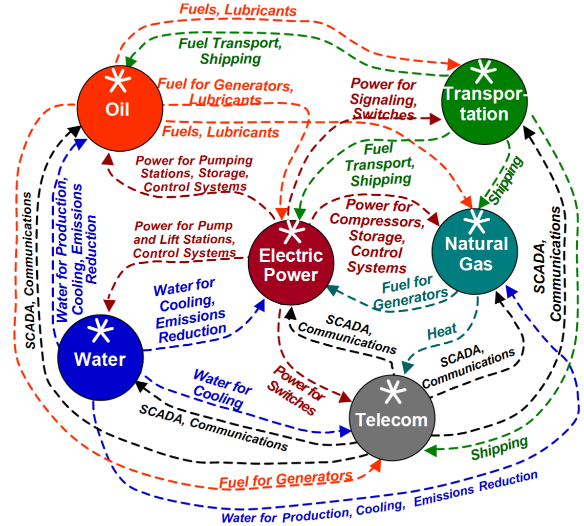
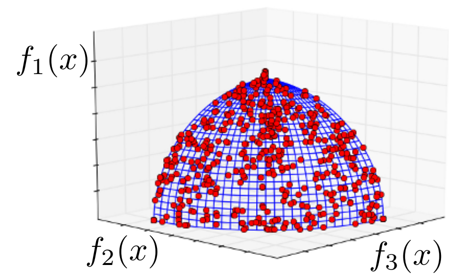
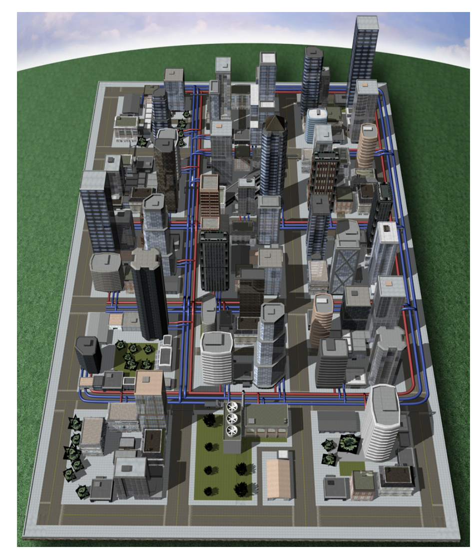
Un Marco de CVaR para Optimización con Intereses Múltiples

(A CVaR Framework for Multi-Stakeholder Optimization)





Victor M. Zavala
Department of Chemical and Biological Engineering
University of Wisconsin-Madison
http://zavalab.engr.wisc.edu



With: A. Dowling (UWisc)
L.F. Fuentes J.M. Ponce (UMichoacan)
G. Ruiz-Mercado (EPA)

Optimización Multiobjetivo – Multiobjective Optimization

MOO

$$\min_{x \in \mathcal{X}} \left(f_1(x), f_2(x)..., f_n(x)\right)$$

Utopia Point

$$\underline{f}_i := \min_{x \in \mathcal{X}} f_i(x), \quad i \in \mathcal{O} := \{1..n\}$$

$$\underline{x}_i := \operatorname*{argmin}_{x \in \mathcal{X}} f_i(x), \quad i \in \mathcal{O}$$

Nadir Point

$$\overline{f}_i := \max\{f_i(\underline{x}_1), f_i(\underline{x}_2), ..., f_i(\underline{x}_n)\}, \quad i \in \mathcal{O}$$

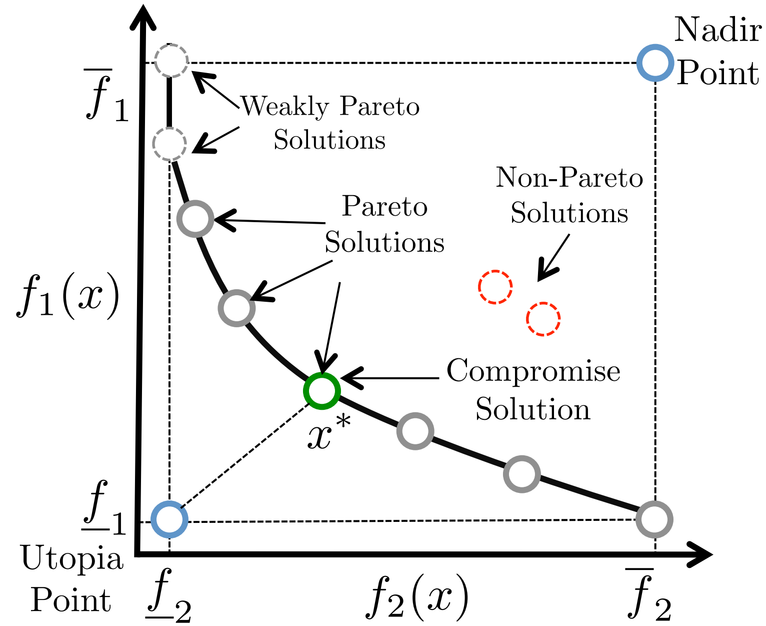
Re-Scaling

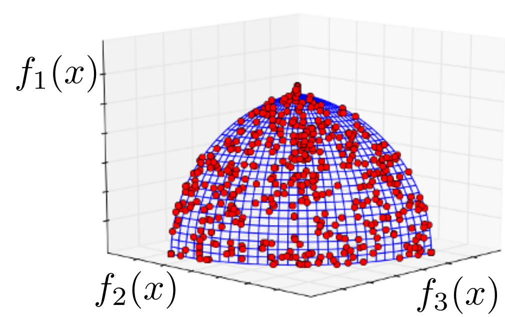
$$\hat{f}_i(x) \leftarrow \frac{f_i(x) - \underline{f}_i}{\overline{f}_i - \underline{f}_i}, \; i \in \mathcal{O}$$

Compromise Solution

$$x^* = \min_{x \in \mathcal{X}} \|\mathbf{f}(x)\|_p$$

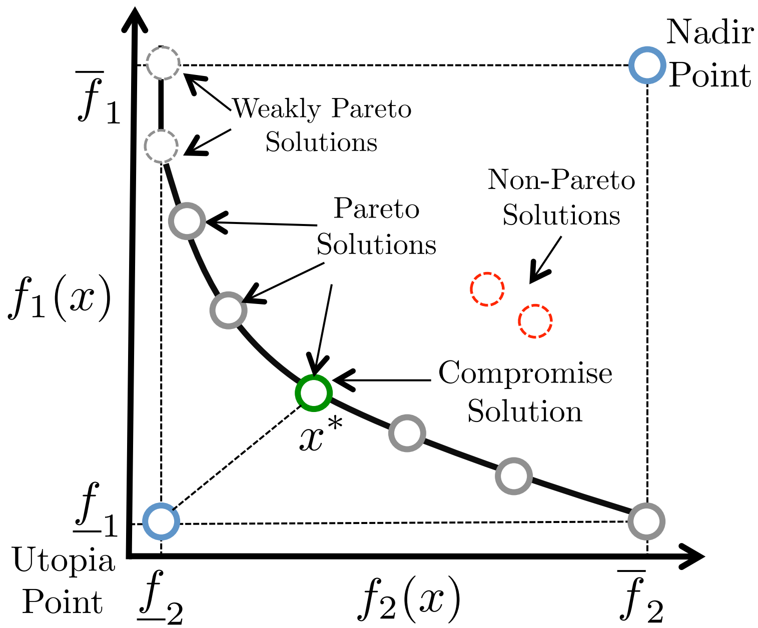
$\mathbf{f}(x) := (f_1(x), f_2(x), ..., f_n(x))$





Issues:
- **Ambiguity:** Meaning of Compromise?
- **Dimensionality:** Construct Pareto Set?

