Measuring progress in branch-and-bound MILP algorithms

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Overview

- Reasons to measure progress of branch-and-bound
- Current measures
- Some graphical representations
- A weighted sum measure of progress
- Conclusions and future work

Reasons to measure the progress of branch-and-bound

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- How good is the best solution so far?
- How much longer until we have a proven optimal solution?
- How likely is it that a better solution will be found, and how much better will it be?
- Should we change any algorithm strategies? (branching, node selection, cuts, ...)

Current measures

- Optimality gap

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- Predicted tree size

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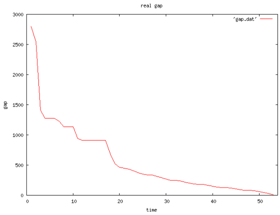
- Number of active nodes
- Predicted tree size
- Some internal measures used for guiding the algorithm

Optimality gap: strengths and weaknesses



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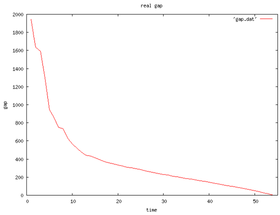

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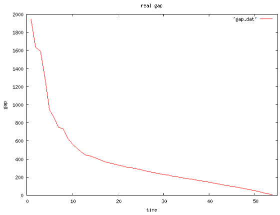

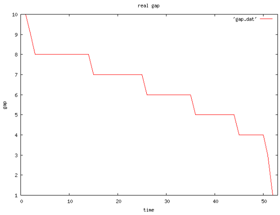

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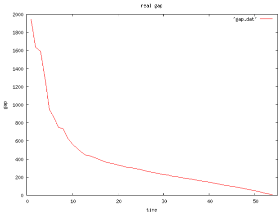

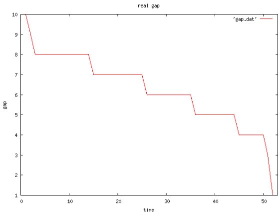

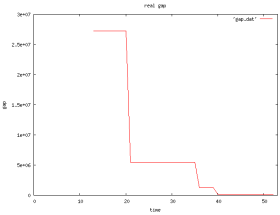

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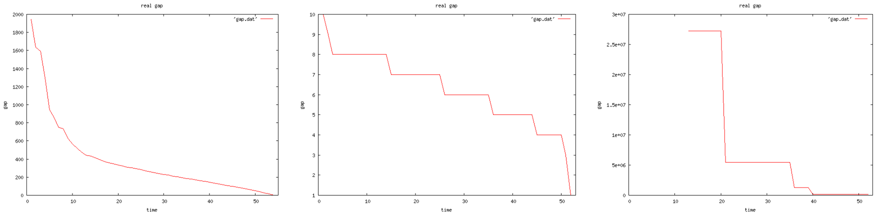

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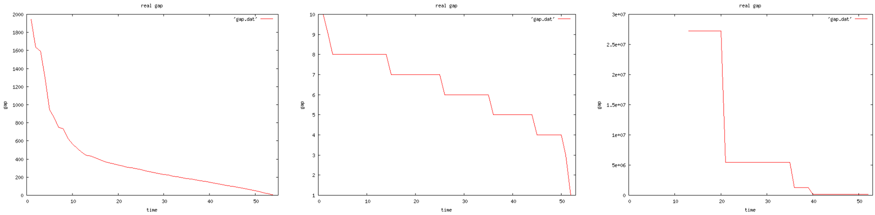

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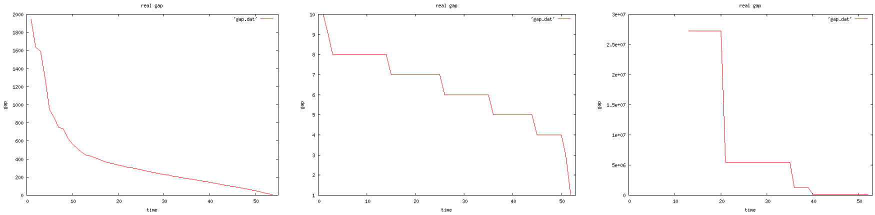
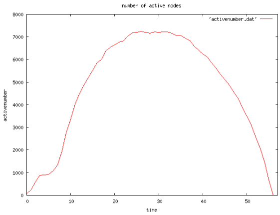


- Strength: guarantee on quality of solution
- Strength: nonincreasing
- Weakness: may remain constant for long periods, then drop suddenly

Number of active nodes: strengths and weaknesses



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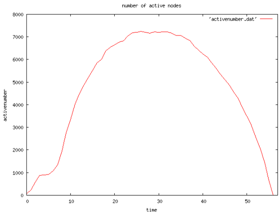

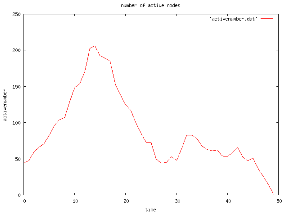

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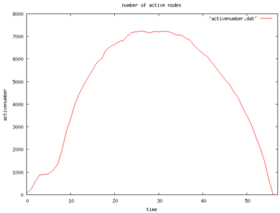

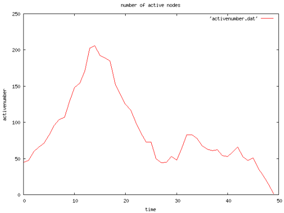

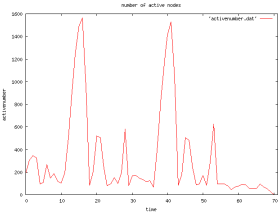

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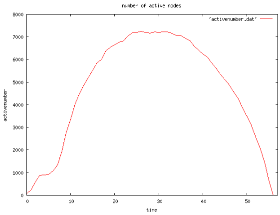

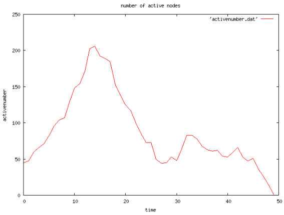

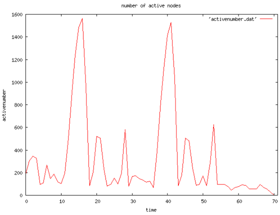

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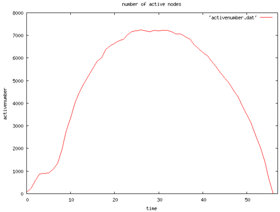

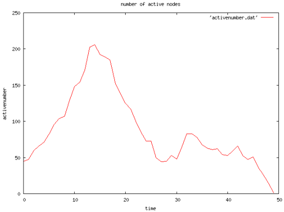

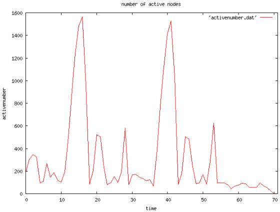

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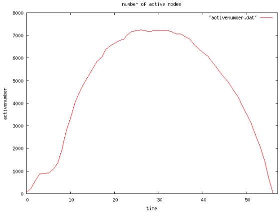

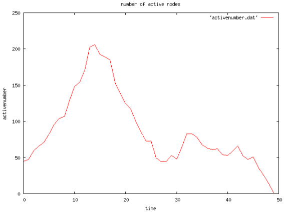

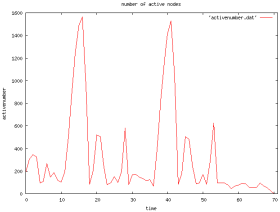

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- Strength: Addresses a key question
- Weakness: Estimate is based on a common tree shape, but this tree shape depends on specific algorithm implementation and parameters

Possible goals in analyzing b&b progress

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- Find a good measure of progress

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- For each active node:

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- So far, we have considered several instances from MIPLIB 3 that take more than 30 seconds but less than an hour

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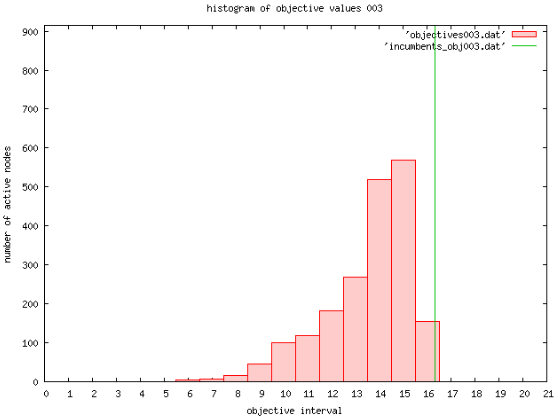
Histogram of active node LP bounds



- Horizontal axis is the LP bound bins
- Vertical axis is number of active nodes
- Green vertical line is the current incumbent value

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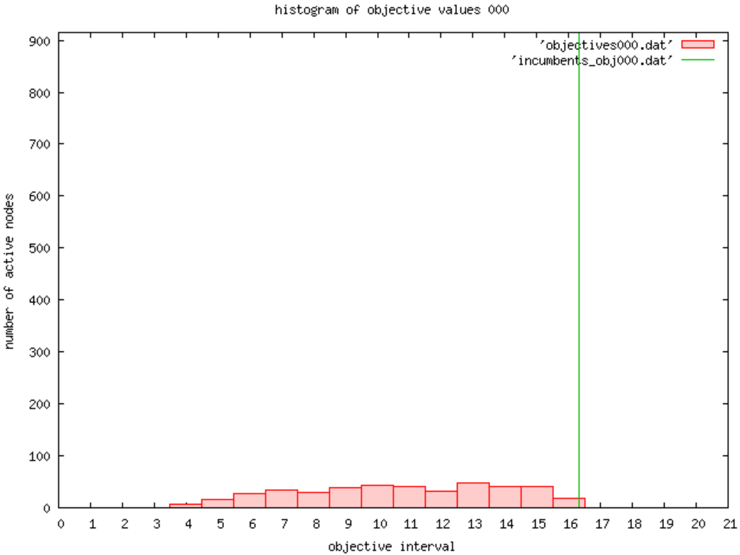

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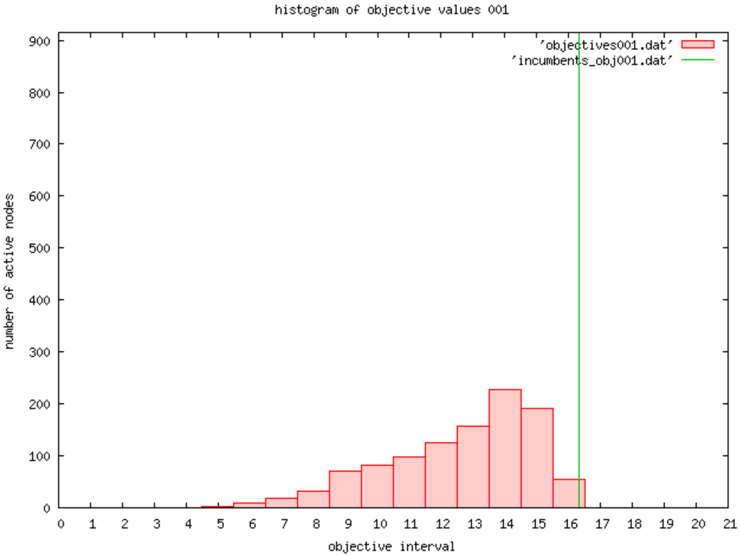

