SYMPHONY 5.0
Callable Library for Mixed Integer Programming
and
Implementation

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Outline of Talk

• Introduction to SYMPHONY 5.0
  – Callable library API
  – OSI interface
  – User callbacks

• Implementation
  – Warm-Starting
    * Resolve
    * Bicriteria solve
    * 2-Stage SIP

• Examples
Brief Introduction to SYMPHONY

• Overview
  – A callable library for solving mixed-integer linear programs with a wide variety of customization options.
  – Core solution methodology is a state of the art implementation of branch, cut, and price.
  – Outfitted as a generic MILP solver.
  – Extensive documentation available.
  – Source can be downloaded from www.branchandcut.org

• SYMPHONY Solvers
  - Generic MILP
  - Traveling Salesman Problem
  - Vehicle Routing Problem
  - Mixed Postman Problem
  - Set Partitioning Problem
  - Matching Problem
  - Network Routing
What is COIN-OR?

Fully integrated with the Computational Infrastructure for Operations Research (COIN-OR) libraries.

- **The COIN-OR Project**
  - An *initiative* promoting the development and use of interoperable, open-source software for operations research.
  - A * consortium * of researchers in both industry and academia dedicated to improving the state of computational research in OR.
  - A non-profit corporation known as the COIN-OR Foundation

- **The COIN-OR Repository**
  - A *library* of interoperable software tools for building optimization codes, as well as some stand-alone packages.
  - A *venue for peer review* of OR software tools.
  - A *development platform* for open source projects, including a CVS repository.
  - Soon to be hosted by INFORMS.
Supported Formats and Architectures

• Input formats
  – MPS (COIN-OR parser)
  – GMPL/AMPL (GLPK parser)
  – User defined

• Output/Display formats
  – Text
  – IGD
  – VbcTool

• Supported architectures
  – Single-processor Linux, Unix, or Windows
  – Distributed memory parallel (message-passing)
  – Shared memory parallel (OpenMP)
SYMPHONY C Callable Library

• Primary subroutines
  – sym_open_environment()
  – sym_parse_command_line()
  – sym_load_problem()
  – sym_find_initial_bounds()
  – sym_solve()
  – sym_mc_solve()
  – sym_resolve()
  – sym_close_environment()

• Auxiliary subroutines
  – Accessing and modifying problem data
  – Accessing and modifying parameters
  – User callbacks
Implementing a MILP Solver with SYMPHONY

- Using the callable library, we only need a few lines to implement a solver.
- The file name and other parameters are specified on the command line.
- The code is the same for any configuration or architecture, sequential or parallel.

Command line would be

```
symphony -F model.mps
```

```c
int main(int argc, char **argv)
{
    sym_environment *p = sym_open_environment();
    sym_parse_command_line(p, argc, argv);
    sym_load_problem(p);
    sym_solve(p);
    sym_close_environment(p);
}
```
OSI interface

• The COIN-OR Open Solver Interface is a standard C++ class for accessing solvers for mathematical programs.

• Each solver has its own derived class that translates OSI calls into those of the solver’s library.

• For each method in OSI, SYMPHONY has a corresponding method.

• The OSI interface is implemented as wrapped C calls.

• The constructor calls `sym_open_environment()` and the destructor calls `sym_close_environment()`.

• The OSI initialSolve() method calls `sym_solve()`.

• The OSI resolve() method calls `sym_resolve()`.
Using the SYMPHONY OSI interface

- Here is the implementation of a simple solver using the SYMPHONY OSI interface.

```c
int main(int argc, char **argv)
{
    OsiSymSolverInterface si;
    si.parseCommandLine(argc, argv);
    si.loadProblem();
    si.branchAndBound();
}
```

- Again, the code is the same for any configuration or architecture, sequential or parallel.
Customizing

• The main avenues for advanced customization are the parameters and the user callback subroutines.

• There are more than 50 callbacks and over 100 parameters.

• The user can override SYMPHONY’s default behavior in a variety of ways.
  – Custom input
  – Custom displays
  – Branching
  – Cut/column generation
  – Cut pool management
  – Search and diving strategies
  – LP management
Warm Starts for MILP

- To allow resolving from a warm start, we have defined a SYMPHONY warm start class, which is derived from CoinWarmStart.

- The class stores a snapshot of the search tree, with node descriptions including:
  - lists of active cuts and variables,
  - branching information,
  - warm start information, and
  - current status (candidate, fathomed, etc.).

