Exploiting orbits in symmetric IL

A review of François Margot's pap

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Preliminaries

- Let Π^n be the set of all permutations of $I^n = \{1, \ldots$
- Π^n is the symmetric group o I^n
- $\pi \in \Pi$ is an n-vector
- $\pi[i]$ is the image of i under π:
- let w be the vector obtained by permuting v accor

$$w[\pi[i]] = v[i] \; for \; all \; i \; \in \; I^n$$

Preliminaries

- Consider the following ILP

$$min\ c^T x$$

$$s.t.\ Ax \geq b,$$

$$x \in \{0,1\}^n,$$

- WLOG we can assume A, b, and c are all integers
- Let π be a permutation of n variable, σ a permutat
- Let $A(\pi, \sigma)$ be the matrix obtained from A by perm and columns
- let $G = \{\pi | \pi(c) = c\ and\ \exists\ \sigma\ s.t.\ \sigma(b) = b, A(\pi, \sigma) =$

Preliminaries

- Definition: The orbit of S under G is

$$orb(S, G) = \{S' \subseteq I^n | S' = g(S)\ for\ some$$

- Definition: the stabilizer of S in G is:

$$stab(S, G) = \{g \in G | g(S) = S\}$$

- Denote F_k^a to be the set of variables fixed to k at n
- N^a the set of variables not fixed at node a

Ranked Branching Rule

- Subproblems are isomorphic if $\exists$ a permutation g
  $g(F_k^a) = F_k^b$ for $k = 0, 1$
- Using this definition is difficult
  - ⋆ How do you find g for given nodes a and b?
  - ⋆ This will have to be done a lot

Ranked Branching Rule

- Goal: evaluate a single node, not pairs of nodes
- Let R be a rank vector, indicating the order in whic have been used for branching
- The rule to select the branching variable x_f at a is

$$(i)\ If\ \exists\ j \in N^a withR[j] < n+1,\ then\ f = arg\ mi$$

$$(ii)\ Else,\ choose\ f \in N^a$$

Ranked Branching Rule

- Let $J = \{j_1, \ldots, j_p\}$ be unordered multiset of I^{n+1}
- Let J^* be the ordered multiset formed by listing J non-decreasing order
- Given set $J_i,\ J_j,\ J_i \preceq J_j$ if J_i is lexicographically s
- For a given R, J is a representative of the sets in its min. under G:

$$R(j) \preceq R(g(J))\ \forall g \in G$$

Lemma 1

- Let R_1 and R_2 be two rank vectors obtained durin
cut, assume R_2 is obtained after R_1, then:

◇ (i) If J is not a representative w.r.t. R_1, then J is no
representative w.r.t. R_2

◇ (ii) If J is a rep. w.r.t. R_1 and all entries in R(j) are
J is the unique rep. to R_1

◇ (iii) if J is a rep w.r.t. R_1 and all entries in R(j) are
then J is a rep w.r.t. R_2

Lemma 2

Let $J \subseteq I^n$ be a rep under G w.r.t. R. Let $j' := J - j$ w
$j \in arg\ max\{R[i] | i \in J\}$. Then J' is also a rep w.r.t. R.

- Isomorphism Pruning: If F_1^a is not a representative
  prune node a.

Variable Setting

- At node a, all variables in the orbit of $stab(F_1^a)$ can soon as we know any variable can be set to k
- standard setting algorithms let you set a variable t can show $\exists$ an optimal solution with that variable that does not work, that solution can be pruned by pruning
- If you are able to set $x_i = k$, then for any $g \in G$, w to k