recherche Opérationnelle Sud Ouest (ROSO) - Thomas Liebling
- Recherche Opérationnelle Sud Est (ROSE) - Dominique de Werra

Student Proposals - COIN-or.org
- Preprocessing Techniques for Mixed Integer Programming
- A Cut Generator Library for Integer Programming

Seminar Series: Martine Labbé - Université Libre De Bruxelles
- Branch-and-Cut for Network Design in Telecommunications
- Two-Connected Network with Bounded Rings

3ème Cycle Romand de Recherche Opérationnelle - Zinal, Switzerland
- Alexander Schrijver of CWI (National Research Institute for Mathematics and Computer Science in the Netherlands), Amsterdam
- Kurt Mehlhorn of Max-Planck-Institut für Informatik, Saarbrücken
Common Optimization INterface for Operations Research

- An initiative to spur the development of open source software for the operations research community.

- Current projects in the COIN repository [www.coin-or.org]:
  - **BCP**: a parallel branch-cut-price framework
  - **CGL**: a cut generation library
  - **DFO**: a package for solving general nonlinear optimization problems when derivatives are unavailable
  - **VOL**: the volume algorithm
  - **OSI**: an open solver interface layer
  - **OTS**: an open framework for tabu search
  - **IPOPT**: an interior point algorithm for general large-scale nonlinear optimization
  - **CLP**: a native simplex solver
ROSO Seminar Series (May 2002):  
*Decomposition-based Methods for Large-scale Discrete Optimization*

Galenica Pharmaceutical - Logistics Optimization  
- Current system uses a greedy insertion heuristic

Contribution  
- Improve the quality of the solutions provided by the heuristic methods  
- Provide some measurement or proof of optimality
École Polytechnique Fédérale de Lausanne, Suisse (EPFL)

Galenica Pharmaceutical - Logistics

Vehicle Routing Problem with Time Windows (VRPTW)

Branch Cut and Price (BCP)

BCP for VRPTW
  - Formulation
  - Pricing Problem
  - Cutting Planes
  - Branching

Implementation of BCP for VRPTW

(Preliminary) Computational Results

Current Research
Major pharmaceutical corporation headquartered in Bern, Switzerland

Several distribution centers (DCs) across the country

Apriori set of customers (and demand) to be serviced each day of the week (from a given DC)

Each customer has a (tight) fixed time window when delivery may occur (considered hard constraints).

Routing between customers calculated by a street-level route generator.

Cost is a linear combination of distance and time.
Design
- Delivery length is typically small so the same truck can make several cycles per day (load, depart from DC, delivery, return to DC).
- It is possible to use trailers with some trucks.
- What is the effect of soft time windows?

Depot
- Trucks sometimes require 60m maintenance on return to depot.
- During loading there are only \( n \) slots available at one time.

Trucks
- Some trucks are only available within a specified time window.
- Trucks are capacitated by both weight and volume.

Route
- If the route exceeds 4 hours, the driver must take a 15m break.
- Idle time at the customer must be less than 15m.
Outline

- École Polytechnique Fédérale de Lausanne, Suisse (EPFL)
- Galenica Pharmaceutical - Logistics
- Vehicle Routing Problem with Time Windows (VRPTW)
- Branch Cut and Price (BCP)

  BCP for VRPTW
  - Formulation
  - Pricing Problem
  - Cutting Planes
  - Branching

- Implementation of BCP for VRPTW
- (Preliminary) Computational Results
- Current Research
Set of customers $C$ set of homogenous vehicles $K$ with capacity $q$.

Let $N$ define the set of nodes $0, 1, 2, \ldots, n + 1$.

The set of arcs $A$ is defined as $\{(i, j) : i \neq j, i \neq n + 1, j \neq 0\}$.

For each arc $(i, j) \in A$, we have cost $c_{ij}$ and time $t_{ij}$ which includes the service time at customer $i$.

Each customer $i \in C$ has a demand $d_i$ and a time window $[a_i, b_i]$.