- The tree is stored in a compact form by storing the node descriptions as differences from the parent.

- Other auxiliary information is also stored, such as the current incumbent.

- A warm start can be saved at any time and then reloaded later.

- The warm starts can also be written to and read from disk.
Warm Starting Procedure

• After modifying parameters
  – If only parameters have been modified, then the candidate list is recreated and the algorithm proceeds as if left off.
  – This allows parameters to be tuned as the algorithm progresses if desired.

• After modifying problem data
  – We limit modifications to those that do not invalidate the node warm start information.
  – Currently, we only allow modification of rim vectors.
  – After modification, all leaf nodes must be added to the candidate list.
  – After constructing the candidate list, we can continue the algorithm as before.
Warm Starting Example (Parameter Modification)

• The following example shows a simple use of warm starting to create a dynamic algorithm.

```c
int main(int argc, char **argv)
{
    OsiSymSolverInterface si;
    si.parseCommandLine(argc, argv);
    si.loadProblem();
    si.setSymParam(OsiSymFindFirstFeasible, true);
    si.setSymParam(OsiSymSearchStrategy, DEPTH_FIRST_SEARCH);
    si.initialSolve();
    si.setSymParam(OsiSymFindFirstFeasible, false);
    si.setSymParam(OsiSymSearchStrategy, BEST_FIRST_SEARCH);
    si.resolve();
}
```
Warm Starting Example (Problem Modification)

- The following example shows how to warm start after problem modification.

```c
int main(int argc, char **argv)
{
    OsiSymSolverInterface si;
    CoinWarmStart ws;
    si.parseCommandLine(argc, argv);
    si.loadProblem();
    si.setSymParam(OsiSymNodeLimit, 100);
    si.initialSolve();
    ws = si.getWarmStart();
    si.setSymParam(OsiSymNodeLimit, 10000);
    si.resolve();
    si.setObjCoeff(0, 1);
    si.setObjCoeff(200, 150);
    si.setWarmStart(ws);
    si.resolve();
}
```
Bicriteria MILPs

- We limit the discussion here to pure integer programs (ILPs), but generalization to MILPs is straightforward.
- The general form of a bicriteria ILP is
  \[ \text{vmax} [cx, dx], \]
  \[ \text{s.t.} \quad Ax \leq b, \]
  \[ x \in \mathbb{Z}^n. \]
- Solutions don’t have single objective function values, but pairs of values called outcomes.
- A feasible \( \hat{x} \) is called efficient if there is no feasible \( \bar{x} \) such that \( cx \geq c\hat{x} \) and \( dx \geq d\hat{x} \), with at least one inequality strict.
- The outcome corresponding to an efficient solution is called Pareto.
- The goal is to enumerate Pareto outcomes.

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Supported Outcomes

• A bicriteria ILP can be converted to a single-criteria ILP by substituting a \textit{weighted sum objective}

\[
\max_{x \in X} (\beta c + (1 - \beta) d) x
\]

for the bicriteria objective to obtain a parameterized family of ILPs.

• Optimal solutions to members of this family are extreme points of the convex lower envelope of outcomes and are called \textit{supported}.

• Supported outcomes are Pareto, but the converse is not true.

• It is straightforward to generate all \textit{supported outcomes} by solving a sequence of ILPs.
Illustration of Pareto and Supported Outcomes
Generating Pareto Outcomes

- To generate Pareto outcomes, we must replace the weighted sum objective with a weighted Chebyshev norm (WCN) objective.
- Let $x^c$ be a solution to the original ILP with objective $c$ and $x^d$ be a solution with objective $d$.
- Then the WCN objective is

$$\min_{x \in X} \max \{\beta(cx - cx^c), (1 - \beta)(dx - dx^d)\}.$$ 

- This objective can be linearized to obtain another family of ILPs.
- Assuming uniform dominance, Bowman showed solutions are efficient if and only if they optimal for some member of this family.
- The mild condition is uniform dominance, which states that all the points in Pareto set are strongly Pareto: $c\tilde{x} > cx$ and $d\tilde{x} > dx$!
The WCN algorithm

- The algorithm maintains a list of Pareto outcomes found so far, ordered by corresponding $\beta$ value.
- We choose a pair $(p, q)$ from the list and determine whether there is a Pareto outcome between them by solving an ILP with WCN objective and weight
  \[ \beta_{pq} = (dx - dx^d)/(cy - cy^c + dx - dx^d), \]
- If the result is a known outcome, then $\beta_{pq}$ is a breakpoint.
- Otherwise, the result is a new efficient solution $r$ and we add $(p, r)$ and $(r, q)$ to the list.
- This algorithm is asymptotically optimal.
Implementing the WCN algorithm

• Because the WCN algorithm involves solving a sequence of slightly modified MILPs, warm starting can be used.