Optimización Multiobjetivo – Multiobjective Optimization



Definition: (**Weak Pareto Optimality**) A decision x^* with $f_i(x^*)$, $i \in \mathcal{O}$ is a *weakly Pareto optimal* solution of MOO if there does not exist an alternative solution $\bar{x}$ with objectives $f_i(\bar{x})$, $i \in \mathcal{O}$ satisfying $f_i(\bar{x}) < f_i(x^*)$ for all $i \in \mathcal{O}$.

Definition: (**Pareto Optimality**) A decision x^* with $f_i(x^*)$, $i \in \mathcal{O}$ is a *Pareto optimal* solution of MOO if there does not exist an alternative solution $\bar{x}$ with objectives $f_i(\bar{x})$, $i \in \mathcal{O}$ satisfying $f_i(\bar{x}) \leq f_i(x^*)$ for all $i \in \mathcal{O}$ and at least one index i satisfying $f_i(\bar{x}) < f_i(x^*)$.

Toma de Decisiones con Intereses Múltiples – Multistakeholder Optimization

Ideal Stakeholder Solution

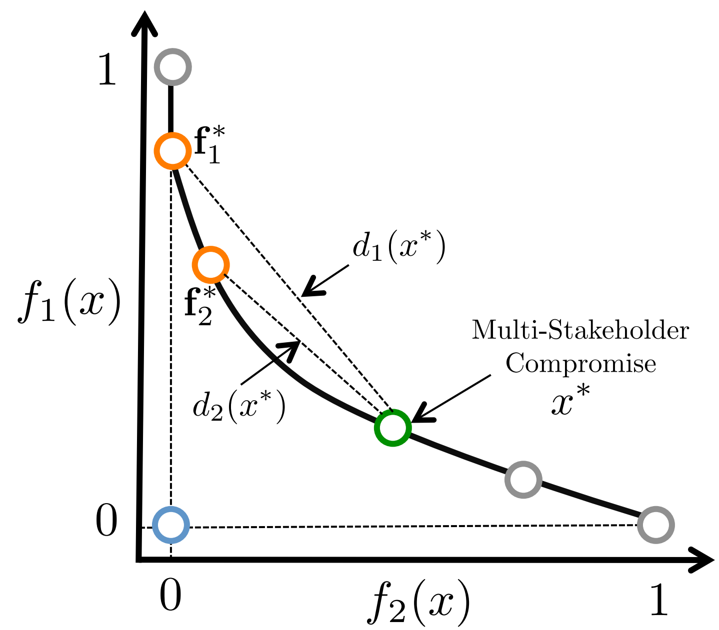
$$x_j^* := \operatorname*{argmin}_{x \in \mathcal{X}} \mathbf{w}_j^T \mathbf{f}(x), \; j \in \mathcal{S} := \{1..m\}$$

$\mathbf{w}_j$: Stakeholder Priority Vector

Stakeholder Dissatisfaction Function

$$\begin{aligned} d_j(x) &:= \mathbf{w}_j^T \left(\mathbf{f}(x) - \mathbf{f}_j^* \right) \\ &= \mathbf{w}_j^T \mathbf{f}(x) - \mathbf{w}_j^T \mathbf{f}_j^* \end{aligned}$$

$$\mathbf{f}_j^* := \mathbf{f}(x_j^*)$$



Average Dissatisfaction *Dyer, 1992*

$$\min_{x \in \mathcal{X}} \frac{1}{m} \sum_{j \in \mathcal{S}} d_j(x)$$

Worst-Case Dissatisfaction *Mehrotra, 2012*

$$\min_{x \in \mathcal{X}} \max_{j \in \mathcal{S}} \{d_j(x)\}$$

Conditional Value-at-Risk *This Work*

$$\min_{x \in \mathcal{X}} \; \mathrm{CVaR}_\alpha \left(d(x) \right)$$

Key Observation: Interpret Opinions as Samples from Population

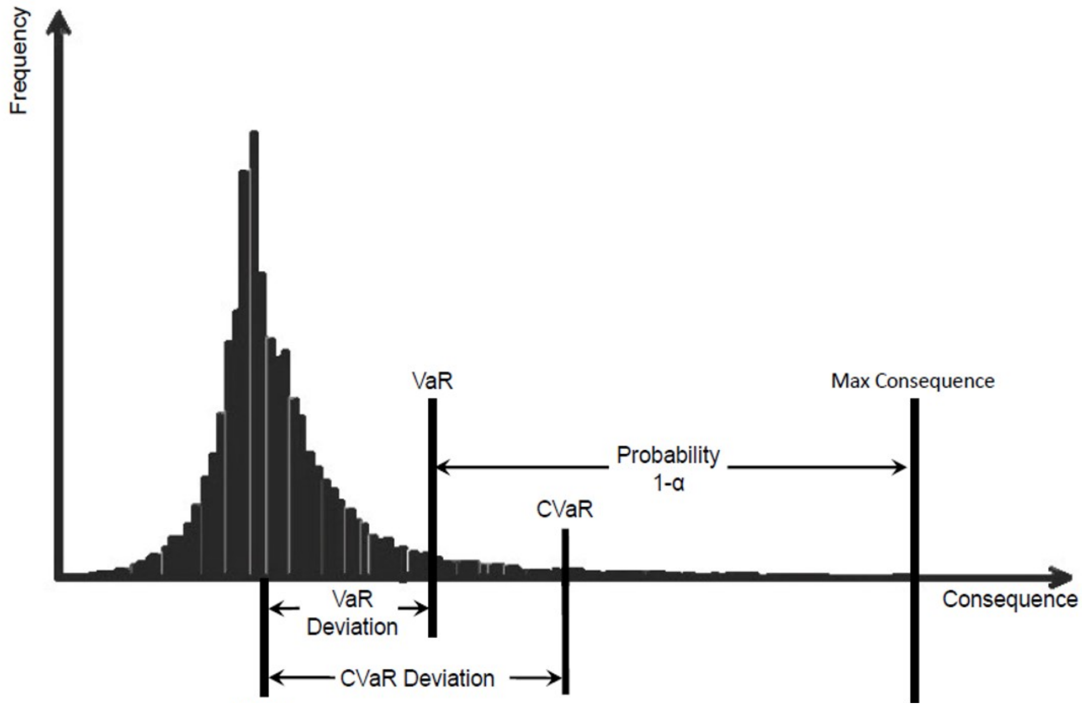
Valor en Riesgo Condicional - Conditional Value At Risk (CVaR) *Rockafellar & Uryasev, 2000*

$$\mathrm{CVaR}_\alpha[d(x)] = \min_\nu \frac{1}{m}\sum_{j=1}^{m}\left[\nu + \frac{1}{1-\sigma}\left[d_j(x)-\nu\right]_+\right]$$

Key Property:

$$\lim_{\alpha\to 0} \mathrm{CVaR}_\alpha[d(x)] = \frac{1}{m}\sum_{j\in\mathcal{S}} d_j(x)$$

$$\lim_{\alpha\to 1} \mathrm{CVaR}_\alpha[d(x)] = \max_{j\in\mathcal{S}}\{d_j(x)\}$$



Question: Are CVaR Solutions Pareto Optimal?