Define the variable $x_{ijk} = 1$ if vehicle $k \in K$ drives along arc $(i, j) \in A$.
Let $s_{ik}$ define the time vehicle $k$ starts to service customer $i$. 
Vehicle Routing with Time Windows

Design a set of minimal cost routes, one for each vehicle,

$$\min \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ijk}$$

such that, each customer is serviced exactly once,

$$\sum_{k \in K} \sum_{j \in N} x_{ijk} = 1 \quad \forall i \in C$$  \hspace{1cm} (1)$$

every route originates at vertex 0 and ends at vertex \(n + 1\),

$$\sum_{j \in N} x_{ojk} = 1 \quad \forall k \in K$$  \hspace{1cm} (2)$$

$$\sum_{i \in N} x_{ihk} - \sum_{j \in N} x_{hjk} = 0 \quad \forall h \in C, k \in K$$  \hspace{1cm} (3)$$

$$\sum_{i \in N} x_{i,n+1,k} = 1 \quad \forall k \in K$$  \hspace{1cm} (4)$$
the time windows and capacity constraints are observed and

\[ \sum_{i \in C} d_i \sum_{j \in N} x_{ijk} \leq q \quad \forall k \in K \] (5)

\[ s_{ik} + t_{ij} - M (1 - x_{ijk}) \leq s_{jk} \quad \forall i, j \in N, k \in K \] (6)

\[ a_i \leq s_{ik} \leq b_i \quad \forall i \in N, k \in K \] (7)

assignments are integral.

\[ x_{ijk} \in \{0, 1\} \quad \forall i, j \in N, k \in K \] (8)
École Polytechnique Fédérale de Lausanne, Suisse (EPFL)

Galenica Pharmaceutical - Logistics

Vehicle Routing Problem with Time Windows (VRPTW)

Branch Cut and Price (BCP)

BCP for VRPTW
  - Formulation
  - Pricing Problem
  - Cutting Planes
  - Branching

Implementation of BCP for VRPTW

(Preliminary) Computational Results

Current Research
Consider problem $P$:

$$\begin{align*}
\min & \quad c^T x \\
\text{s.t.} & \quad Ax \leq b \\
& \quad x_i \in \mathbb{Z} \forall i \in I
\end{align*}$$

where $(A, b) \in \mathbb{R}^{m \times n+1}$, $c \in \mathbb{R}^n$.

Let $P = \text{conv}\{x \in \mathbb{R}^n : Ax \leq b, x_i \in \mathbb{Z} \forall i \in I\}$.

Basic Algorithmic Approach
- Use LP relaxations to produce lower bounds.
- Branch using hyperplanes.

Basic Algorithmic Elements
- A method for producing and tightening the LP relaxations.
- A method for branching.
Weyl-Minkowski

\[ \exists (\bar{A}, \bar{b}) \in \mathbb{R}^{m \times n+1} \text{ s.t. } \mathcal{P} = \{ x \in \mathbb{R}^n : \bar{A}x \leq \bar{b} \} \]

We want the solution to \( \min \{ c^T x : \bar{A}x \leq \bar{b} \} \).

Solving this LP isn’t practical (or necessary).

BCP Approach

Form LP relaxations using submatrices of \( \bar{A} \).

The submatrices are defined by sets \( \mathcal{V} \subseteq [1..n] \) and \( \mathcal{C} \subseteq [1..m] \).

BCP Elements

Pricing Algorithm - Variable Generation

Cutting Plane Algorithm - Constraints Generation
Outline

- École Polytechnique Fédérale de Lausanne, Suisse (EPFL)
- Galenica Pharmaceutical - Logistics
- Vehicle Routing Problem with Time Windows (VRPTW)
- Branch Cut and Price (BCP)
  - BCP for VRPTW
    - Formulation
    - Pricing Problem
    - Cutting Planes
    - Branching
  - Implementation of BCP for VRPTW
  - (Preliminary) Computational Results
  - Current Research
Set Partitioning Formulation

- Set of elementary feasible routes $\hat{R}$
- For each route $r \in \hat{R}$, let $\beta_{ir}$ be the number of times route $r$ visits customer $i$ with cost $c_r$.

\[
\begin{align*}
\min & \sum_{r \in \hat{R}} c_r y_r \\
\sum_{r \in \hat{R}} \beta_{ir} y_r &= 1 & \forall i \in C \\
\left[ \sum_{i \in C} \frac{d_i}{q} \right] &\leq \sum_{r \in \hat{R}} y_r & \leq |K| \\
y_r &\in \{0, 1\} & \forall r \in \hat{R}
\end{align*}
\]
Pricing Problem

- Reduced cost of variable $y_r$ is $c_r - \sum_{i \in C \cup \{0\}} \pi_i \beta_{ir}$

- Corresponds to the modified arc costs $\hat{c}_{ij} = c_{ij} - \pi_i$

- Does there exist a member of $\hat{R}$ with negative reduced cost?