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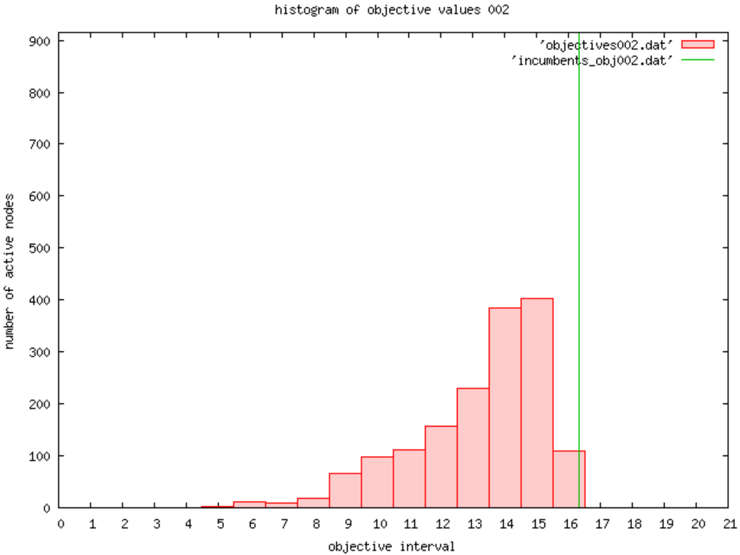

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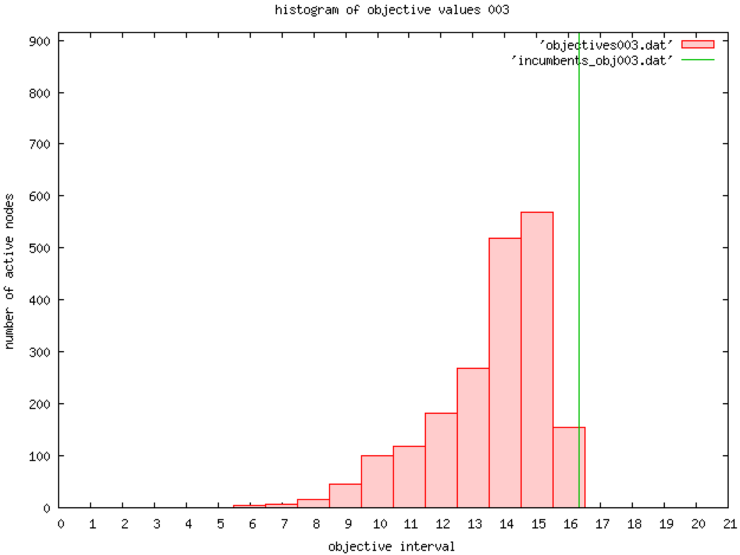

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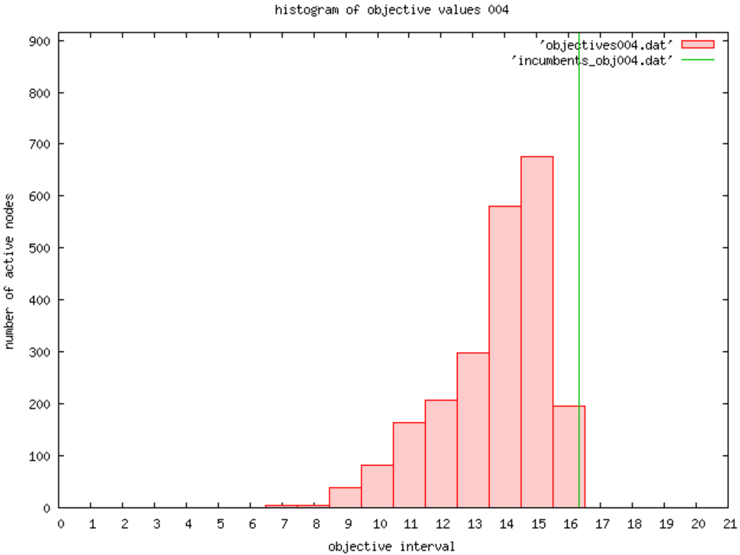

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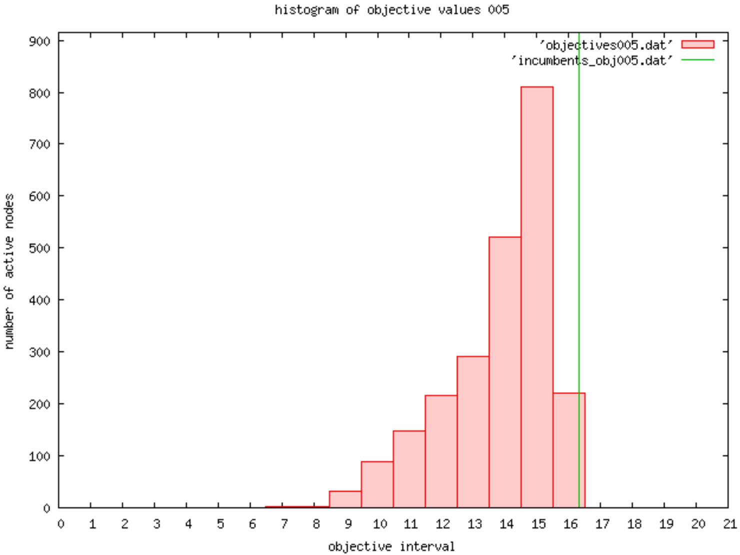

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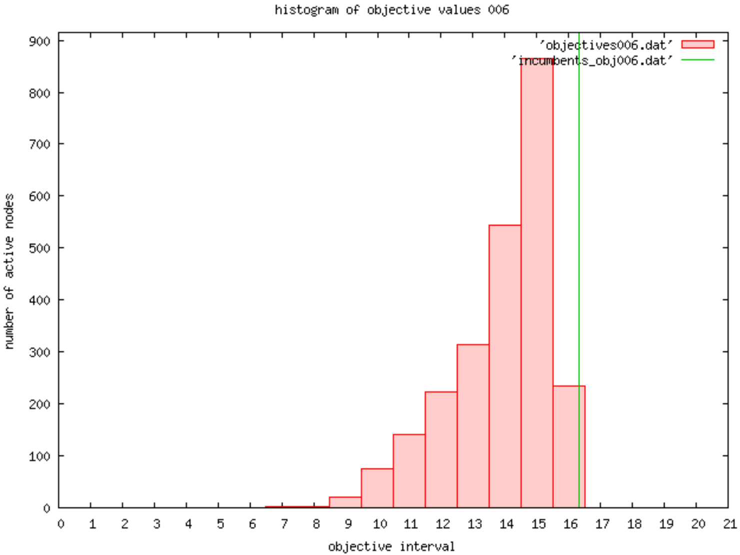

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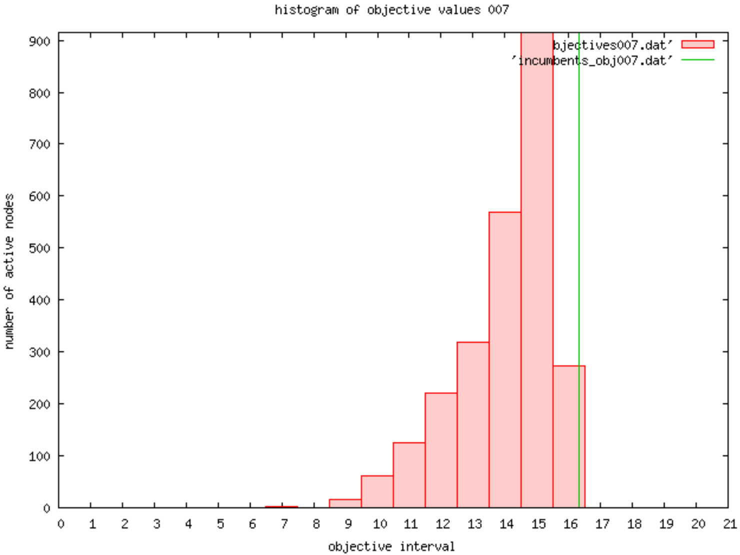

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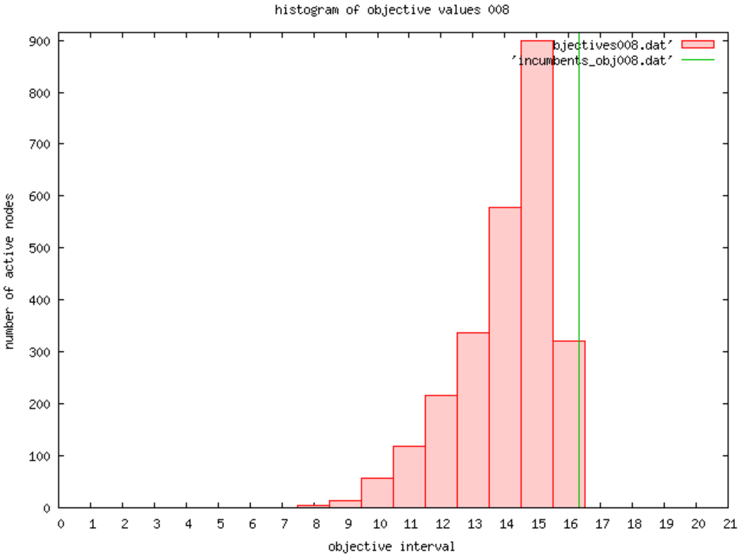

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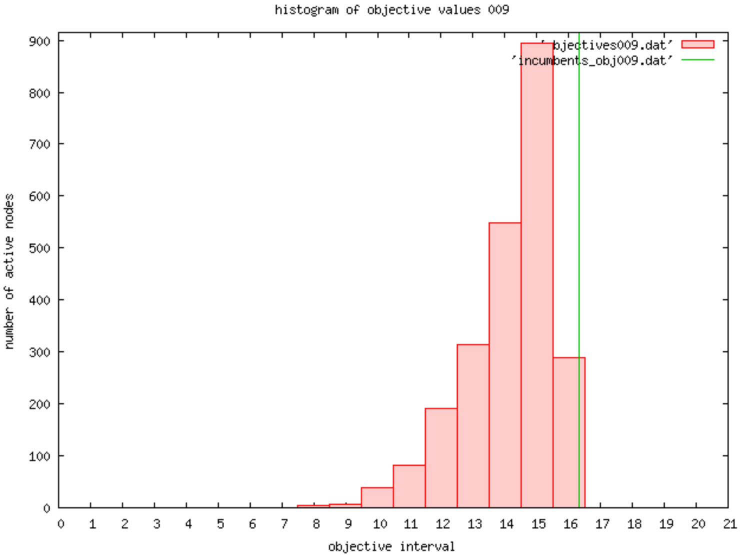

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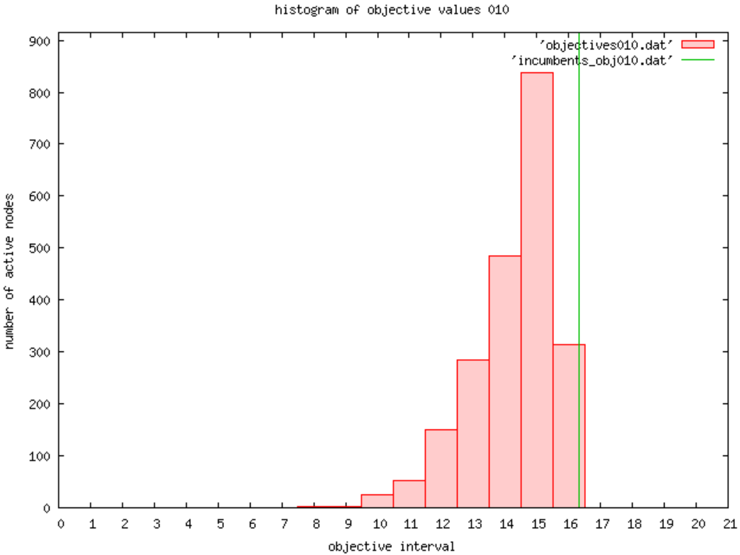