Interpretación Geométrica - Geometric Interpretation

Disagreement Vector

$$d_j(x) := \mathbf{w}_j^T(\mathbf{f}_j(x) - \mathbf{f}_j^*), j \in \mathcal{S}$$

$$\mathbf{d}(x) := [d_1(x), d_2(x), ..., d_m(x)]$$

Definition: (**Scaled L_p norm**). Consider a fixed decision $x \in \Re^{n_x}$ and the dissatisfaction vector $\mathbf{d}(x) \in \Re^m$. The scaled L_p norm (denoted as L_p^m) of $\mathbf{d}(x)$ is defined as,

$$\|\mathbf{d}(x)\|_p^m := \left(\frac{1}{m} \sum_{j=1}^{m} |d_j(x)|^p \right)^{\frac{1}{p}}, \; p \geq 1.$$

The scaled L_p norm has the following extreme cases,

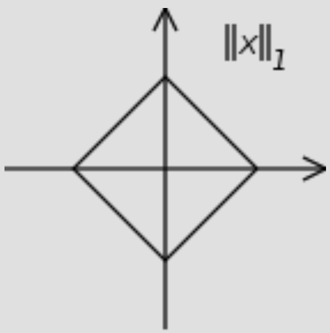
$$\|\mathbf{d}(x)\|_1^m = \frac{1}{m} \sum_{j=1}^{m} |d_j(x)|$$

$$\|\mathbf{d}(x)\|_\infty^m = \max_j |d_j(x)|.$$

Interpretación Geométrica - Geometric Interpretation

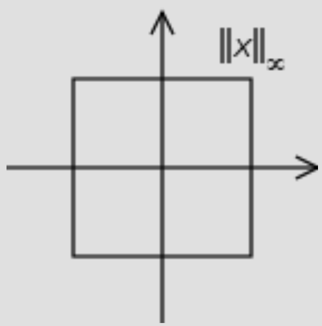
$$\min_{x \in \mathcal{X}} \frac{1}{m} \sum_{j \in \mathcal{S}} d_j(x)$$

$$\min_{x \in \mathcal{X}} \|\mathbf{d}(x)\|_1^m$$



$$\min_{x \in \mathcal{X}} \max_{j \in \mathcal{S}} \{d_j(x)\}$$

$$\min_{x \in \mathcal{X}} \|\mathbf{d}(x)\|_\infty^m$$



$$\min_{x \in \mathcal{X}} \mathrm{CVaR}_\alpha\left(d(x)\right)$$

?

?

La Norma CVaR - The CVaR Norm *Pavlikov & Uryasev, 2014*

Definition: (**Scaled CVaR norm**). Consider the vector $\mathbf{d}(x) \in \Re^m$ and assume (without loss of generality) that $d_1(x) \leq d_2(x) \leq \cdots \leq d_m(x)$ holds. Define also the scalars $\alpha_j := \frac{j}{m},\ j = 0, ..., m-1$. The *scaled CVaR norm* of vector $\mathbf{d}(x)$ with parameter α_j is defined as,

$$\ll \mathbf{d}(x) \gg_{\alpha_j}^m := \frac{1}{m-j} \sum_{i=j+1}^{m} d_i(x).$$

Norm Conditions:

Homogeneity:
$\rho(\lambda \mathbf{x}) = \lambda \rho(\mathbf{x})$

Subadditivity:
$\rho(\mathbf{x}_1, \mathbf{x}_2) \leq \rho(\mathbf{x}_1) + \rho(\mathbf{x}_2)$

Normalized:
$\rho(0) = 0$

CVaR Norm Properties: For fixed x consider the discrete random variable $d(x)$ with outcomes $d_1(x), d_2(x), ..., d_m(x)$, probabilities $p_j = \frac{1}{m},\ j \in \mathcal{S}$, and the corresponding vector $\mathbf{d}(x)$.

i) $\langle\langle \cdot \rangle\rangle_\alpha^m$ is a Norm for $\alpha \in [0,1]$

ii) $\langle\langle \mathbf{d}(x) \rangle\rangle_\alpha^m = CVaR_\alpha(d(x))$ for $\alpha \in [0,1]$.

iii) $\langle\langle \mathbf{d}(\mathbf{x}) \rangle\rangle_0^m = \|\mathbf{d}(x)\|_1^m$

iv) $\langle\langle \mathbf{d}(x) \rangle\rangle_\alpha^m = \|\mathbf{d}(x)\|_\infty^m$ for $\frac{m-1}{m} \leq \alpha \leq 1$.

v) For α such that $\alpha_j < \alpha < \alpha_{j+1},\ j = 0, ..., m-2$:

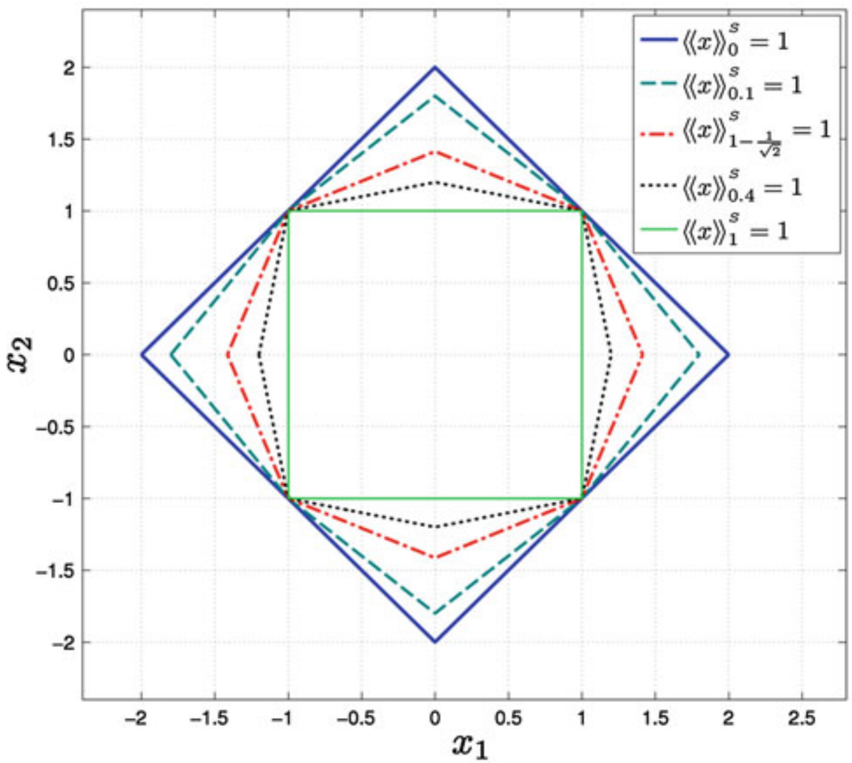
$$\langle\langle \mathbf{d}(x) \rangle\rangle_\alpha^m \ = \ \mu \langle\langle \mathbf{d}(x) \rangle\rangle_{\alpha_j}^m + (1-\mu) \langle\langle \mathbf{d}(x) \rangle\rangle_{\alpha_{j+1}}$$

with $\mu := \frac{(\alpha_{j+1}-\alpha)(1-\alpha_j)}{(\alpha_{j+1}-\alpha_j)(1-\alpha)}$.