- Instance of *Elementary Shortest Path Problem with Time Windows and Capacity Constraints* (ESPPTWCC).

- ESPPTWCC is *NP-hard* and there is no known efficient algorithm.

- Let $R$ be the set of all feasible routes (nonelementary routes are allowed).

- Now an instance of SPPTWCC (also *NP-hard*) but there exists an efficient pseudo-polynomial dynamic programming [Desrochers et al].
**SPP with Resource Constraints**

- Ideas of label setting and dominance from *Dijkstras* for SPP.

- Assumption: time and capacity are discretized.

- Define a state as $c(i, t, d)$ for vertex $i$, current time $t$ and accumulated demand $d$.

  $$c(0, 0, 0) = 0$$

  $$c(j, t, d) = \min_i \{ \hat{c} + c(i, t', d') | t' + t_{ij} = t \land d' + d_i = d \}$$

- The number of possible states is $\Gamma = \sum_{i \in N} (b_i - a_i)(q - 1)$.

- Dominance: $(i, t_1, d_1) \prec (i, t_2, d_2)$ if and only if $c(i, t_1, d_1) \leq c(i, t_2, d_2)$, $t_1 \leq t_2$ and $d_1 \leq d_2$. 

---

IGERT Experience In Switzerland
Logistics at Galenica Pharmaceutical – p.19
**SPP with Resource Constraints**

<Initialization>

\[ NPS = \{(0,0,0)\}, c(0,0,0) = 0 \]

repeat

\( (i,t,d) = \text{BestLabel}(NPS) \)

for \( j := 1 \) to \( n + 1 \) do

if \( i \neq j \) and \( t + t_{ij} \leq b_j \land d + d_j \leq q \) then

<Label Feasible>

if \( c(j, \max\{t + t_{ij}, a_j\}, d + d_j) > c(i, t, d) + \hat{c}_{ij} \) then

<New Label Better>

\[ \text{InsertLabel}(NPS, (j, \max\{t + t_{ij}, a_j\}, d + d_j)) \]

\[ c(j, \max\{t + t_{ij}, a_j\}, d + d_j) = c(i, t, d) + \hat{c}_{ij} \]

until \( i = n + 1 \)

return
SPP with Resource Constraints

- Can find a lot of path cycling between “good” vertices.

- Houck et al: 2-cycle Elimination $i \rightarrow j \rightarrow i$ extend the label to $(i, t, d, pred)$ which increases the number of states by 2.

- Irnich: $k$-cycle Elimination can increase the number of states by $\binom{k}{2} + 1$.
  - Example: $k = 4 : i \rightarrow j \rightarrow k \rightarrow l \rightarrow i$

- Data Structures
  - $N$ Linked Lists
  - Generalized Buckets
Let \( k(S) \) denote the least number of vehicles needed to serve each customer in \( S \): NP-hard.

Define the flow on an arc \( x_{ij} = \sum_{r \in R} \gamma^r_{ij} y_r \).

Let \( k \) be some integer no larger than \( k(S) \).

Define the general \( k \)-Path Cut as

\[
x(S) = \sum_{(i,j) \in \delta(S)} x_{ij} \geq k(S) \geq k
\]

\( x(S) \geq 1 \): Directed version of TSP subtour elimination constraints.

Separation: Polynomial using max flow / min cut.

Appelgate, Bixby, Chvátal and Cook - CONCORDE
Separation of $k$-Path Cuts

$k = 2$ : Is it valid? Determine a set $S$ where $x(S) < 2$.

- Larsen: Greedy search of neighborhood until $x(S) \geq 2$.
- Rich/Cook: Random contraction algorithm of Karger - with high probability, finds all cuts with weight within a multiplicative factor $\alpha$ of the min cut in $O(n^{2\alpha} \log^3 n)$.

$k = 2$ : Is it violated? Check if $k(S) \geq 2$.