• Two approaches
  – Warm start from the result of the previous iteration.
  – Solve a “base” problem first and warm each subsequent problem from there.

• In addition, we can optionally save the global cut pool from iteration to iteration, using SYMPHONY’s persistent cut pools.

• If the uniform dominance assumption is not satisfied, then we have to filter out weakly dominated solutions.

• Both the callable library and the OSI interface allow the user to define a second objective function and call the bicriteria solver.
Network Routing Problems

• Using SYMPHONY, we developed a custom solver for a class of network design and routing problems.

• A single commodity is supplied to a set of customers from a single supply point.

• We must design the network and route the demand, obeying capacity and other side constraints.

• We wish to consider both
  – the cost of construction (the sum of lengths of all links), and
  – the latency of the resulting network (the sum of length multiplied by demand carried for all links).

• These are competing objectives, so we can analyze the tradeoff by using the SYMPHONY multicriteria solver.
2-Stage Stochastic Programming Solver Using Dual Decomposition

- Consider the following two stage stochastic programming instance with fixed, relatively complete, integer recourse:

\[ z = \min \{ cx + Q(x) : Ax \leq b, \ x \in X \} \quad \text{where} \]
\[ Q(x) = E \xi \phi(h(\xi) - T(\xi)x) \quad \text{and} \]
\[ \phi(s) = \min \{ q(\xi)y : Wy \leq s, y \in Y \} \]

with appropriate dimensions.

- If we define:

\[ S^j := \{(x, y^j) : Ax \leq b, x \in X, T^j x + Wy^j \leq h^j, y^j \in Y \} \]

then, the deterministic equivalent of the problem would be:

\[ z = \min \{ cx + \sum_j p^j q^j y^j : (x, y^j) \in S^j \} \quad j = 1, \ldots, r \]
• Furthermore, we can introduce the copies of first stage variables: \( x^1, \ldots x^r \) and rewrite the equation as:

\[
\begin{align*}
    z &= \min \left\{ \sum_j p^j (c x^j + q^j y^j) : (x^j, y^j) \in S^j \right\} \quad j = 1, \ldots, r \\
    \text{s.t } x^1 &= x^2 = \ldots x^r \quad \text{(Non-anticipativity constraint)}
\end{align*}
\]

• Assume that we represent the non-anticipativity constraint by the equality:

\[
\sum_j H^j x^j = 0
\]

with appropriate dimensions.
• The Lagrangian relaxation with respect to the non-anticipativity condition is the problem of finding $x^j, y^j, j = 1, \ldots, r$ such that:

$$D(u) = \min \left\{ \sum_j L_j(x^j, y^j, u) : (x^j, y^j) \in S^j \right\} \quad \text{where}$$

$$L_j(x^j, y^j, u) = p^j(cx^j + q^jy^j) + u(H^jx^j) \quad j = 1, \ldots, r$$

• Now on, we have converted our initial problem to find:

$$Z_{LD} = \max_u D(u)$$

• The main advantage of this formulation is that we can separate the problem into subproblems for each scenario:

$$D(u) = \sum_j^r D_j(u) \quad \text{where}$$

$$D_j(u) = \min \left\{ L_j(x^j, y^j, u) : (x^j, y^j) \in S^j \right\}$$
Branch and Bound Algorithm

• Each of these $r$ subproblems is an MILP problem.

• So, at step $t$ of Subgradient Optimization, $r$ subproblems defined as $D_j(u^t) = \min \{L_j(x^j, y^j, u^t) : (x^j, y^j) \in S^j\} j = 1, ..., r$ need to be solved.

• Solving $Z_{LD}$ will give an upper bound which in general is larger than $z$. That is because of the duality gap.

• A branch and bound algorithm is presented. Basically, we solve Lagrangian dual relaxation of each node branched on some component of $x$.