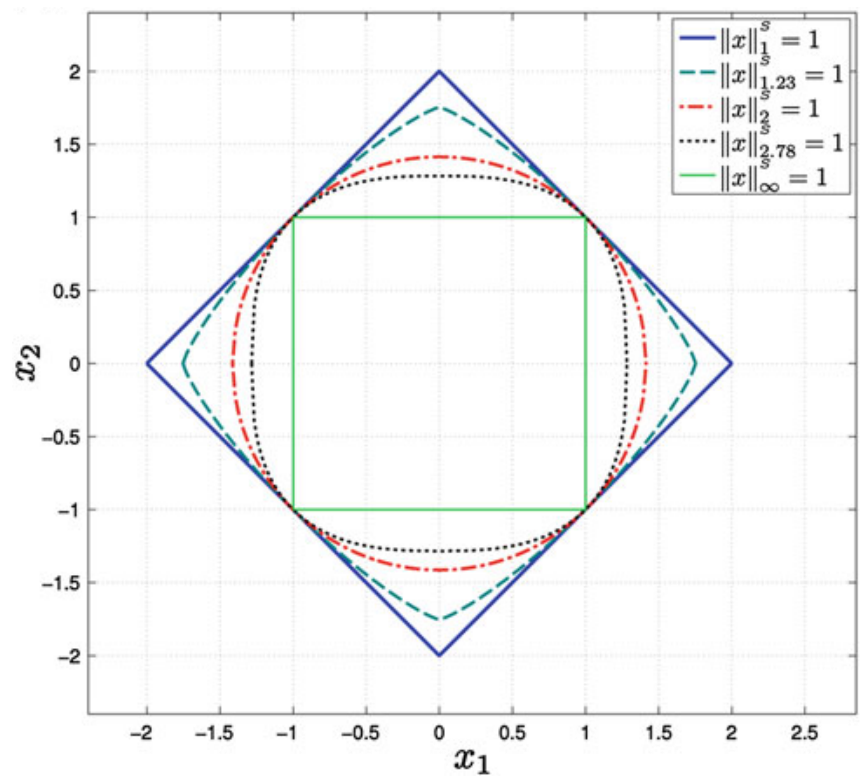
vi) $\langle\langle \mathbf{d}(x) \rangle\rangle_\alpha^m$ is a nondecreasing function of $\alpha \in [0,1]$.

La Norma CVaR - The CVaR Norm *Pavlikov & Uryasev, 2014*

CVaR Norm



L_p^S Norm



CVaR Norm Combinatorial But Can be Computed Using <u>Continuous Formulation</u>

$$\min_{x \in \mathcal{X}} \ \mathrm{CVaR}_\alpha\left[d(x)\right] \quad \Longleftrightarrow \quad \min_{x \in \mathcal{X}} \ll \mathbf{d}(x) \gg^m_\alpha \quad \Longleftrightarrow \quad \min_{(x,y) \in \mathcal{X} \times \Re} y + \frac{1}{(1-\alpha)m} \sum_{j=1}^{m} (d_j(x) - y)_+$$

Optimalidad de Pareto de las Soluciones de CVaR (Pareto Optimality of CVaR Solutions)

MOO

$$\min_{x\in\mathcal{X}} \; (f_1(x), f_2(x)..., f_n(x))$$

CVaR Problem

$$\min_{x\in\mathcal{X}} \ll \mathbf{d}(x) \gg_{\alpha}^{m} \quad\Longleftrightarrow\quad \min_{(x,y)\in\mathcal{X}\times\Re} y + \frac{1}{(1-\alpha)m}\sum_{j=1}^{m}(d_j(x)-y)_+$$

Lemma: Consider decisions $\bar{x}, x^*$ with corresponding $d_j(\bar{x}), d_j(x^*)$. We have:

$$d_j(\bar{x}) < d_j(x^*),\ j\in\mathcal{S} \quad\Longrightarrow\quad \langle\!\langle \mathbf{d}(\bar{x})\rangle\!\rangle_{\alpha}^{m} \;<\; \langle\!\langle \mathbf{d}(x^*)\rangle\!\rangle_{\alpha}^{m},\ \alpha\in[0,1].$$

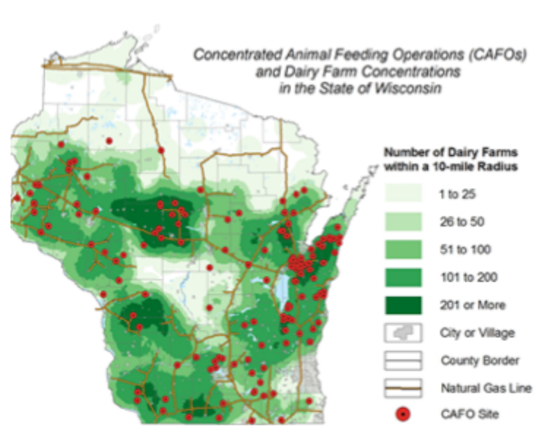
Theorem: Let x^* be a solution of the CVaR problem. We have:

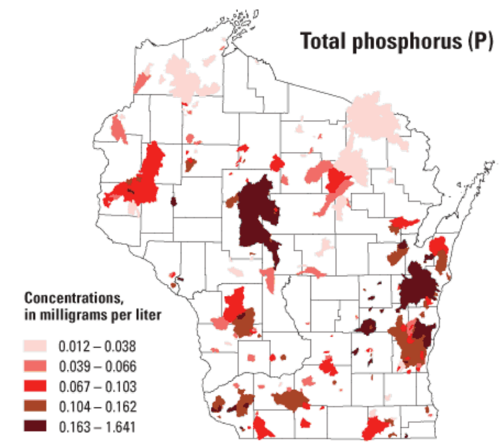
1. If $\mathbf{w}_j^{(i)} \geq 0,\ j\in\mathcal{S}, i\in\mathcal{O}$ then x^* is weak Pareto for MOO $\forall\alpha\in[0,1]$.
2. If $\mathbf{w}_j^{(i)} > 0,\ j\in\mathcal{S}, i\in\mathcal{O}$ then x^* is Pareto for MOO $\forall\alpha\in[0,1]$.

Proof of Weak Pareto: x^* is optimal for CVaR and thus $\langle\!\langle \mathbf{d}(x)\rangle\!\rangle_{\alpha}^{m} \geq \langle\!\langle \mathbf{d}(x^*)\rangle\!\rangle_{\alpha}^{m}$ for any $x\in\mathcal{X}$. Assume x^* is *not* weakly Pareto optimal. This implies that there exists an alternative $\bar{x}\in\mathcal{X}$ such that $f_i(\bar{x}) < f_i(x^*)$ for all $i\in\mathcal{O}$. We thus have that $\mathbf{w}_j^T\mathbf{f}(\bar{x}) < \mathbf{w}_j^T\mathbf{f}(x^*)$ for any $\mathbf{w}_j$ with $\mathbf{w}_j^{(i)} \geq 0$ and $\sum_{i\in\mathcal{O}}\mathbf{w}_j^{(i)} = 1$. Consequently, $d_j(\bar{x}) < d_j(x^*)$ for all $j\in\mathcal{S}$. From previous Lemma we also have that $\langle\!\langle \mathbf{d}(\bar{x})\rangle\!\rangle_{\alpha}^{m} < \langle\!\langle \mathbf{d}(x^*)\rangle\!\rangle_{\alpha}^{m}$. We thus have that the alternative $\bar{x}$ cannot exist and we have a contradiction.

Localización de Instalaciones de Biogas - BioGas Facility Location

Source	Methane Potential (tonnes/yr)
Wastewater	2,339,339
Landfills*	2,454,974
Animal manure	1,905,253
IIC organic waste	1,157,883
Total	**7,857,449**





Some Info:

U.S. Farm Animals Produce <u>2 Times</u> the Amount of Waste of Entire Human Population
Single Dairy Cow Generates 20 tons of Waste/year
There are 9 Million Cows in the U.S.
From EPA: 2,000 Farms Could Support Biogas from Waste (Less than 200 Installations)

Challenges:

How to Reconcile Priorities (Emissions/Water/Health/Investment/Not-in-my-Backyard)?
How to Derive Fair Incentives/Regulations?