- Step 1: Is there sufficient capacity on a single vehicle to service $S$?
- Step 2: Is there a feasible route that leaves the depot, visits each customer in $S$ once and returns to the depot?

Step 2 is an instance of a TSPTW-feasibility problem: NP-complete.

TSPTW can be solved efficiently using a DP similar to SPPTWCC.

$k > 2$ : Rich/Cook

- Is it valid? Greedy or Karger
- Is it violated? Instance of VRPTW-feasibility!!
If number of vehicles $\sum_{r \in R} y_r$ is fractional branch on constraint (10).

Else, choose the arc $(i, j)$ which maximizes the $c_{ij} \min(x_{ij}, 1 - x_{ij})$.

- $x_{ij} = 0$, Fix routes containing arc $(i, j)$ to 0.
- $x_{ij} = 1$, Fix any route that visits $i$ or $j$ without using arc $(i, j)$ to 0.

Alternatives: Branching on resource constraints (TWs, capacity).
Outline

- École Polytechnique Fédérale de Lausanne, Suisse (EPFL)
- Galenica Pharmaceutical - Logistics
- Vehicle Routing Problem with Time Windows (VRPTW)
- Branch Cut and Price (BCP)
- BCP for VRPTW
  - Formulation
  - Pricing Problem
  - Cutting Planes
  - Branching
- Implementation of BCP for VRPTW
- (Preliminary) Computational Results
- Current Research
Implementation - COIN/BCP

- Summer Sessions: [http://sagan.ie.lehigh.edu/coin/coin.html](http://sagan.ie.lehigh.edu/coin/coin.html)
- Modules: TM, LP, VG, CG
- Tree Manager
  - `create_root` - initial set of variables (routes)
- Linear Program
  - `cuts_to_rows` - expand constraints given the current LP
  - `vars_to_cols` - expand variables given the current LP
  - `select_branching_candidates` - branching
- Variable Generator
  - `generate-vars` - generate variables (SPPTWCC)
- Cut Generator
  - `generate_cuts` - generate cuts (k-Path Cuts)
Galenica Logistics - Challenges

Design

- Delivery length is typically small so the same truck can make several cycles per day (load, depart from DC, delivery, return to DC).
- 3 deliveries per day: morning, noon, night

- It is possible to use trailers with some trucks. ??

- What is the effect of soft time windows?
  - Increase the TW and set a cost $p(s_i)$. Dominance criterion remains if cost is non-decreasing, i.e., penalty on late, not idle.

Depot

- Trucks sometimes require 60m maintenance on return to depot.

- During loading there are only $n$ slots available at one time.
  - Allow lead time between deliveries for needed maintenance.
Galenica Logistics - Challenges

- Trucks
  - Some trucks are only available within a specified time window.
  - Trucks are capacitated by both weight and volume.
  - Heterogenous fleet means solving $K$ instances of SPPRC at each iteration.
  - $2$ dimensional resource constraints (weight, volume):
    $$\Gamma = \sum_{i \in N} (b_i - a_i)(q_1 - 1)(q_2 - 1)$$

- Route
  - If the route exceeds $4$ hours, the driver must take a $15$m break.
  - Idle time at the customer must be less than $15$m.
  - As a hard constraint can be restricted in column generator.
Outline

- École Polytechnique Fédérale de Lausanne, Suisse (EPFL)
- Galenica Pharmaceutical - Logistics
- Vehicle Routing Problem with Time Windows (VRPTW)
- Branch Cut and Price (BCP)
- BCP for VRPTW
  - Formulation
  - Pricing Problem
  - Cutting Planes
  - Branching
- Implementation of BCP for VRPTW
- (Preliminary) Computational Results
- Current Research
Computational Results

Benchmark Solomon Instances
- **c1 instances**: Clustered geography, short horizon (5-10 customers per route)
- **r1 instances**: Random geography, short horizon
- **rc1 instances**: Random and Clustered geography, short horizon
- **c2 instances**: Clustered geography, long horizon (more than 30 customers per route)
- **r2 instances**: Random geography, long horizon
- **rc2 instances**: Random and Clustered geography, long horizon

- Rich/Cook: 300 MHz Pentium II with 256MB - Cplex 5.0
- Galati: - OSL V3.0
### Computational Results