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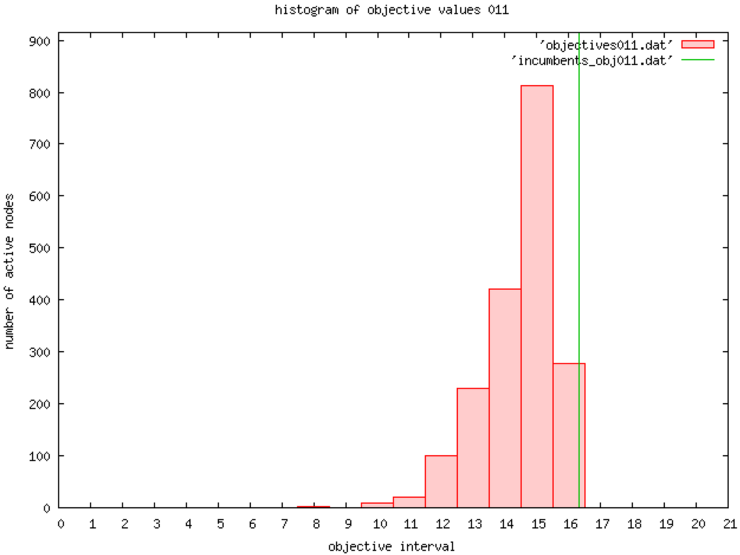

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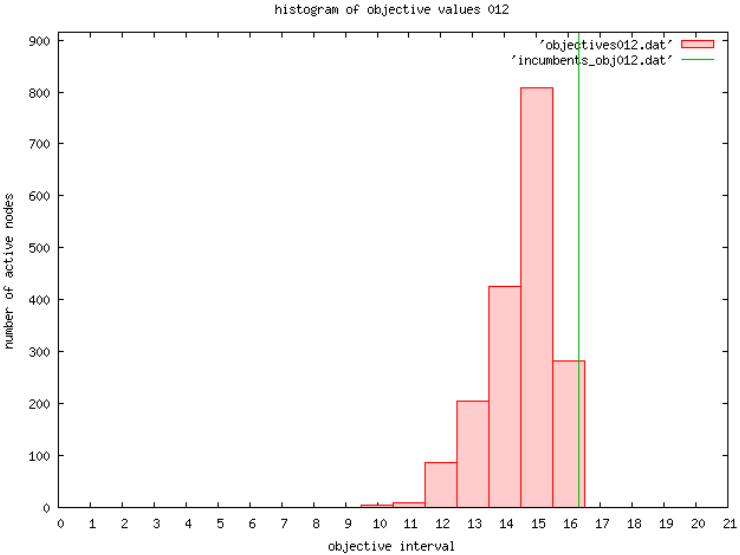

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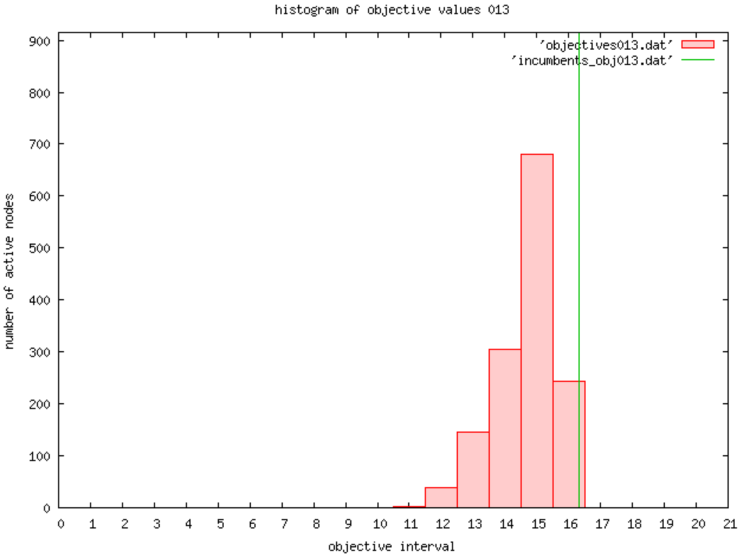

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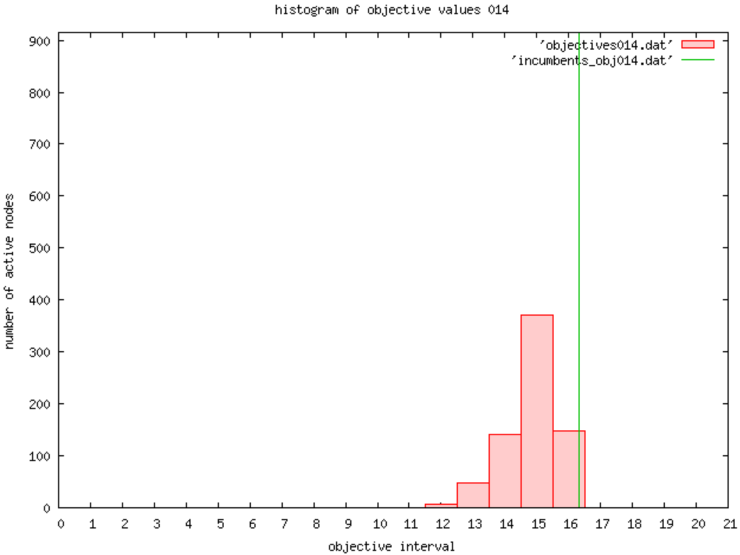

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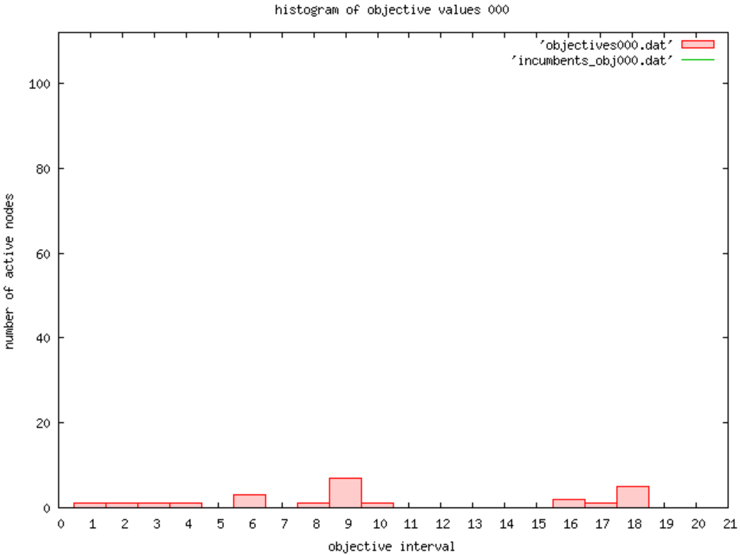

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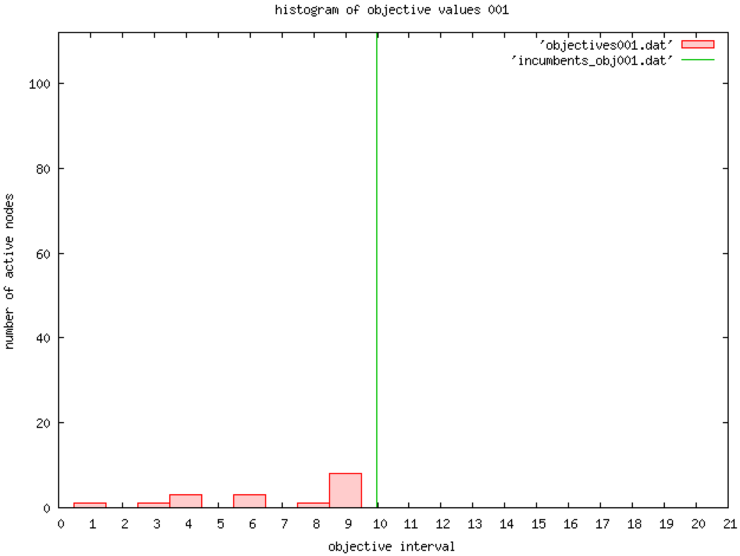

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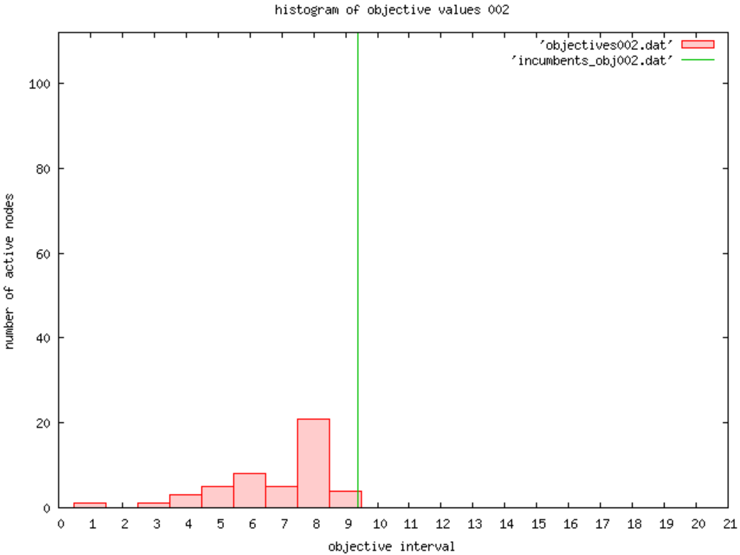

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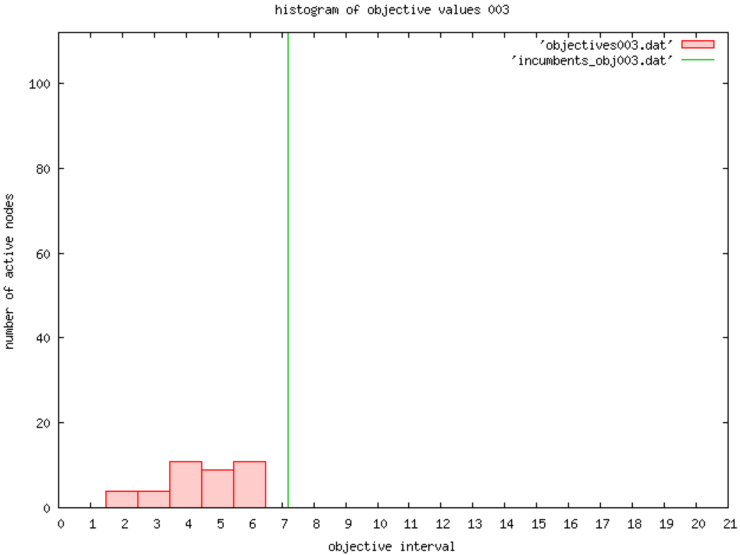