• Because the algorithm involves solving a sequence of modified MILP’s in each node, SYMPHONY’s warm starting can be used.
Example: Warm Starting

- Consider the simple warm-starting code from earlier in the talk.
- Applying this code to the MIPLIB 3 problem p0201, we obtain the results below.
- Note that the warm start doesn’t reduce the number of nodes generated, but does reduce the solve time dramatically.

<table>
<thead>
<tr>
<th></th>
<th>CPU Time</th>
<th>Tree Nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generate warm start</td>
<td>28</td>
<td>100</td>
</tr>
<tr>
<td>Solve orig problem (from warm start)</td>
<td>3</td>
<td>118</td>
</tr>
<tr>
<td>Solve mod problem (from scratch)</td>
<td>24</td>
<td>122</td>
</tr>
<tr>
<td>Solve mod problem (from warm start)</td>
<td>6</td>
<td>198</td>
</tr>
</tbody>
</table>
Example: Bicriteria ILP

• Consider the following bicriteria ILP:

\[ \text{vmax} \quad [8x_1, x_2] \]

\[ \text{s.t.} \quad 7x_1 + x_2 \leq 56 \]

\[ 28x_1 + 9x_2 \leq 252 \]

\[ 3x_1 + 7x_2 \leq 105 \]

\[ x_1, x_2 \geq 0 \]

• For this ILP, we get the set of Pareto outcomes pictured on the next slide.
Example: Pareto and Supported Outcomes for Example

Non-dominated Solutions
Example: Bicriteria Solver

- Consider the simple ILP from our earlier example.
- By examining the supported solutions and break points, we can easily determine $p(\theta)$, the objective function value as a function of $\theta$.

<table>
<thead>
<tr>
<th>$\theta$ range</th>
<th>$p(\theta)$</th>
<th>$x_1^*$</th>
<th>$x_2^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(-\infty, 1.333)$</td>
<td>64</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>$(1.333, 2.667)$</td>
<td>$56 + 6\theta$</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>$(2.667, 8.000)$</td>
<td>$40 + 12\theta$</td>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>$(8.000, 16.000)$</td>
<td>$32 + 13\theta$</td>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>$(16.000, \infty)$</td>
<td>$15\theta$</td>
<td>0</td>
<td>15</td>
</tr>
</tbody>
</table>
Example: Graph of Price Function
Example: 2-Stage SIP problem

• The problem was obtained from Caroe:

\[
\max \left\{ \frac{3}{2} x_1 + 4x_2 + Q(x_1, x_2) : 0 \leq x_1, x_2 \leq 5 \text{ and integer} \right\}
\]

where \( Q(x_1, x_2) \) is the expected value of the multi-knapsack problem:

\[
\max \ \{16y_1 + 19y_2 + 23y_3 + 28y_4\}
\]

\[s.t\quad 2y_1 + 3y_2 + 4y_3 + 5y_4 \leq \xi_1 - x_1,\]

\[6y_1 + y_2 + 3y_3 + 2y_4 \leq \xi_2 - x_2, \ y_i \in \{0, 1\}, \ i = 1, \ldots, 4\]

and the random variable \( \xi = (\xi_1, \xi_2) \) is uniformly distributed on \( \Psi = \{(5, 5), (5, 6), \ldots, (5, 15), (6, 5), \ldots, (15, 15)\} \), giving a total of 121 scenarios.
• SUTIL (provided by Prof. Linderoth) was used to read SMPS files.

• The non-anticipativity constraints used are:

\[
\sum_{j \neq k} p^j x^j + (p^k - 1)x^k = 0 \quad k = 1, \ldots, r
\]

• Initial Lagrangian multipliers are picked to be 0.

<table>
<thead>
<tr>
<th></th>
<th>Running Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>without warm start</td>
<td>298.92</td>
</tr>
<tr>
<td>with warm start from the previous iter.</td>
<td>160.87</td>
</tr>
<tr>
<td>with warm start from the first iter.</td>
<td>160.48</td>
</tr>
</tbody>
</table>
Conclusion

- We presented a new version of the SYMPHONY solver with an OSI interface supporting warm starting for MILPs.
- We have shown how this capability can be used to implement an efficient bicriteria solver for ILPs.
- We have shown how this solver can in turn be used to perform sensitivity analysis and analyze tradeoffs for competing objectives.
- In future work, we plan on refining SYMPHONY’s warm start and sensitivity analysis capabilities.
- Two papers covering the contents of this talk are available.
- Full computational results will be available in a future paper.