**Solomon Instances - \textit{c1.25} instances**

<table>
<thead>
<tr>
<th>problem</th>
<th>Rich/Cook</th>
<th></th>
<th></th>
<th></th>
<th>Galati</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>nodes</td>
<td>cols</td>
<td>cuts</td>
<td>time</td>
<td>nodes</td>
<td>cols</td>
<td>cuts</td>
<td>time</td>
</tr>
<tr>
<td>c101.25</td>
<td>1</td>
<td>499</td>
<td>0</td>
<td>0.57</td>
<td>1</td>
<td>1146</td>
<td>0</td>
<td>0.57</td>
</tr>
<tr>
<td>c102.25</td>
<td>1</td>
<td>726</td>
<td>0</td>
<td>1.83</td>
<td>1</td>
<td>3713</td>
<td>0</td>
<td>2.63</td>
</tr>
<tr>
<td>c103.25</td>
<td>1</td>
<td>616</td>
<td>0</td>
<td>3.34</td>
<td>1</td>
<td>3430</td>
<td>0</td>
<td>4.41</td>
</tr>
<tr>
<td>c104.25</td>
<td>1</td>
<td>570</td>
<td>0</td>
<td>5.41</td>
<td>1</td>
<td>2898</td>
<td>0</td>
<td>15.62</td>
</tr>
<tr>
<td>c105.25</td>
<td>1</td>
<td>445</td>
<td>0</td>
<td>0.58</td>
<td>1</td>
<td>1487</td>
<td>0</td>
<td>0.66</td>
</tr>
<tr>
<td>c106.25</td>
<td>1</td>
<td>363</td>
<td>0</td>
<td>0.27</td>
<td>1</td>
<td>993</td>
<td>0</td>
<td>0.42</td>
</tr>
<tr>
<td>c107.25</td>
<td>1</td>
<td>472</td>
<td>0</td>
<td>0.50</td>
<td>1</td>
<td>1227</td>
<td>0</td>
<td>0.45</td>
</tr>
<tr>
<td>c108.25</td>
<td>1</td>
<td>380</td>
<td>0</td>
<td>0.69</td>
<td>1</td>
<td>2029</td>
<td>0</td>
<td>1.02</td>
</tr>
<tr>
<td>c109.25</td>
<td>1</td>
<td>383</td>
<td>0</td>
<td>1.20</td>
<td>3</td>
<td>1357</td>
<td>9</td>
<td>3.05</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>9</strong></td>
<td><strong>4454</strong></td>
<td><strong>0</strong></td>
<td><strong>14.39</strong></td>
<td><strong>11</strong></td>
<td><strong>18280</strong></td>
<td><strong>9</strong></td>
<td><strong>28.83</strong></td>
</tr>
</tbody>
</table>
Solomon Instances - *c1.50* instances

<table>
<thead>
<tr>
<th>problem</th>
<th>Rich/Cook</th>
<th>Galati</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>nodes</td>
<td>cols</td>
</tr>
<tr>
<td>c101.50</td>
<td>1</td>
<td>956</td>
</tr>
<tr>
<td>c102.50</td>
<td>1</td>
<td>1150</td>
</tr>
<tr>
<td>c103.50</td>
<td>1</td>
<td>1192</td>
</tr>
<tr>
<td>c104.50</td>
<td>4</td>
<td>2043</td>
</tr>
<tr>
<td>c105.50</td>
<td>1</td>
<td>1222</td>
</tr>
<tr>
<td>c106.50</td>
<td>1</td>
<td>744</td>
</tr>
<tr>
<td>c107.50</td>
<td>1</td>
<td>1176</td>
</tr>
<tr>
<td>c108.50</td>
<td>1</td>
<td>1142</td>
</tr>
<tr>
<td>c109.50</td>
<td>1</td>
<td>999</td>
</tr>
<tr>
<td>Total</td>
<td>12</td>
<td>10624</td>
</tr>
<tr>
<td>Total/104</td>
<td>7</td>
<td>7625</td>
</tr>
</tbody>
</table>
### Computational Results

