

IE 426

Optimization models and applications

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Evaluation

Homework: 20% (four)
Quiz #1: 10% (\approx Sep 25)
Quiz #2: 15% (\approx Nov 5)
Case study: 25%
Final exam: 30%

Case studies

Groups of three-four people

- ▶ study an Optimization problem,
- ▶ propose a model, and
- ▶ solve it using a tool¹ of their choice.

¹An optimization tool...

Material

- ▶ Select chapters of “Introduction to Operations Research” by F.S. Hillier and G.J. Lieberman, McGraw-Hill: New York, NY, 1990;
- ▶ Select chapters of “Introduction to Mathematical Programming: Applications and Algorithms”, Volume 1, by W.L. Winston and M. Venkataramanan;
- ▶ Select chapters of “Operations Research: Applications and Algorithms” by Wayne L. Winston, PWS-Kent Pub. Co., 1991;
- ▶ three chapters of a book on modeling by Robert Fourer, available for download.
- ▶ modeling language: “AMPL: A Modeling Language for Mathematical Programming” by Robert Fourer, David M. Gay, and Brian W. Kernighan.
- ▶ Some handouts.

Modeling languages

They are similar, and each has its own pros/cons. All have limited version available to students.

Mosel: very nice Graphical User Interface (GUI)

AMPL: preferred. No GUI, but I know it better (read: I can help)

GAMS: Has version with even nicer GUI (Aimms)

Optimization models...

- ▶ Optimization aims at finding the best configuration of processes, systems, products, etc.
- ▶ It relies on a theory developed mostly in the past 50 years
- ▶ Applying Optimization in an industrial, financial, logistic context yields a better use of budget/resources (\$\$\$) or a higher revenue (\$\$\$)

...and applications

Source: <http://www.informs.com>

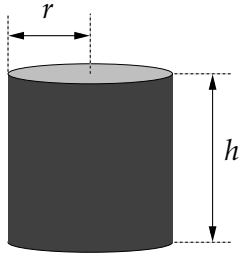
(see also <http://www.ScienceOfBetter.org>)

yr	company	result
86	Eletrobras (hydroelectric energy)	43M\$ saved
90	Taco Bell (human resources)	7.6M\$ saved
92	Harris semicond. prod. planning	50% → 95% orders "on time"
95	GM – Car Rental	+50M\$
96	HP printers — re-designed prod.	2x production
99	IBM — supply chain	750M\$ saved
00	Syngenta — corn production	5M\$ saved

An example

- ▶ You work at a company that sells food in glass containers only. Today, your boss has a bright idea! The **tin can**[®]. It's a cylinder made of tin.
 - ▶ The can must contain $V = 20 \text{ cu.in.}$ (11 fl.oz., 33 cl)
 - ▶ Cut and solder tin foil to produce cans
 - ▶ Tin (foil) is expensive, use as little as possible
- Boss:** "What is the ideal can? Tall and thin or short and fat?"
- You:** A cylinder with volume V using as little tin as possible.

Example



If we knew radius r and height h ,

▶ the volume would be $\pi r^2 h$

▶ qty of tin would be $2\pi r^2 + 2\pi r h$

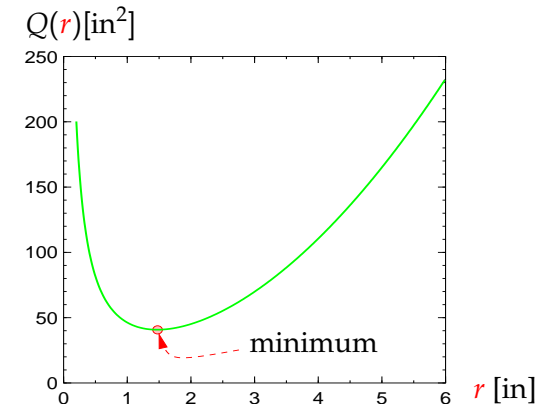
$\pi r^2 h$ must be $V = 20 \text{ in}^3 \Rightarrow h = \frac{V}{\pi r^2}$

Rewrite the quantity of tin as $Q(r) = 2\pi r^2 + 2\pi r \frac{V}{\pi r^2}$, or

$$Q(r) = 2\pi r^2 + \frac{2V}{r}$$

\Rightarrow Find the minimum of $Q(r)$!

Minimize the quantity of tin



$$r = \boxed{1.471 \text{ in}}$$

$$h = \frac{V}{\pi(1.471)^2} = 2.942 \text{ in}$$

Aims of this course

- ▶ model Optimization problems
- ▶ so that they can be solved
- ▶ learn a modeling language
- ▶ apply modeling languages to real-world problems

Your first Optimization model

Variables	r : ray of the can's base h : height of the can
Objective	$2\pi r h + 2\pi r^2$ (minimize)
Constraints	$\pi r^2 h = V$ $h > 0$ $r > 0$

Optimization Models, in general, have:

Variables: Height and radius, number of trucks, ... The *unknown* (and desired) part of the problem (one thing your boss cares about).

Constraints: Physical, explicit ($V = 20\text{in}^3$), imposed by law, budget limits. ... They define **all** and **only** values of the variables that give possible solutions.

Objective function: what the boss really cares about. Quantity of tin, total cost of trucks, total estimated revenue, ... a function of the **variables**

Schedule

- ▶ Models, convexity, relaxations
- ▶ Linear Programming (and Optimization on graphs)
- ▶ Integer Programming (and logical constraints)
- ▶ Nonlinear and Stochastic Programming

Reading for today

- ▶ Chapter 1 of Fourer's book (online), **or**
- ▶ Winston, chapter 1, **or**
- ▶ Winston & Venkataramanan, chapter 1, **or**
- ▶ Hillier & Lieberman, chapter 2.

Next lecture:

- ▶ Convexity
- ▶ Relaxations
- ▶ Lower bounds, Upper bounds