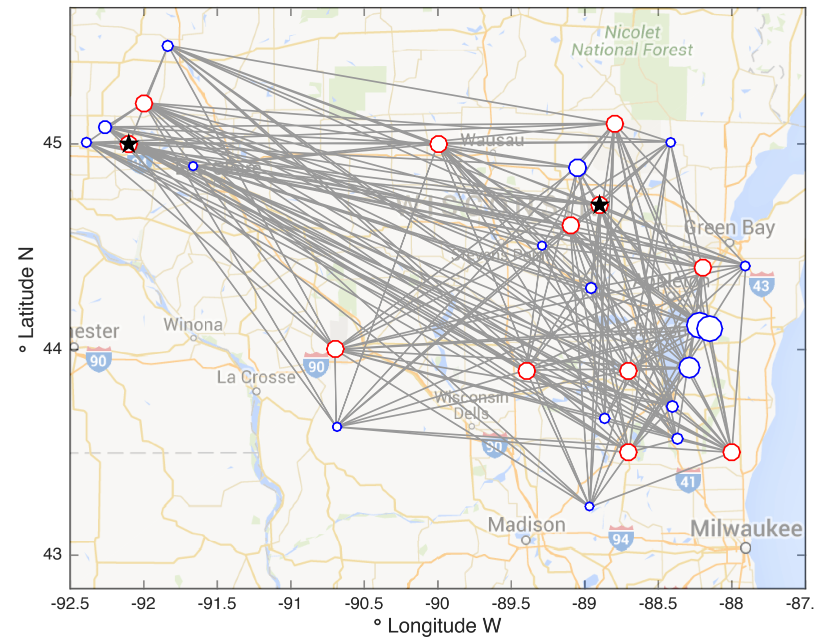
Localización de Instalaciones de Biogas - BioGas Facility Location

$$\max \quad E = \sum_{j\in\mathcal{F}} E_j^P - \sum_{i\in\mathcal{F},j\in\mathcal{B}} E_{i,j}^T - \sum_{j\in\mathcal{F}} E_j^U$$

Emissions

$$\max \quad C = C^{e^-} - C^I - C^O - C^T$$

Economics



Transportation

$$E^T = \sum_{i\in\mathcal{F}}\sum_{j\in\mathcal{B}} \alpha_{CO_2 Diesel} T_{i,j} d_{i,j}$$

Round Trips

$$T_{i,j} = S_{i,j}/\bar{S}, \; i\in\mathcal{F}, j\in\mathcal{B}$$

Processed Waste

$$E^P = \sum_{j\in\mathcal{F}} \alpha_{CO_2 CH_4}\cdot\alpha_{CH_4 H} H_j^P$$

Unprocessed Waste

$$E^U = \sum_{j\in\mathcal{F}} \alpha_{CO_2 CH_4}\cdot\alpha_{CH_4 H} H_j^U$$

Investment

$$C^I = \sum_{i\in\mathcal{T}}\sum_{j\in\mathcal{B}} c_i^I \cdot y_{i,j}$$

Processing Cost

$$C^O = \sum_{i\in\mathcal{T}}\sum_{j\in\mathcal{B}} c_i^O \cdot W_{i,j}$$

Transportation Cost

$$C^T = \sum_{i\in\mathcal{F}}\sum_{j\in\mathcal{B}} c_{i,j}^T \cdot S_{i,j}$$

Electricity Profit

$$C^{e^-} = \sum_{i\in\mathcal{T}}\sum_{j\in\mathcal{B}} c_i^O \cdot c^{e^-} G_{i,j}$$

Balances

$$\sum_{j\in\mathcal{B}} S_{i,j} \le \bar{F}_i, \; i\in\mathcal{F}$$

$$\sum_{i\in\mathcal{F}} S_{i,j} = \sum_{i\in\mathcal{T}} W_{i,j}, \; j\in\mathcal{B}$$

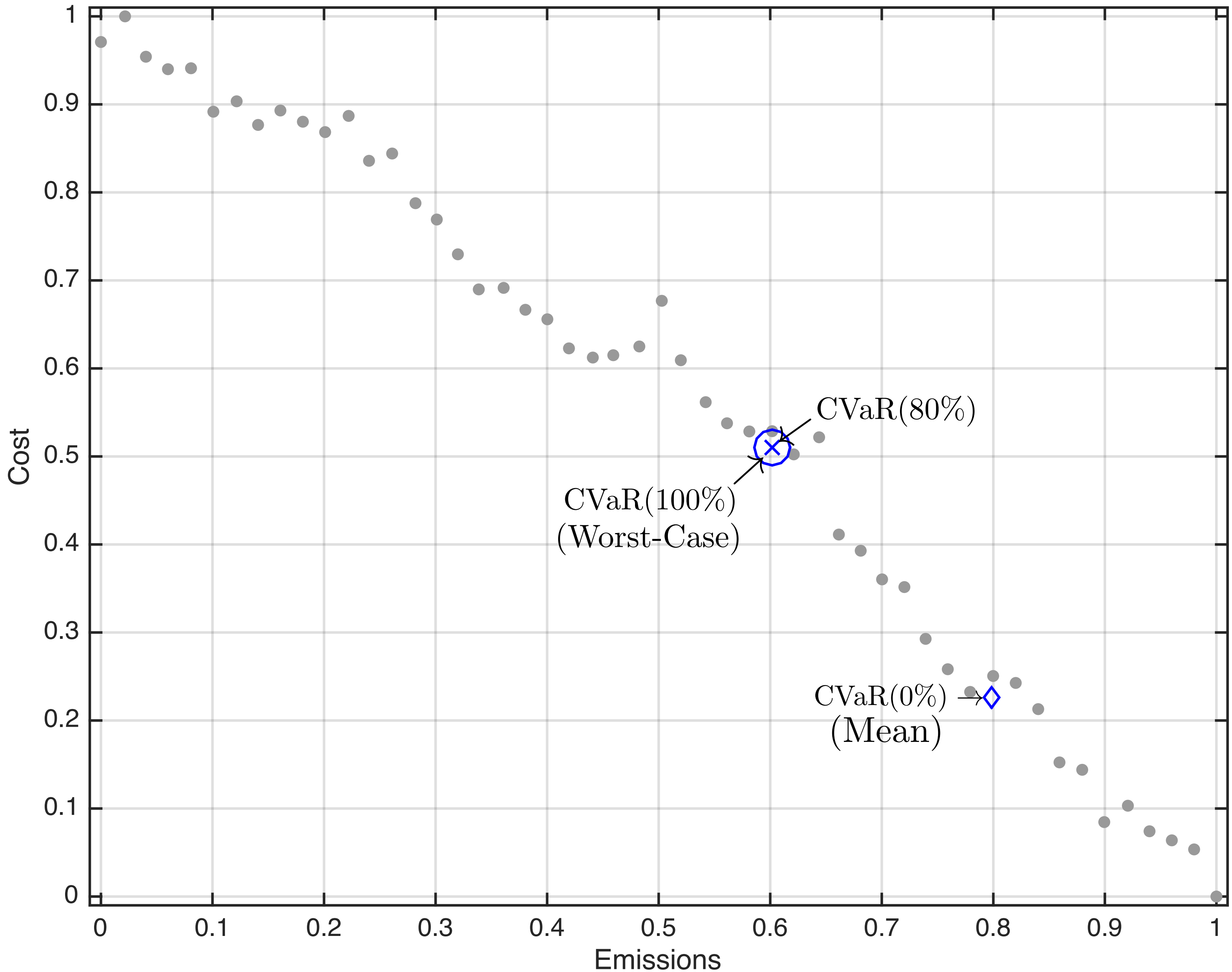
$$W_i^P = \sum_{j\in\mathcal{B}} S_{i,j}, \; i\in\mathcal{F}$$