**Solomon Instances - r1.25 instances**

<table>
<thead>
<tr>
<th>problem</th>
<th>Rich/Cook</th>
<th>Galati</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>nodes</td>
<td>cols</td>
</tr>
<tr>
<td>r101.25</td>
<td>1</td>
<td>84</td>
</tr>
<tr>
<td>r102.25</td>
<td>1</td>
<td>152</td>
</tr>
<tr>
<td>r103.25</td>
<td>1</td>
<td>218</td>
</tr>
<tr>
<td>r104.25</td>
<td>1</td>
<td>277</td>
</tr>
<tr>
<td>r105.25</td>
<td>1</td>
<td>124</td>
</tr>
<tr>
<td>r106.25</td>
<td>1</td>
<td>236</td>
</tr>
<tr>
<td>r107.25</td>
<td>1</td>
<td>278</td>
</tr>
<tr>
<td>r108.25</td>
<td>2</td>
<td>305</td>
</tr>
<tr>
<td>r109.25</td>
<td>1</td>
<td>192</td>
</tr>
<tr>
<td>r110.25</td>
<td>16</td>
<td>603</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>26</td>
<td>2469</td>
</tr>
</tbody>
</table>
## Computational Results

### Solomon Instances - *r1.50* instances

<table>
<thead>
<tr>
<th>problem</th>
<th>nodes</th>
<th>cols</th>
<th>cuts</th>
<th>time</th>
<th>nodes</th>
<th>cols</th>
<th>cuts</th>
<th>time</th>
</tr>
</thead>
<tbody>
<tr>
<td>r101.50</td>
<td>1</td>
<td>292</td>
<td>1</td>
<td>1.67</td>
<td>3</td>
<td>680</td>
<td>1</td>
<td>1.46</td>
</tr>
<tr>
<td>r102.50</td>
<td>1</td>
<td>418</td>
<td>0</td>
<td>1.43</td>
<td>1</td>
<td>1847</td>
<td>0</td>
<td>1.19</td>
</tr>
<tr>
<td>r103.50</td>
<td>49</td>
<td>2051</td>
<td>2</td>
<td>47.37</td>
<td>182</td>
<td>10431</td>
<td>0</td>
<td>78.56</td>
</tr>
<tr>
<td>r105.50</td>
<td>8</td>
<td>609</td>
<td>2</td>
<td>5.51</td>
<td>68</td>
<td>1929</td>
<td>15</td>
<td>29.00</td>
</tr>
<tr>
<td>r106.50</td>
<td>1</td>
<td>499</td>
<td>5</td>
<td>4.19</td>
<td>1</td>
<td>1584</td>
<td>3</td>
<td>2.11</td>
</tr>
<tr>
<td>r107.50</td>
<td>42</td>
<td>2262</td>
<td>3</td>
<td>67.07</td>
<td>180</td>
<td>10997</td>
<td>0</td>
<td>89.79</td>
</tr>
<tr>
<td>r109.50</td>
<td>140</td>
<td>1955</td>
<td>8</td>
<td>91.88</td>
<td>1200</td>
<td>25340</td>
<td>0</td>
<td>271.00</td>
</tr>
<tr>
<td>r110.50</td>
<td>3</td>
<td>666</td>
<td>3</td>
<td>12.16</td>
<td>690</td>
<td>18098</td>
<td>0</td>
<td>213.00</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>245</td>
<td>8752</td>
<td>24</td>
<td>231.28</td>
<td>2325</td>
<td>70906</td>
<td>19</td>
<td>686.11</td>
</tr>
<tr>
<td>r104.50</td>
<td>74</td>
<td>3382</td>
<td>12</td>
<td>489.87</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>3600.00</td>
</tr>
<tr>
<td>108.50</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>
École Polytechnique Fédérale de Lausanne, Suisse (EPFL)

Galenica Pharmaceutical - Logistics

Vehicle Routing Problem with Time Windows (VRPTW)

Branch Cut and Price (BCP)

BCP for VRPTW
  - Formulation
  - Pricing Problem
  - Cutting Planes
  - Branching

Implementation of BCP for VRPTW

(Preliminary) Computational Results

Current Research
**Current Research**

- **DECOMP**: A general separation algorithm for large-scale discrete optimization problems using decomposition methods.

- Optimization Seminar Series - [http://sagan.ie.lehigh.edu/ipseminar](http://sagan.ie.lehigh.edu/ipseminar)