Multi-Echelon Inventory Management

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Outline

- Introduction
  - Overview
  - Network topology
  - Assumptions
  - Deterministic models
- Stochastic models
- Decentralized systems
Overview

- System is composed of **stages** (nodes, sites, …)
- Stages are grouped into **echelons**
- Stages can represent
  - Physical locations
  - BOM
  - Processing activities
Overview

- Stages to the left are *upstream*
- Those to the right are *downstream*
- Downstream stages face customer demand
Network Topology

- Serial system:
Network Topology

Assembly system:
Network Topology

- Distribution system:
Network Topology

- Mixed system:
Assumptions

- **Periodic review**
  - Period = week, month, ...

- **Centralized decision making**
  - Can optimize system globally
  - Later, I will talk about decentralized systems

- **Costs**
  - Holding cost
  - Fixed order cost
  - Stockout cost (vs. service level)
Deterministic Models

- Suppose everything in the system is **deterministic** (not random)
  - Demands, lead times, …
  - Possible to achieve 100% service
- If no fixed costs, explode BOM every period
- If fixed costs are non-negligible, key tradeoff is between fixed and holding costs
  - Multi-echelon version of EOQ
  - MRP systems (optimization component)
Outline

- Introduction
- **Stochastic models**
  - Base-stock model
  - Stochastic multi-echelon systems
  - Strategic safety stock placement
  - Supply uncertainty
- Decentralized systems
Stochastic Models

- Suppose now that demand is stochastic (random)
  - Still assume supply is deterministic
  - Including lead time, yield, …
- I’ll assume:
  - No fixed cost
  - Normally distributed demand: $N(\mu, \sigma^2)$
- Key tradeoff is between holding and stockout costs
The Base-Stock Model

- Single stage (and echelon)
- Excess inventory incurs holding cost of $h$ per unit per period
- Unmet demand is backordered at a cost of $p$ per unit per period
- Stage follows base-stock policy
  - Each period, “order up to” base-stock level, $y$
  - aka order-up-to policy
  - Similar to days-of-supply policy: $y / \mu$ DOS
The Base-Stock Model

- Optimal base-stock level:
  \[ y^* = \mu + z_\alpha \sigma \]

  where \( z_\alpha \) comes from normal distribution and

  \[ \alpha = \frac{p}{p + h} \]

- \( \alpha \) is sometimes called the “newsboy ratio”
Interpretation

\[ y^* = \mu + z_\alpha \sigma \]

- In other words, base-stock level = mean demand + some # of SD’s worth of demand
- # of SD’s depends on relationship between \( h \) and \( p \)
  - As \( h \uparrow \Rightarrow z_\alpha \downarrow \Rightarrow y^* \downarrow \)
  - As \( p \uparrow \Rightarrow z_\alpha \uparrow \Rightarrow y^* \uparrow \)
- If lead time = \( L \):
  \[ y^* = \mu L + z_\alpha \sigma \sqrt{L} \]
Stochastic Multi-Echelon Systems

- Need to set \( y \) at each stage
- Could use base-stock formula
  - But how to quantify lead time?
  - Lead time is stochastic
  - Depends on upstream base-stock level and stochastic demand
- For serial systems, exact algorithms exist
  - Clark-Scarf (1960)
  - But they are cumbersome
An Approximate Method

- Assume that each stage carries sufficient inventory to deliver product within $S$ periods “most of the time”
  - Definition of “most” depends on service level constant, $z_\alpha$
  - $S$ is called the committed service time (CST)
- We simply ignore the times that the stage does not meet its CST
  - For the purposes of the optimization
  - Allows us to pretend LT is deterministic
Net Lead Time

- Each stage has a processing time $T$ and a CST $S$
- Net lead time at stage $i = S_{i+1} + T_i - S_i$

“bad” LT  “good” LT
Net Lead Time vs. Inventory

- Suppose $S_i = S_{i+1} + T_i$
  - e.g., inbound CST = 4, proc time = 2, outbound CST = 6
  - Don’t need to hold any inventory
  - Operate entirely as pull (make-to-order, JIT) system

- Suppose $S_i = 0$
  - Promise immediate order fulfillment
  - Make-to-stock system
Net Lead Time vs. Inventory