$$W_i^U = \bar{F}_i - \sum_{j\in\mathcal{B}} S_{i,j}, \; i\in\mathcal{F}$$

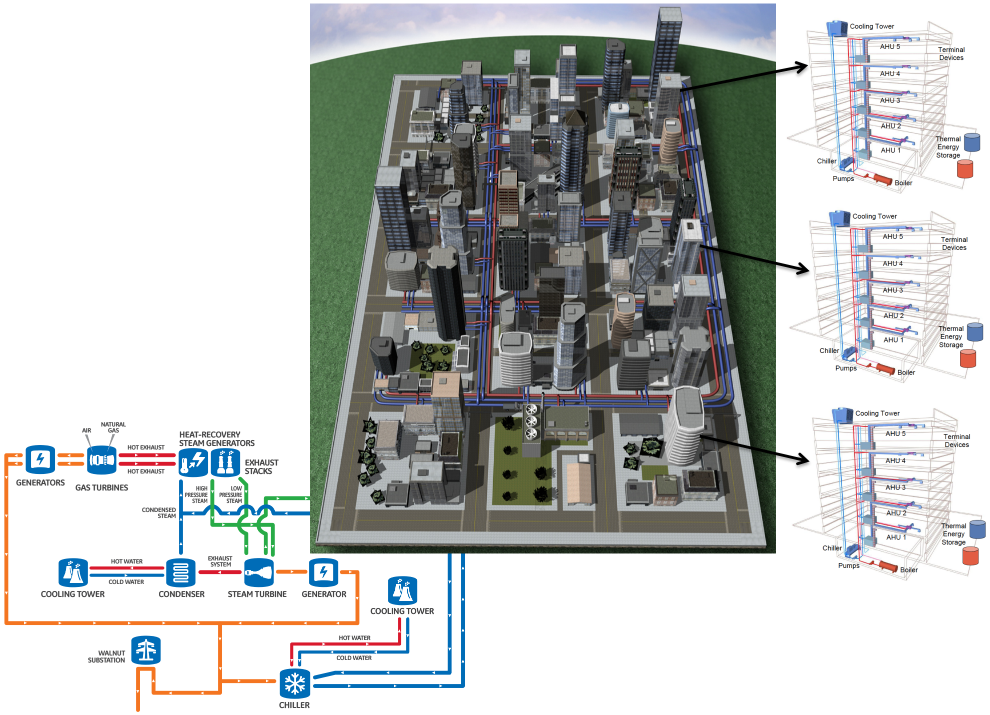
$$G_{i,j} = \alpha_{GW}\cdot W_{i,j}, \; i\in\mathcal{T}, j\in\mathcal{B}$$

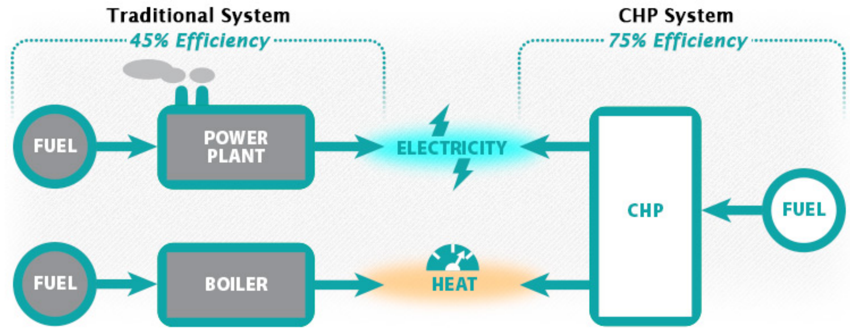
$$G_{i,j} \le \bar{G}_i y_{i,j}, \; i\in\mathcal{T}, j\in\mathcal{B}$$

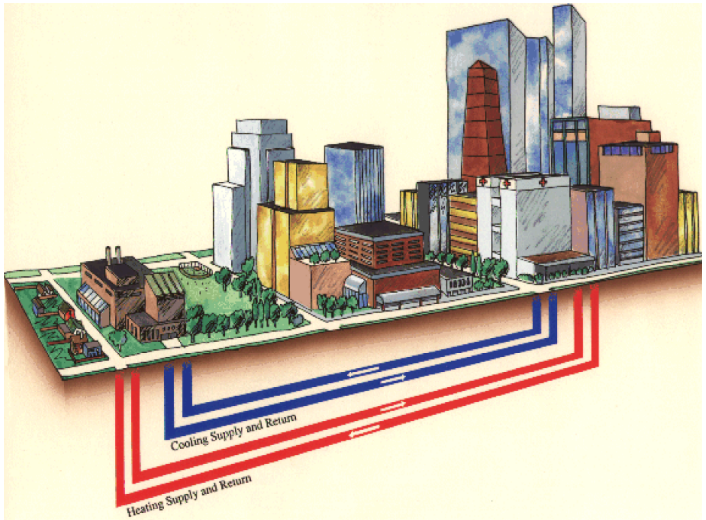
Localización de Instalaciones de Biogas - BioGas Facility Location



Unadades de Calor y Poder Combinado - Combined Heat & Power (CHP) Units







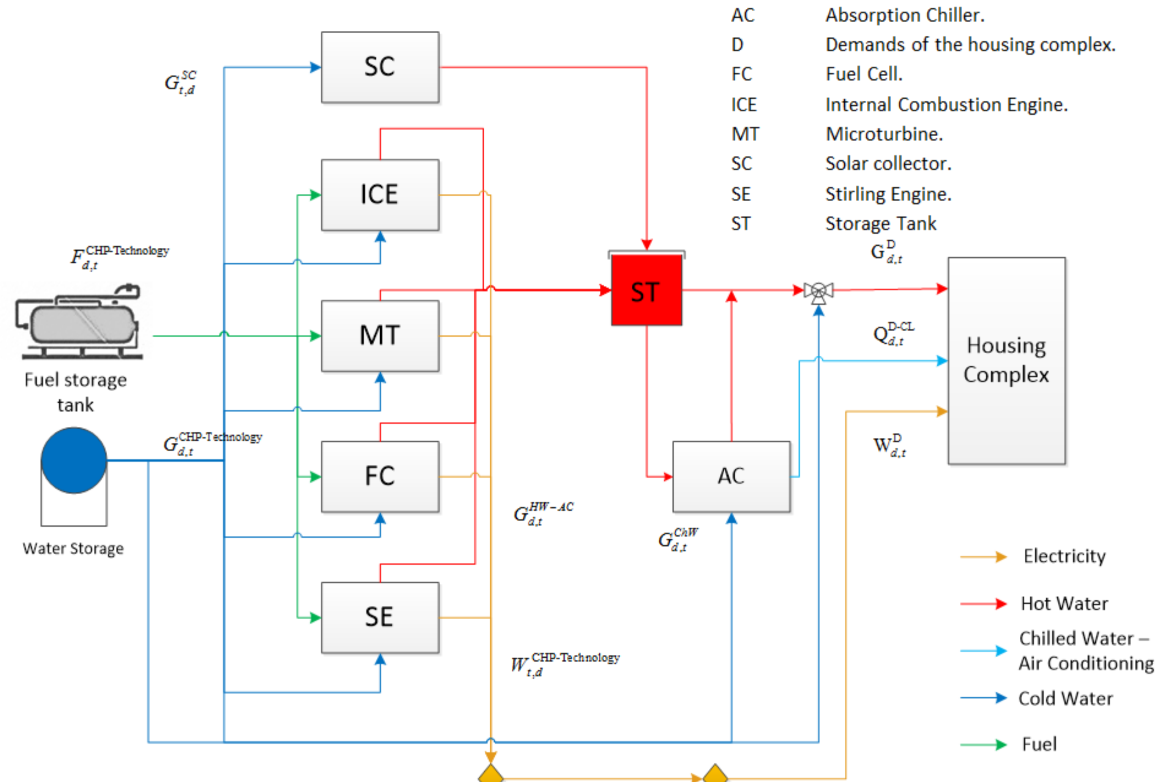
Some Info:

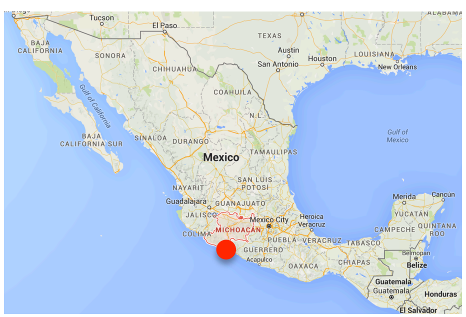
CHP Uses Heat Recovery to Simultaneously Provide Electricity, Heating, and Cooling
CHP Efficiency 70-80% vs. Traditional Power Plant Efficiency 40-50%
U.S. CHP Capacity To Increase from 80GW to 120 GW in 10 Years

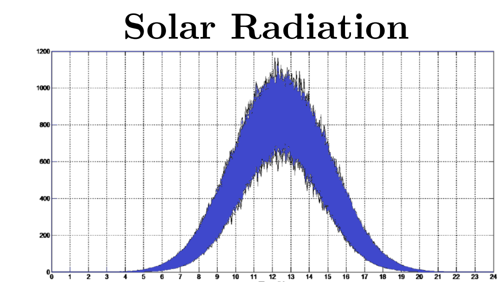
Design Challenges:

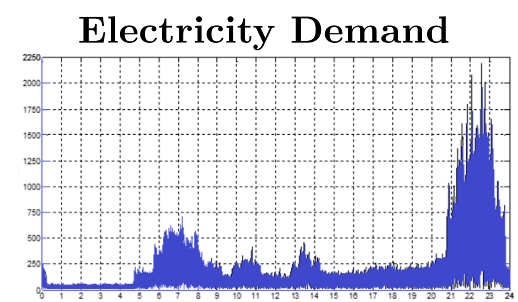
Capture Dynamic Patterns of Electricity/Cooling/Heating Demands
Many Emerging Technologies with Strong Trade-Offs (Investment, Emissions, Water)

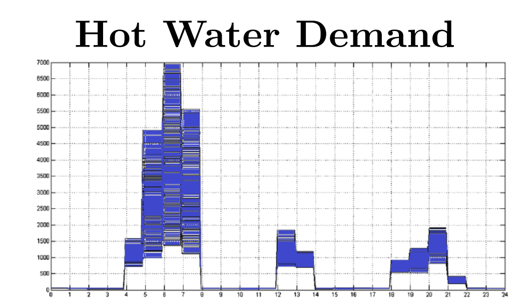
Unidades de Calor y Poder Combinado - Combined Heat & Power (CHP) Units

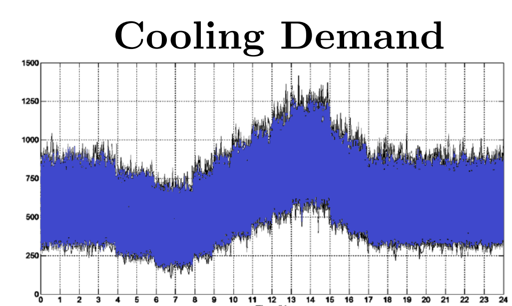












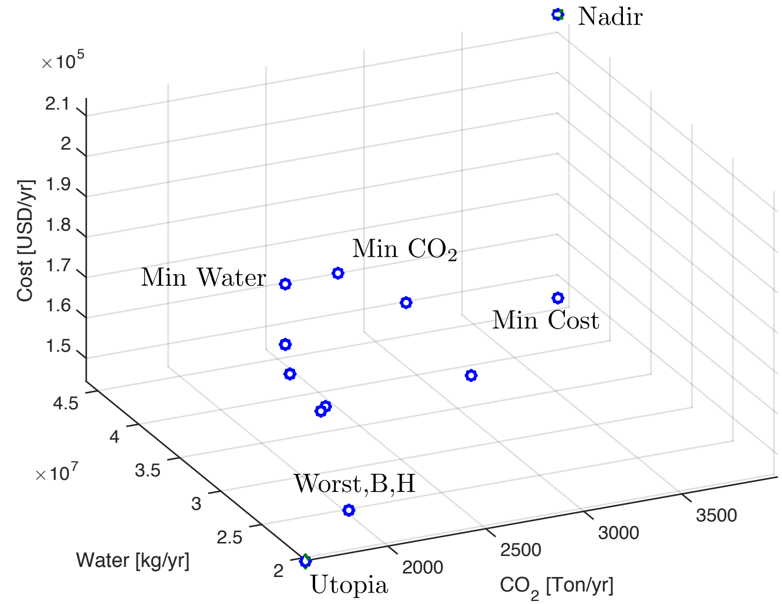
Case Study in Pacific Coast of Mexico:

Real Energy Demands & Weather Data for Housing Complex

Housing Complex with 420 Units and 2,400 Inhabitants

CHP Units

Stakeholder	Cost	Emissions	Water
A	1/3	1/3	1/3
B	-	1/2	1/2
C	1/2	-	1/2
D	1/2	1/2	-
E	-	2/3	1/3
F	-	1/3	2/3
G	1/3	-	2/3
H	1/3	2/3	-
I	2/3	-	1/3
J	2/3	1/3	-



	Cost (USD/yr)	CO_2 (Ton/yr)	Water (Kg/yr)	CHP Tech	CHP Size (kWe)
Min Cost	144,307	3,987	46,411,000	ICE	335
Min Emissions	208,450	1,582	22,186,000	SE	182
Min Water	214,220	1,745	19,602,600	ICE	290
A	182,580	1,679	23,842,000	SE	180
B	144,310	2,016	24,771,000	ICE	285
C	147,120	3,168	37,260,000	ICE	280
D	180,630	1,655	19,603,000	ICE	287
E	193,340	1,582	22,186,000	SE	182
F	193,340	1,582	22,186,000	SE	182
G	184,910	2,482	28,952,000	MT	197
H	144,310	2,016	24,772,000	ICE	285
I	173,190	1,860	23,842,000	SE	181
J	180,630	1,655	19,603,000	ICE	287
Min Average	182,580	1,679	23,842,000	SE	180
Min Worst	144,310	2,016	24,771,000	ICE	285
Utopia	144,307	1,582	19,602,600		
Nadir	214,220	3,987	46,411,000		

Medidas de Riesgo Coherentes – Coherent Risk Measures

Coherency Conditions:

- **Homogeneity:** $\rho(\lambda X) = \lambda\rho(X)$
- **Subadditivity:** $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$
- **Normalized:** $\rho(0) = 0$
- **Monotonicity:** If $X_1 \leq X_2$ a.s. then $\rho(X_1) \leq \rho(X_2)$

$\Longleftrightarrow$

Norm Conditions:

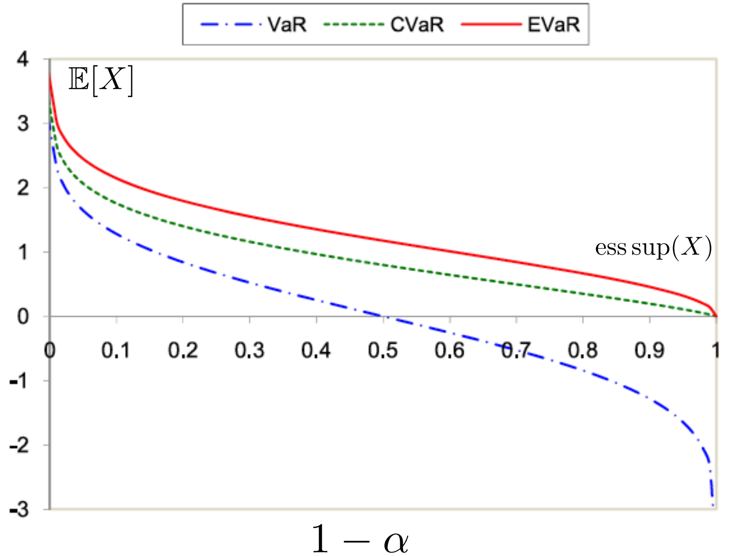
- **Homogeneity:** $\rho(\lambda \mathbf{x}) = \lambda\rho(\mathbf{x})$
- **Subadditivity:** $\rho(\mathbf{x}_1, \mathbf{x}_2) \leq \rho(\mathbf{x}_1) + \rho(\mathbf{x}_2)$
- **Normalized:** $\rho(0) = 0$