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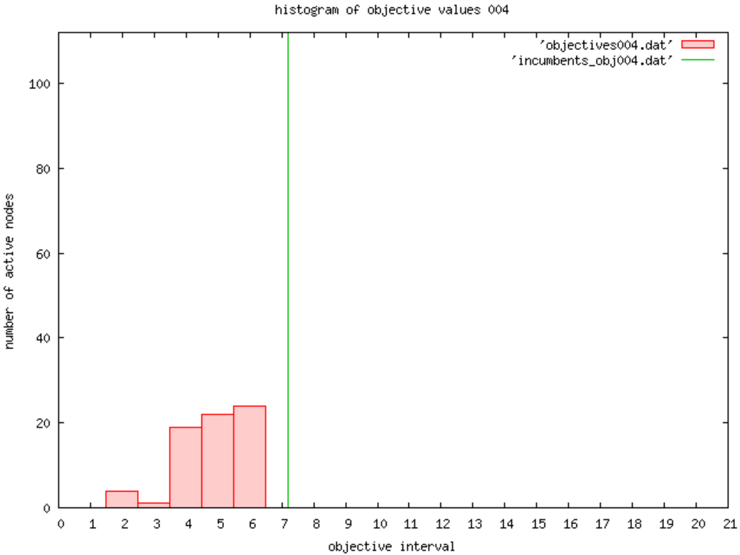

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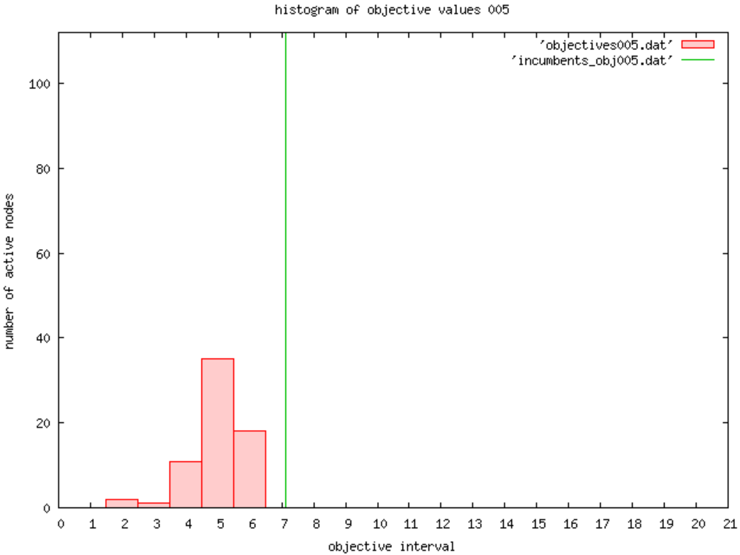

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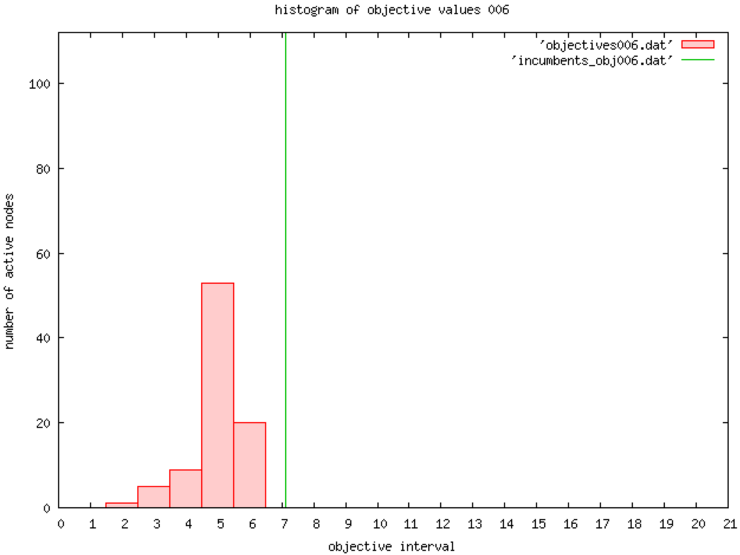

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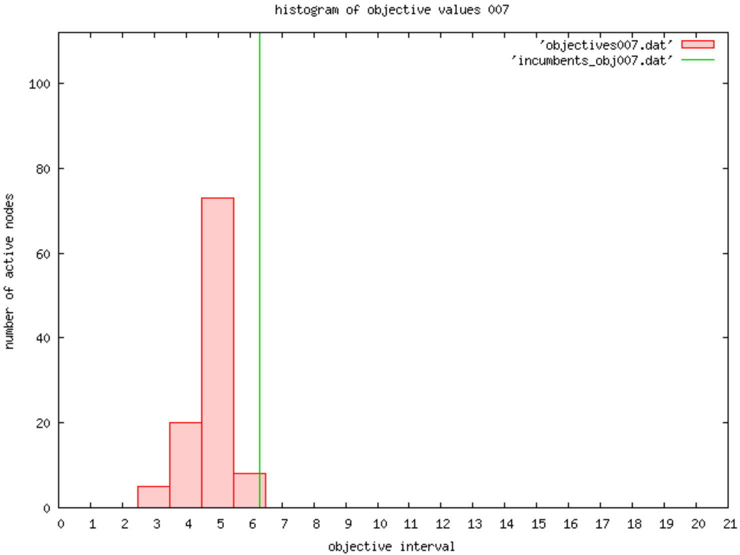

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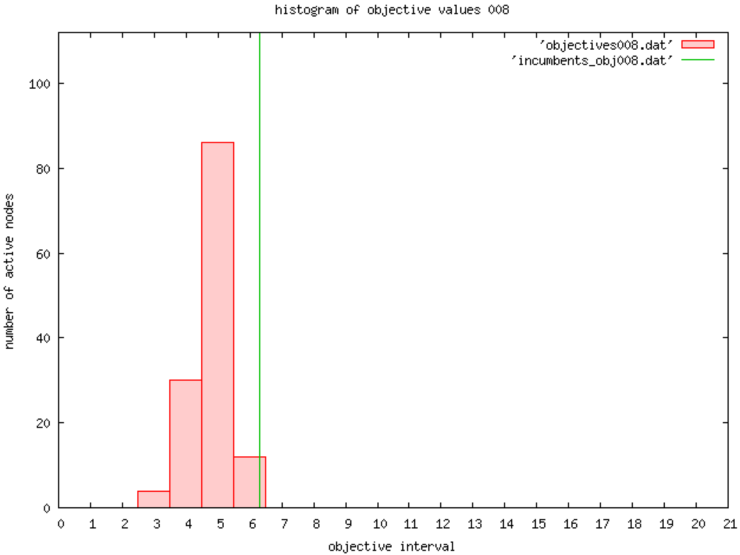

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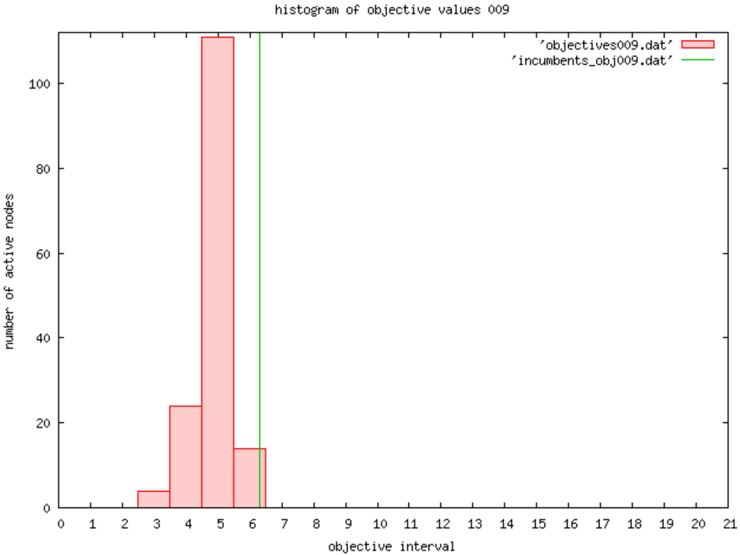

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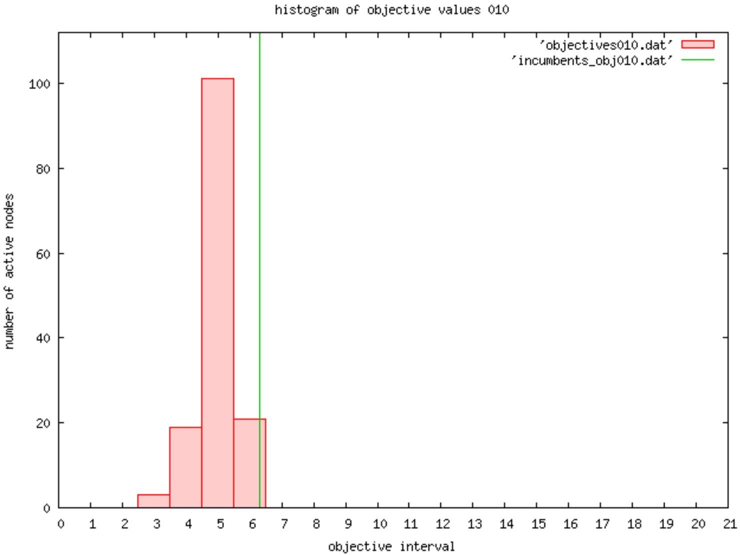

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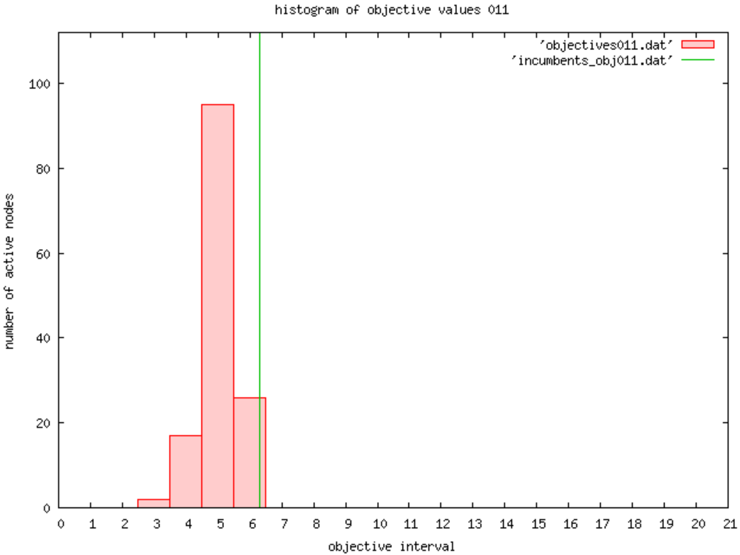

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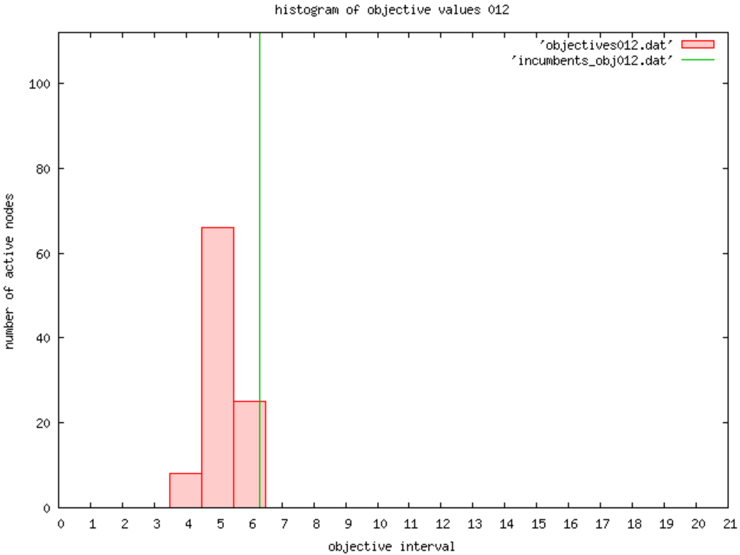