- Precise relationship between NLT and inventory:
  \[ y^* = \mu \times NLT + z_\alpha \sigma \sqrt{NLT} \]

- NLT replaces LT in earlier formula
- So, choosing inventory levels is equivalent to choosing NLTs, i.e., choosing \( S \) at each stage
- Efficient algorithms exist for finding optimal \( S \) values to minimize expected holding cost while meeting end-customer service requirement
Key Insight

- It is usually optimal for only a few stages to hold inventory
  - Other stages operate as pull systems
- In a serial system, every stage either:
  - holds zero inventory (and quotes maximum CST)
  - or quotes CST of zero (and holds maximum inventory)
Case Study


- # below stage = processing time
- # in white box = CST
- In this solution, inventory is held of finished product and its raw materials
A Pure Pull System

- Produce to order
- Long CST to customer
- No inventory held in system
### A Pure Push System

- Produce to forecast
- Zero CST to customer
- Hold lots of finished goods inventory
A Hybrid Push-Pull System

- Part of system operated produce-to-stock, part produce-to-order
- Moderate lead time to customer
CST vs. Inventory Cost

![Graph showing inventory cost ($/year) versus committed lead time to customer (days). The graph compares Push System, Push-Pull System, and Pull System. The Push System has a higher inventory cost compared to the Push-Pull and Pull Systems. The Push-Pull System shows a moderate inventory cost, while the Pull System has the lowest inventory cost.](image-url)
Optimization Shifts the Tradeoff Curve

![Graph showing the shift in the tradeoff curve between committed lead time to customer and inventory cost.](image)
Supply Uncertainty

- Types of supply uncertainty:
  - Lead-time uncertainty
  - Yield uncertainty
  - Disruptions

- Strategies for dealing with demand and supply uncertainty are similar
  - Safety stock inventory
  - Dual sourcing
  - Improved forecasts

- But the two are not the same
Risk Pooling

- One warehouse, several retailers
  - Who should hold inventory?
- If demand is uncertain:
  - Smaller inventory req’t if warehouse holds inv.
  - Consolidation is better
- If supply is uncertain (but demand is not):
  - Disruption risk is minimized if retailers hold inv.
  - Diversification is better
Inventory Placement

Hold inventory upstream or downstream?

Conventional wisdom:
- Hold inventory **upstream**
- Holding cost is smaller

Under supply uncertainty:
- Hold inventory **downstream**
- Protects against stockouts anywhere in system
Outline

- Introduction
- Stochastic models
- Decentralized systems
  - Suboptimality
  - Contracting
  - The bullwhip effect
Decentralized Systems

- So far, we have assumed the system is centralized
  - Can optimize at all stages globally
  - One stage may incur higher costs to benefit the system as a whole
- What if each stage acts independently to minimize its own cost / maximize its own profit?
Suboptimality

- Optimizing locally is likely to result in some degree of suboptimality
- Example: upstream stages want to operate make-to-order
  - Results in too much inventory downstream
- Another example:
  - Wholesaler chooses wholesale price
  - Retailer chooses order quantity
  - Optimizing independently, the two parties will always leave money on the table
Contracting

- One solution is for the parties to impose a contracting mechanism
  - Splits the costs / profits / risks / rewards
  - Still allows each party to act in its own best interest
  - If structured correctly, system achieves optimal cost / profit, even with parties acting selfishly

- There is a large body of literature on contracting
  - In practice, idea is commonly used
  - Actual OR models rarely implemented
Bullwhip Effect (BWE)

- Demand for diapers:

![Graph showing the Bullwhip Effect with time on the x-axis and order quantity on the y-axis.]
Irrational Behavior Causes BWE

- Firms over-react to demand signals
  - Order too much when they perceive an upward demand trend
  - Then back off when they accumulate too much inventory
- Firms under-weight the supply line
- Both are irrational behaviors
- Demonstrated by “beer game”
Rational Behavior Causes BWE

- Famous paper by Hau Lee, et al. (1997)
- BWE can be caused by rational behavior
  - i.e., by acting in “optimal” ways according to OR inventory models
- Four causes:
  - Demand forecast updating
  - Batch ordering
  - Rationing game
  - Price variations
Questions?