Incoherent Risk Measures:

- **Value at Risk:** $\mathrm{VaR}_\alpha(X) := \inf_{t\in\mathbb{R}} \{t : \Pr(X \leq t) \geq \alpha\}$ (Violates Subadditivity)
- **Mean-Standard-Deviation:** $\text{M-SD}_\lambda = \mathbb{E}[X] + \lambda\sigma(X)^2$ (Violates Monotonicity)

Coherent Risk Measures:

- **Expected Value:** $\mathbb{E}[X]$
- **Worst-Case Value:** $\operatorname{ess\,sup}(X)$
- **Conditional Value at Risk:** $\inf_{t\in\mathbb{R}} \left\{t + \frac{1}{1-\alpha}\mathbb{E}[(X-t)_+]\right\}$
- **Entropic Value at Risk:** $\inf_{t>0} \left\{\frac{1}{t}\log \mathbb{E}[\exp(t\,X)]\right\}$



Some Relationships:

$$\mathbb{E}[X] \leq \mathrm{CVaR}_\alpha(X) \leq \mathrm{EVaR}_\alpha(X) \leq \operatorname{ess\,sup}(X)$$

$$\mathrm{VaR}_\alpha(X) \leq \mathrm{CVaR}_\alpha(X) \leq \mathrm{EVaR}_\alpha(X)$$

Índice de Entropía Generalizada - Generalized Entropy Index

Generalized Entropy Index

$$GE_\beta(x) := \frac{1}{m\beta(\beta-1)} \sum_{i\in\mathcal{S}} \left(\left(\frac{s_i(x)}{\bar{s}(x)} \right)^\beta - 1 \right), \quad \beta \in [-1, 2]$$
$$= \frac{1}{m\beta(\beta-1)} \frac{1}{\bar{s}(x)^\beta} \sum_{i\in\mathcal{S}} \left(s_i(x)^\beta - \bar{s}(x)^\beta \right)$$

Mean Log Deviation $\beta = 0$

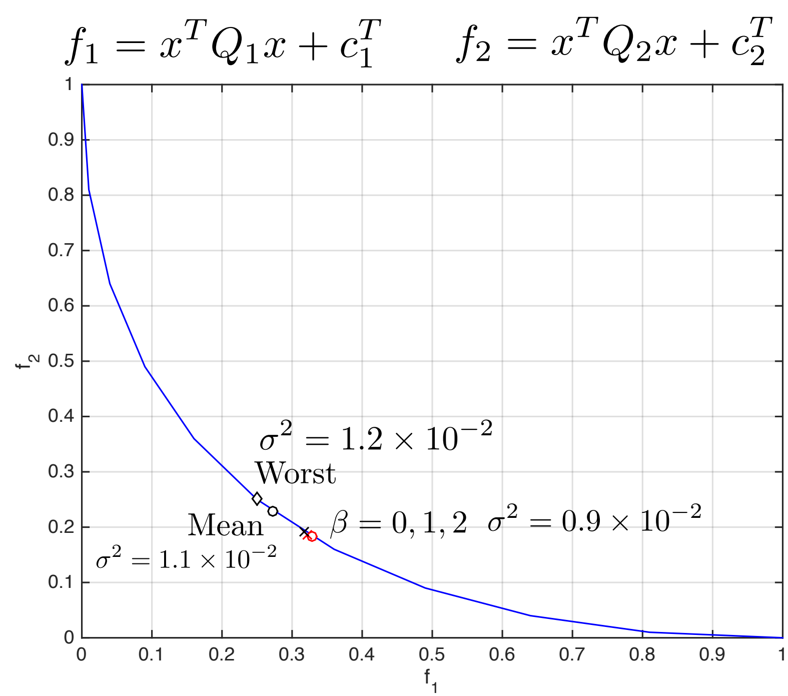
$$GE_0(x) = \log \bar{s}(x) - \frac{1}{m} \sum_{i\in\mathcal{S}} \log s_i(x)$$

Theil Index $\beta = 1$

$$GE_1(x) = \frac{1}{m} \sum_{i\in\mathcal{S}} \frac{s_i(x)}{\bar{s}(x)} \log \frac{s_i(x)}{\bar{s}(x)}$$
$$= \frac{1}{\bar{s}(x)} \left(\frac{1}{m} \sum_{i\in\mathcal{S}} s_i(x) \log s_i(x) - \bar{s}(x) \log \bar{s}(x) \right)$$

Squared Coefficient of Variation $\beta = 2$

$$GE_2(x) = \frac{1}{2} \left(\frac{\sigma(x)}{\bar{s}(x)} \right)^2$$



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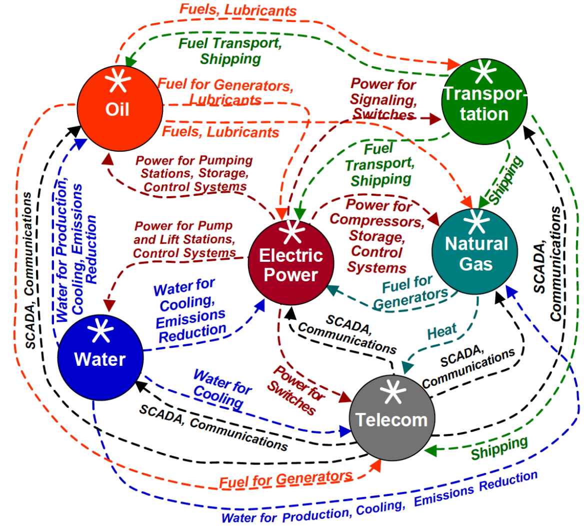
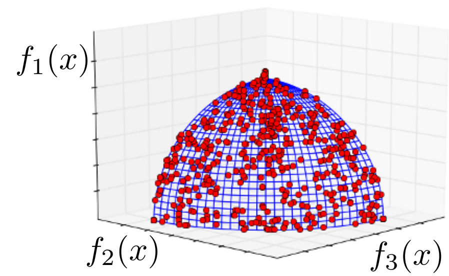
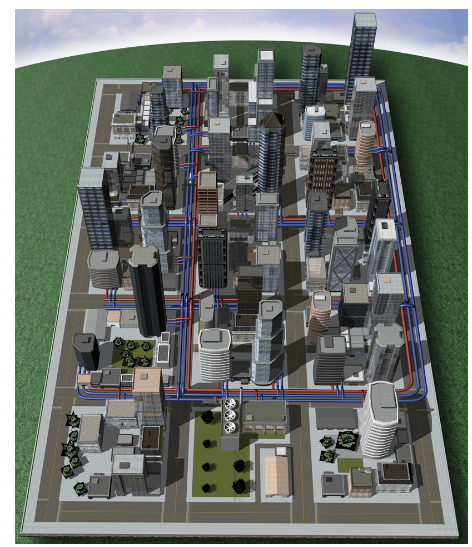


Un Marco de CVaR para Optimización con Intereses Múltiples

(A CVaR Framework for Multi-Stakeholder Optimization)



Victor M. Zavala
Department of Chemical and Biological Engineering
University of Wisconsin-Madison
http://zavalab.engr.wisc.edu



With: A. Dowling (UWisc)
L.F. Fuentes J.M. Ponce (UMichoacan)
G. Ruiz-Mercado (EPA)