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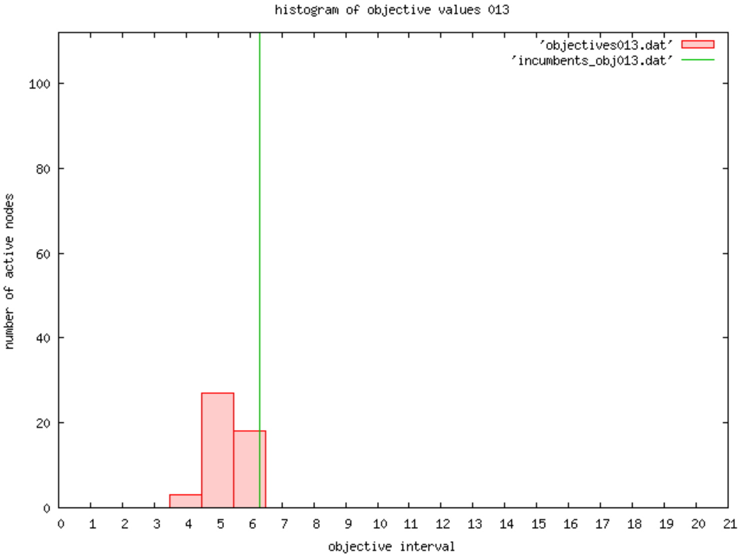

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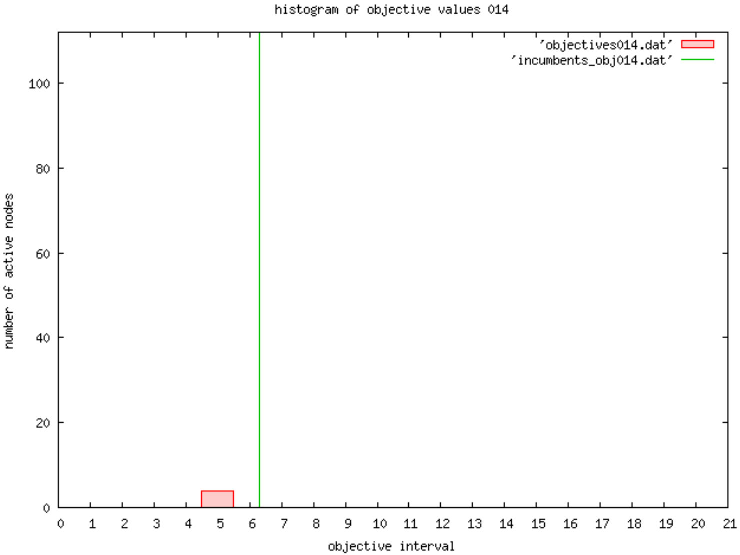
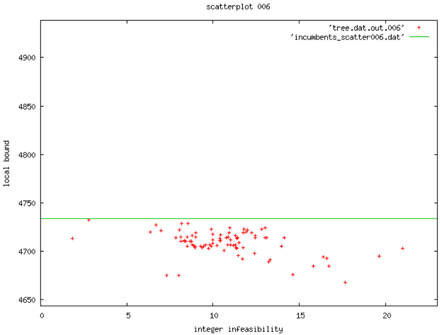
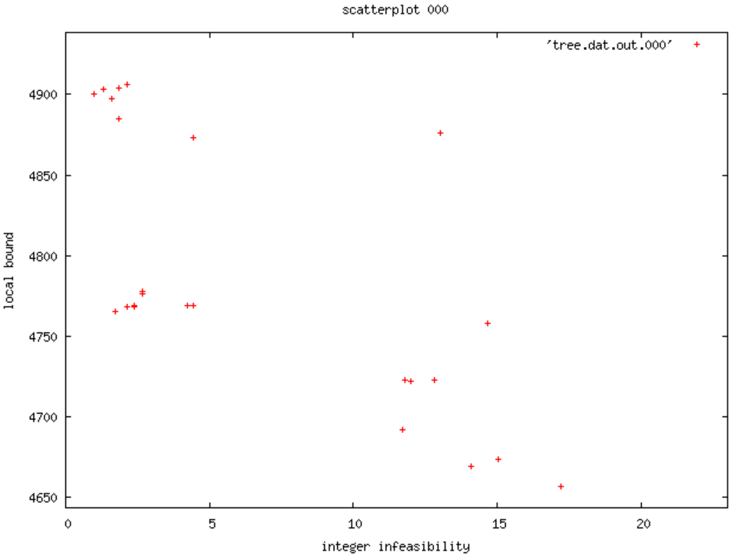


Scatterplot of active node LP bounds and integer infeasibility



- Points represent active nodes
- Vertical axis is the LP bound
- Horizontal axis is the sum of integer infeasibilities
- Green horizontal line is the current incumbent value

Example scatterplot series



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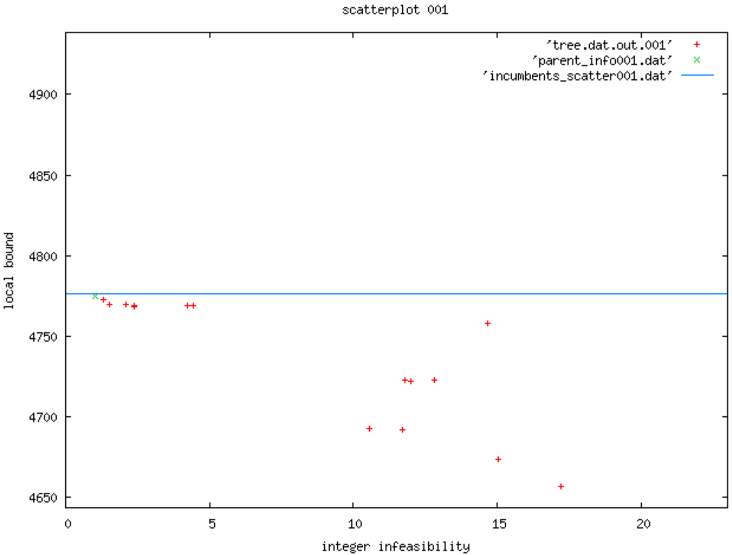

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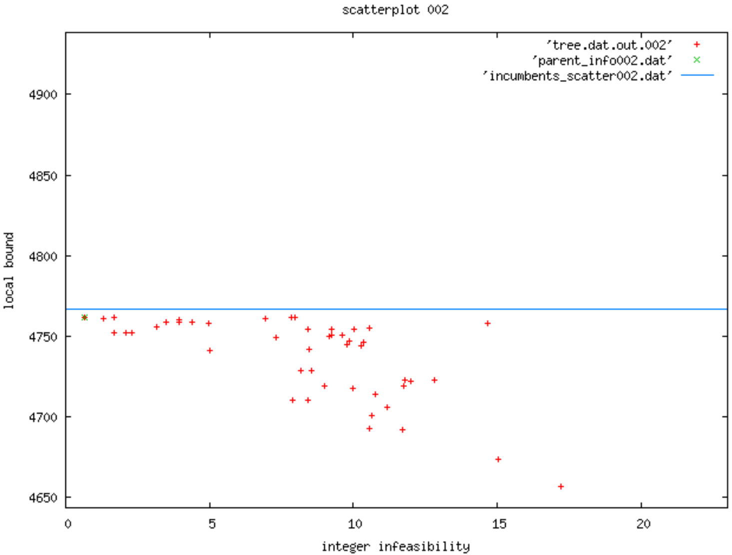

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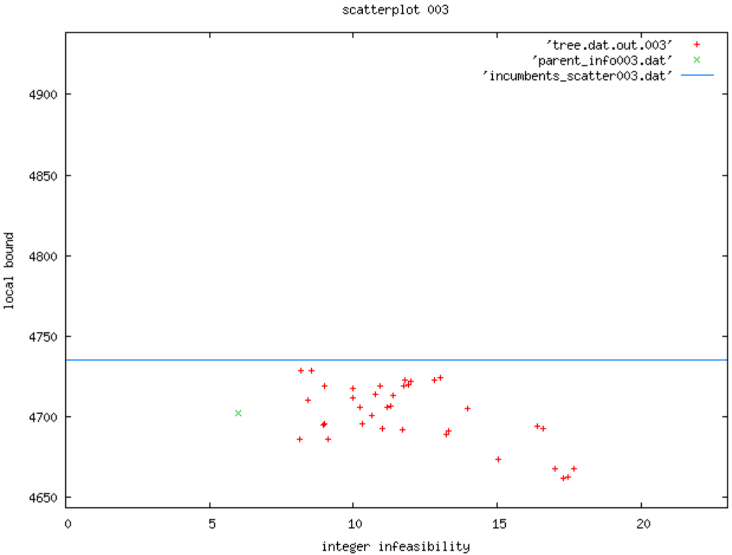

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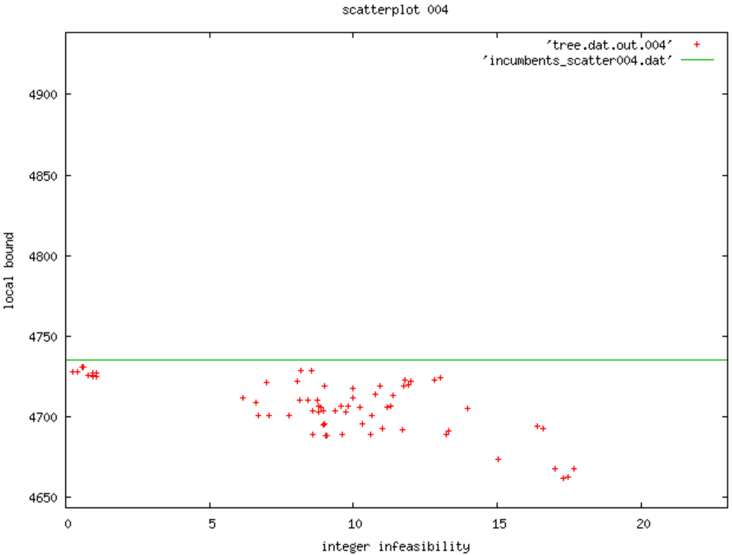

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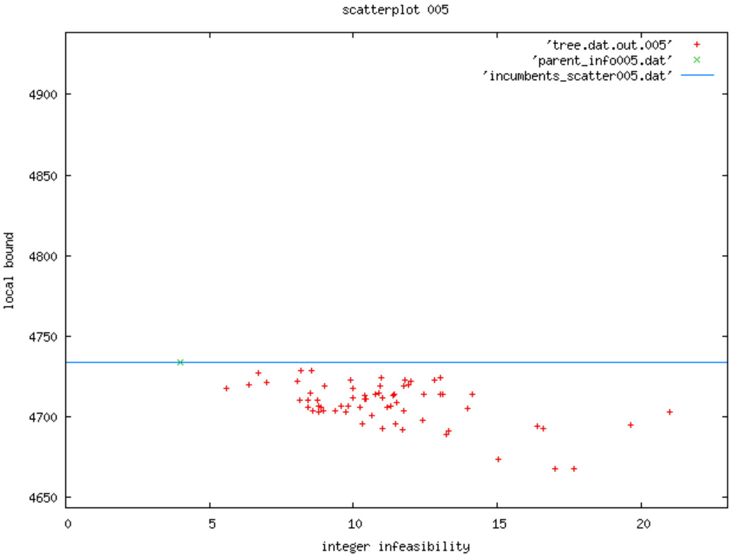

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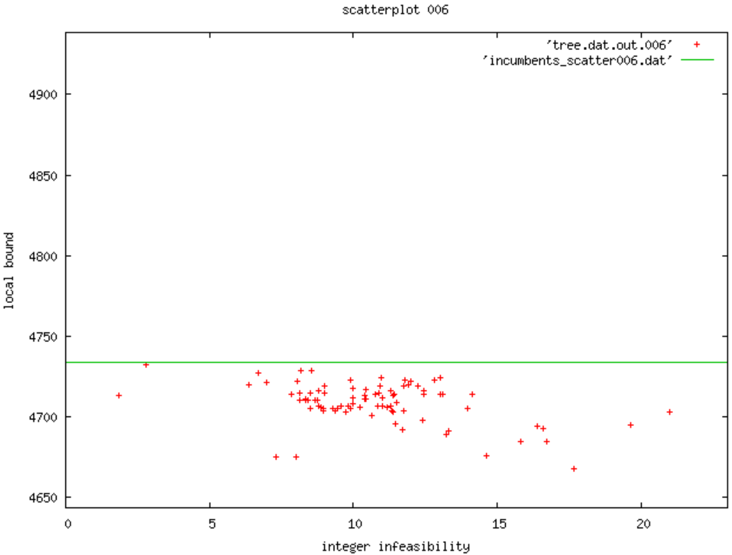

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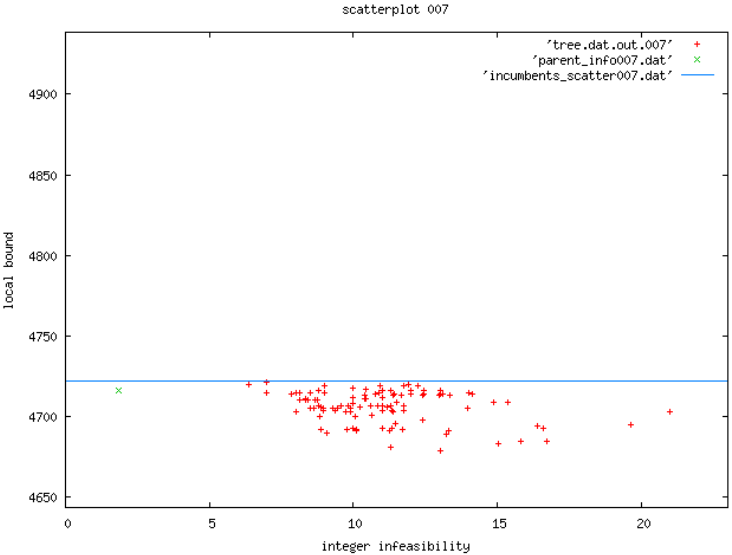

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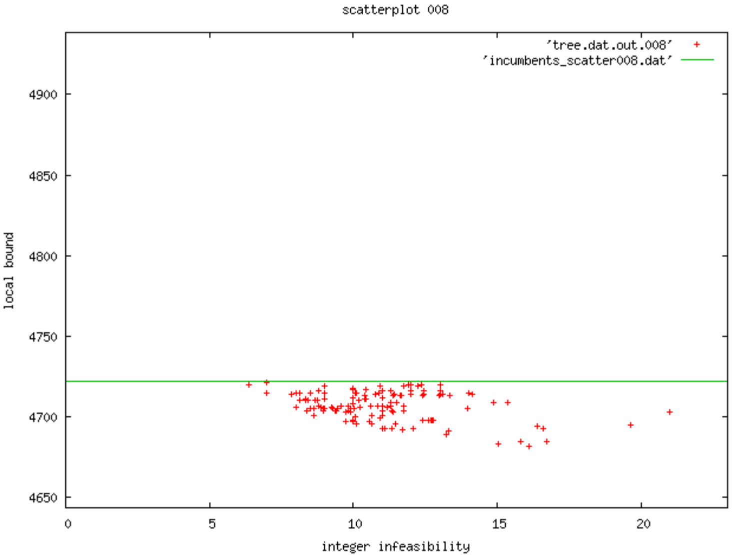

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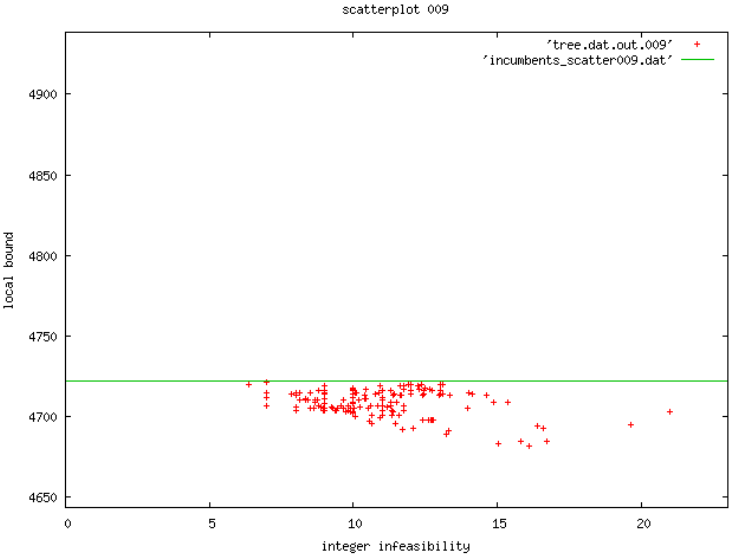

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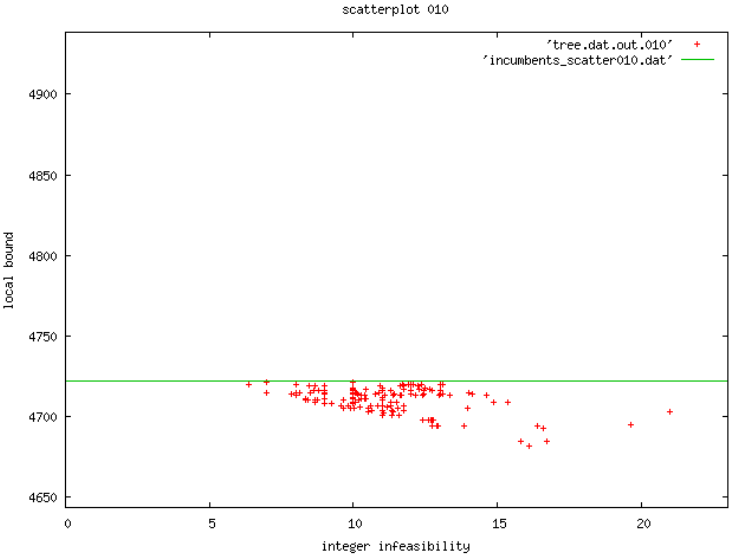

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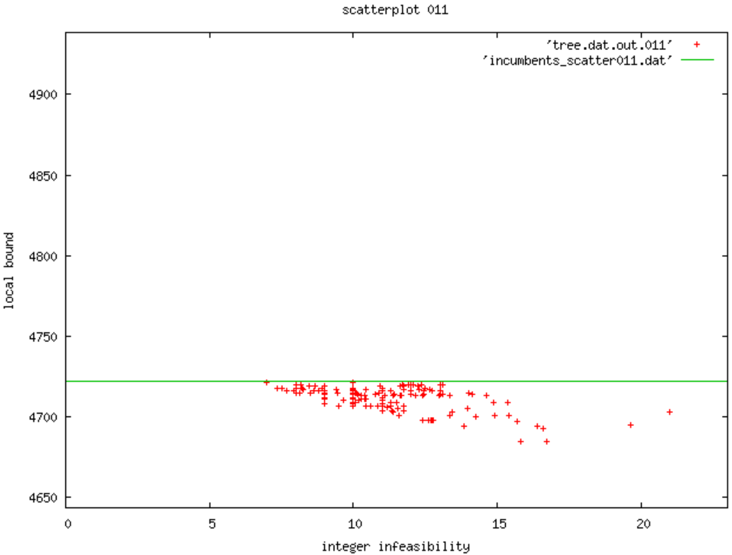

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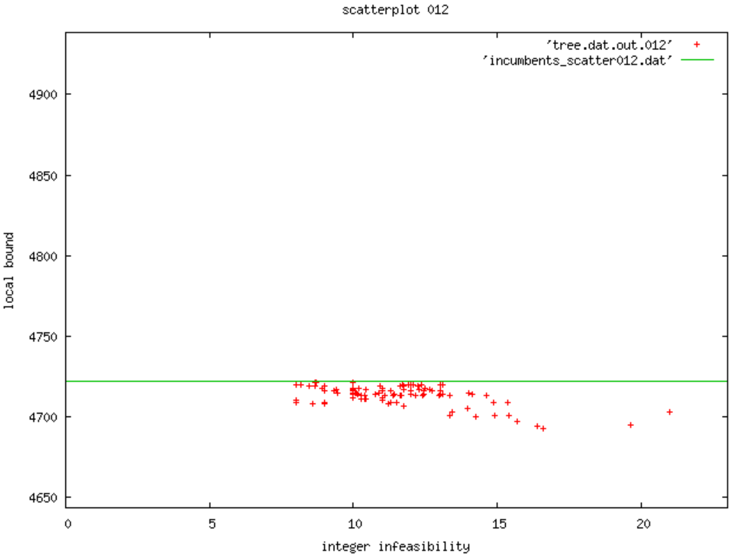

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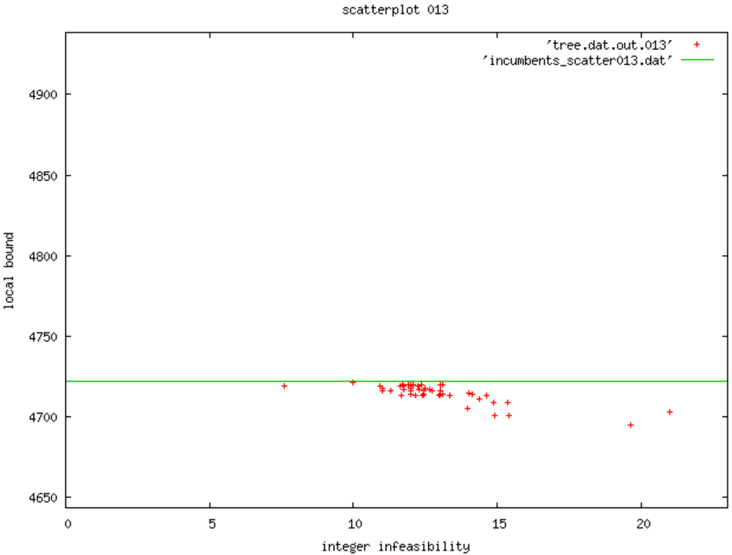

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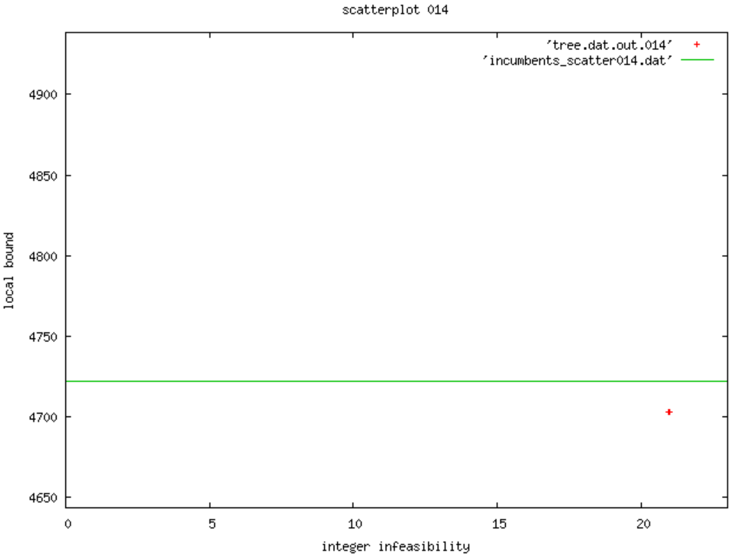
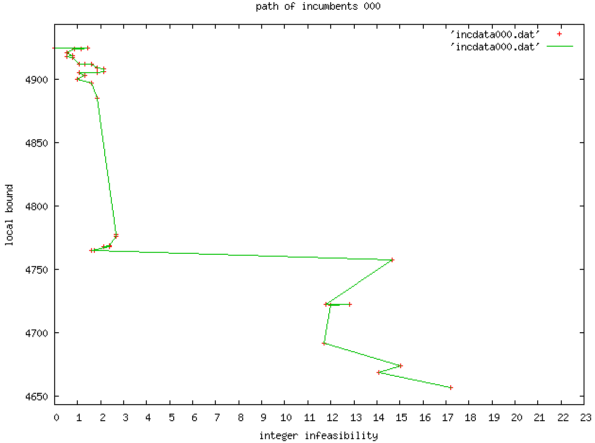
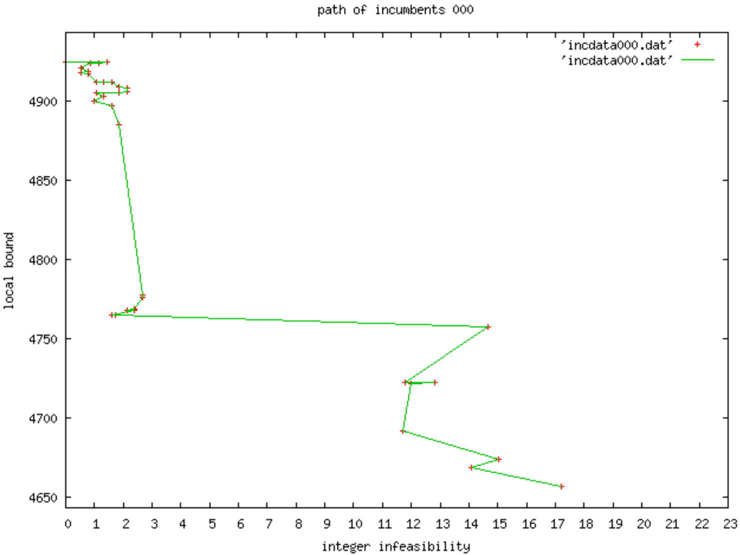


History of active nodes



- Shows the ancestors of the node

Example histories of active nodes



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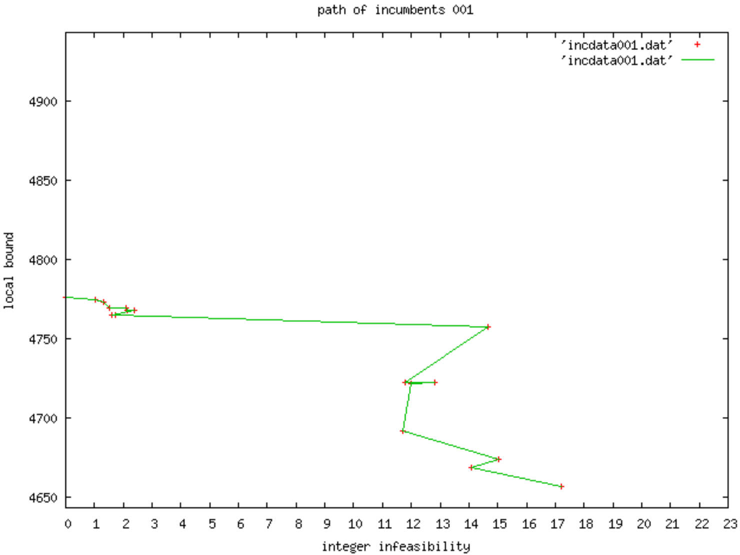

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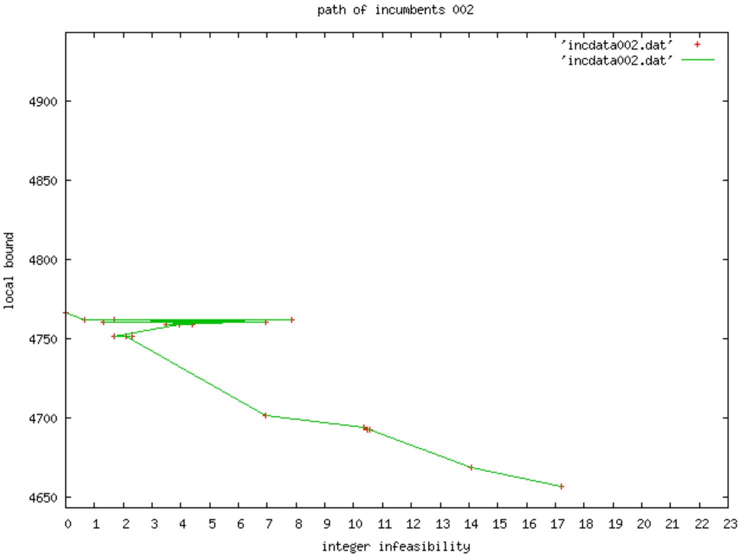

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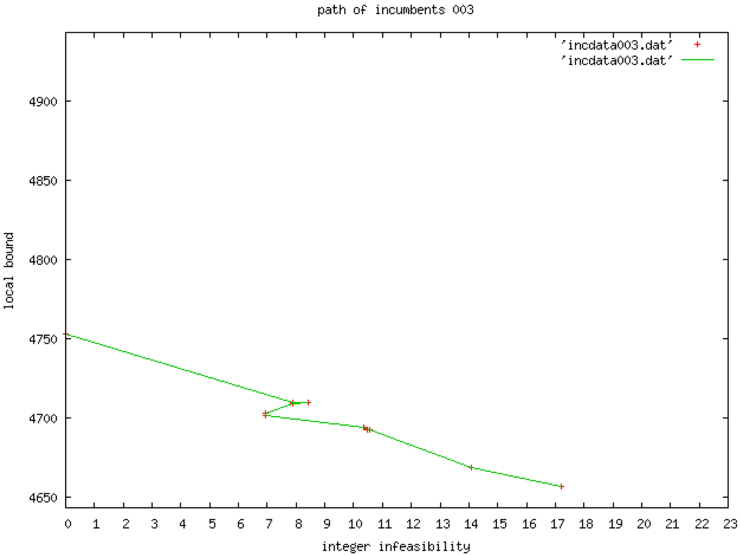

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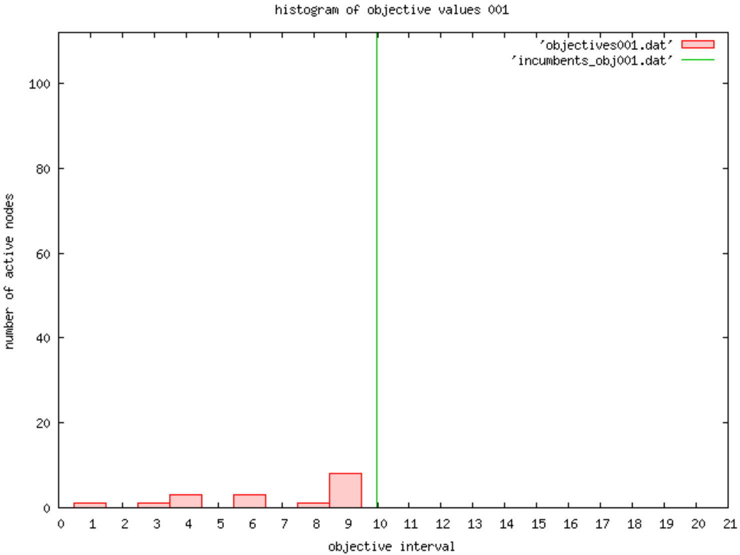

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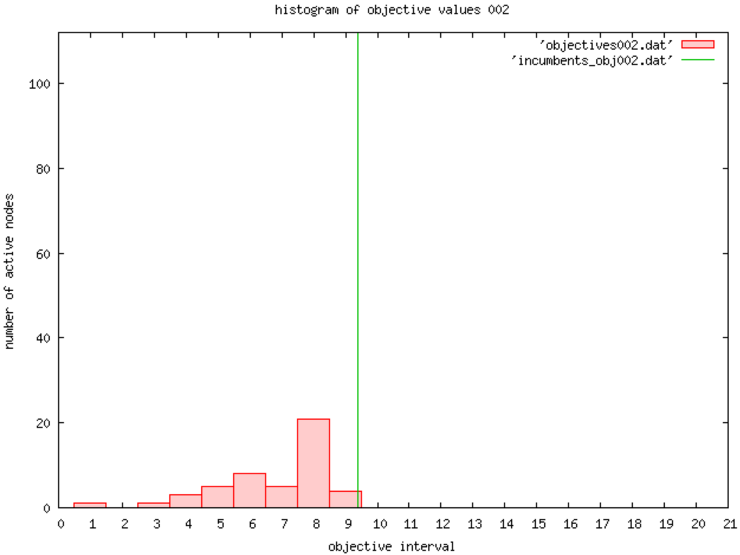

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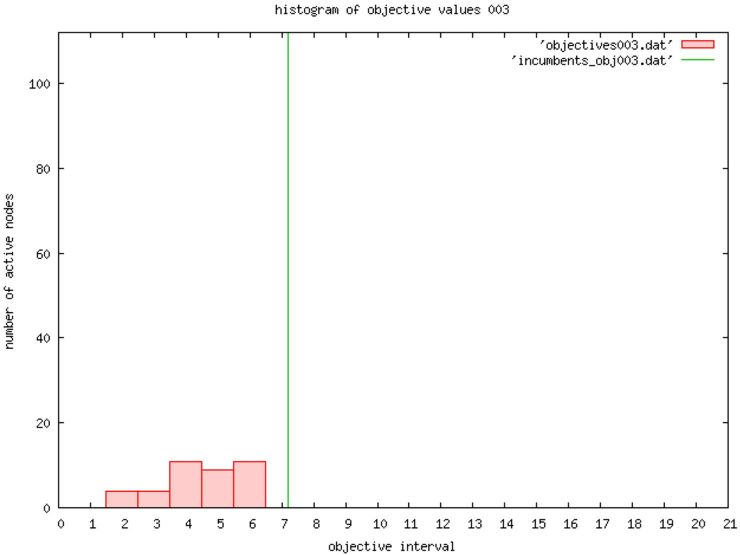

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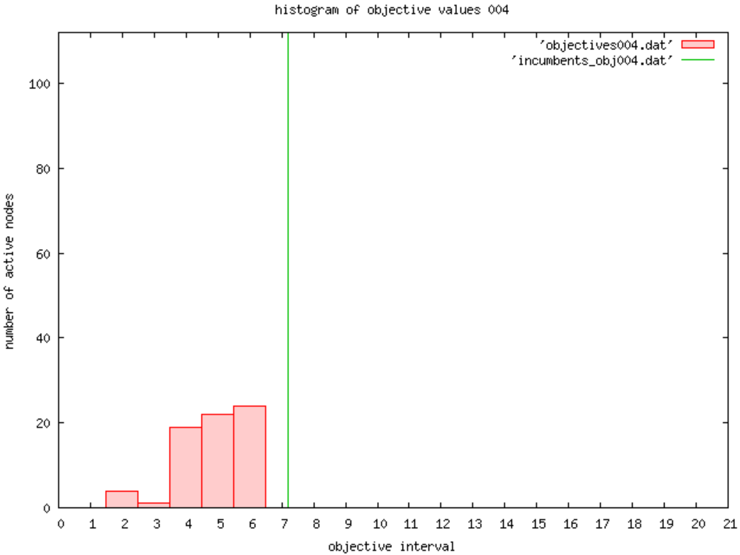

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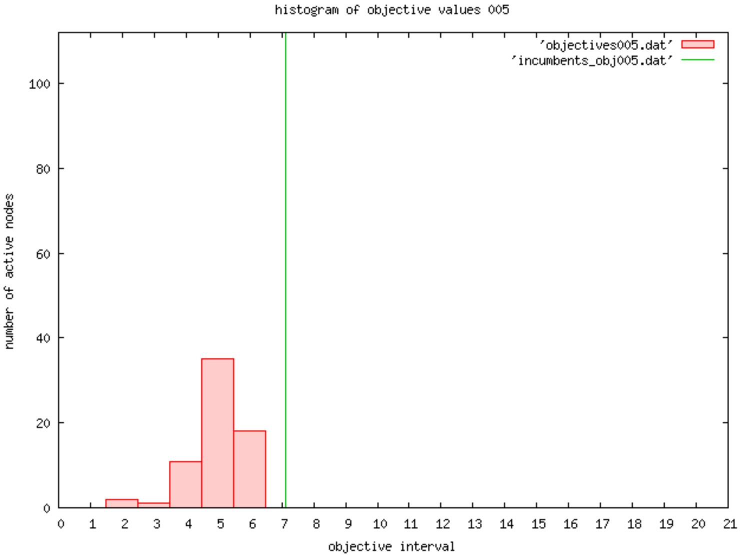

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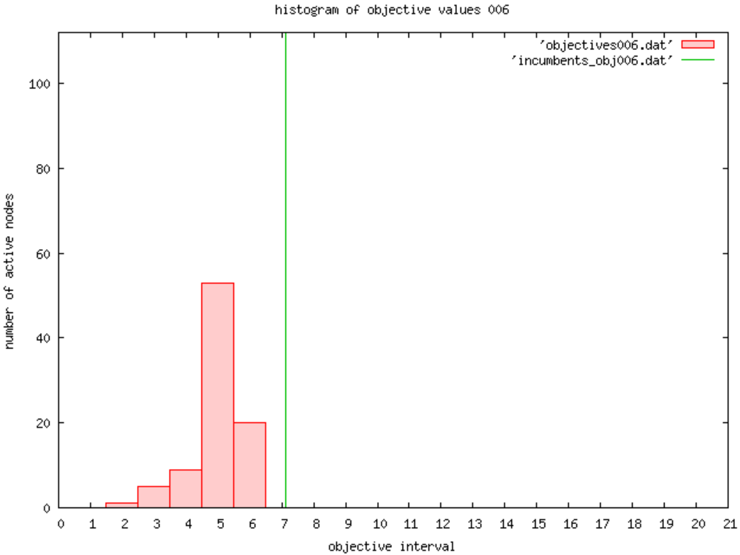

Measure of progress: Weighted sum of active node gaps

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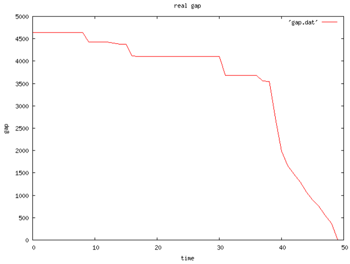
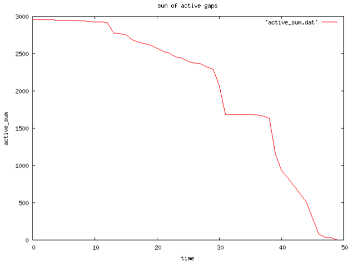
  - Therefore: Monotonic decreasing (as long as lp bounds of parent and child differ)

Example graphics



bell3a, CBC no cuts



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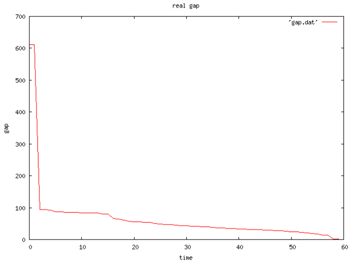
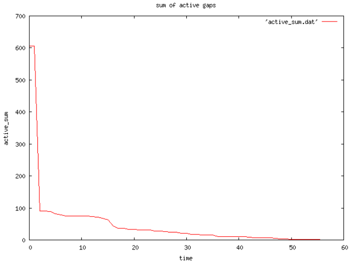


l152lav, CBC default



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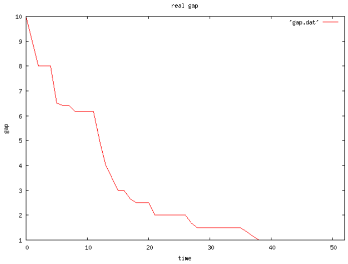
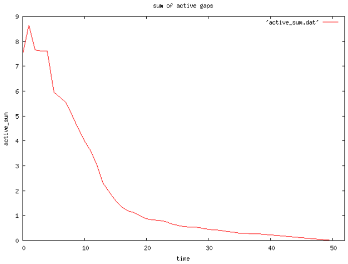


stein45, CBC default



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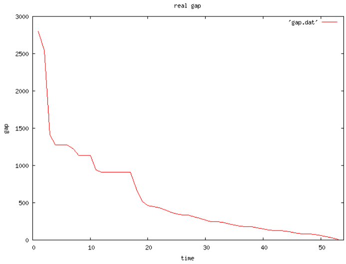
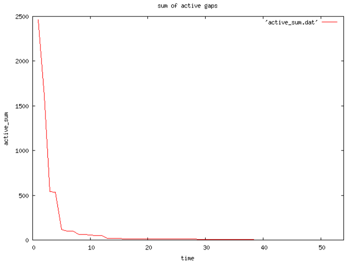


misc07, CBC default



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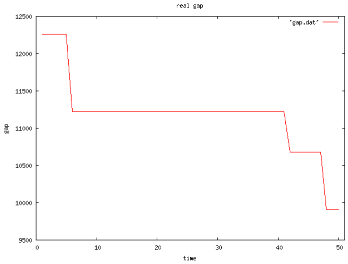
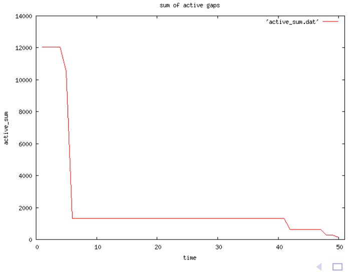


bell3a, GLPK intopt no cuts



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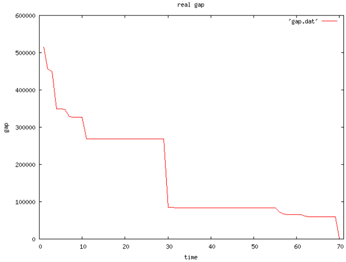
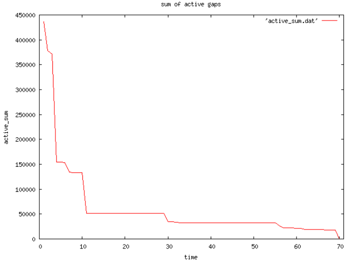
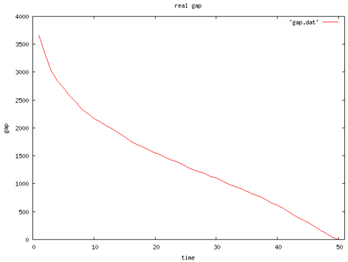


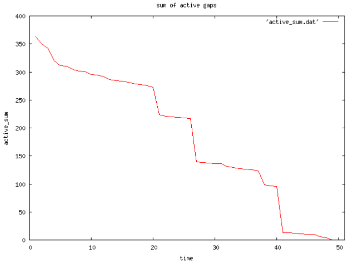
bell5, GLPK intopt no cuts



Example graphics



bell3a, GLPK standard, best bound



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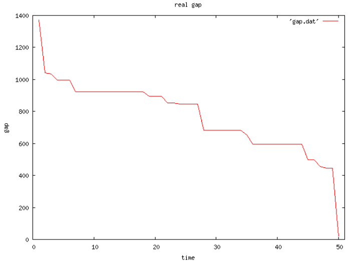
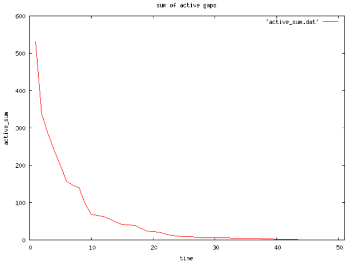


misc07, GLPK standard



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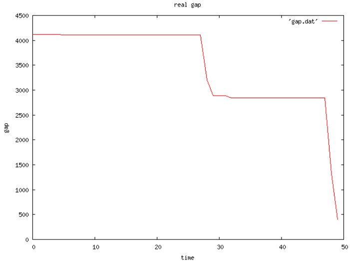

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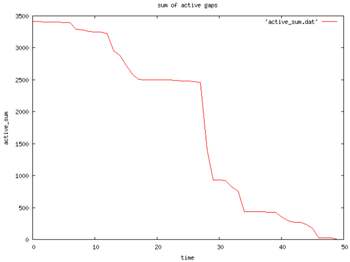

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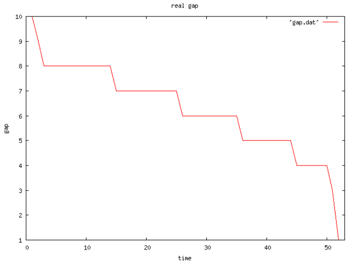
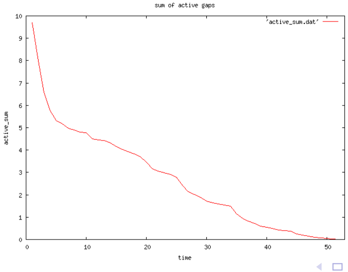


stein45, GLPK intopt with cuts



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- Weakness: still drops significantly when new incumbents found

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- To smooth the graph more, one approach is to anticipate new incumbents

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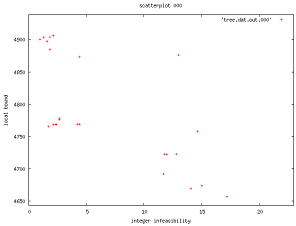

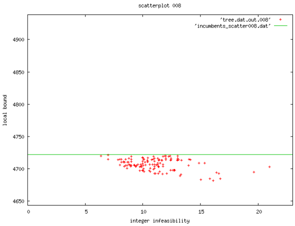

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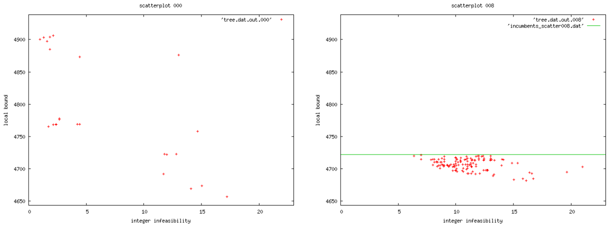

- However, the predictions do not appear to be consistently accurate, especially for big drops

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- Value of open-source codes: proven useful on real-world problems and allow full and easy access to information available during the algorithm

Current and Future work

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- Can other information be extracted: recommended node selection strategy or cuts?