Network Upgrade Design in Tiered CDMA Cellular Networks

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Abstract

Cellular network operators must periodically upgrade their networks by installing new base stations in order to accommodate traffic growth. In this study, an optimization model is developed to determine an optimal location for deploying a new base station in an overloaded, single-macrocell CDMA system. The optimization model explicitly considers inter-cell interference and accounts for both path gain-based terminal-to-base assignments as well as optimal assignments that minimize interference and thereby maximize total system capacity. The results demonstrate that for both assignments schemes, several base station locations may support the maximum capacity when the two-cell system is underloaded. As the traffic demand increases, however, an optimal location for the new base station emerges. For the case of multiple locations, an algorithm is proposed to select from among the alternatives a location that requires low total transmit power. Although the low power locations are similar for the two base assignment schemes, the system can support more capacity with the optimal assignment scheme than with the path gain-based scheme. For these path gain-based assignments, the maximum capacity depends on the *tiered* nature of the deployment, i.e., the typical coverage area offered by the newly-installed base station.

I. Introduction and Motivation

Cellular network operators must periodically upgrade and expand their networks to accommodate traffic growth. The goal of these upgrades is to deploy additional base station equipment at strategic locations either to increase overall network capacity or to provide access in coverage

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deadspots. Due to limited financial resources, service providers often have only a small number of base station deployments at their disposal. Thus, a key challenge in designing such network upgrades lies in *identifying desirable locations for new base station installations*. For example, the City of New York recently made 18,000 locations, primarily street lamps or traffic lights, available for rent to six wireless service providers [1]. The service providers now must identify the subset of these locations that optimally complement and interact with their existing base stations in the region.

Due to spectral and financial limitations, code-division multiple access (CDMA) cellular operators often must tackle such network upgrades while contending with *both* inter- and intra-cell interference in the uplink. Because of full spectral reuse, active user terminals in CDMA systems interfere with each other. While the intra-cell interference is controlled within each cell using a power control algorithm, the inter-cell interference is not and can greatly impact network coverage and *user capacity*, i.e., the total number of simultaneous terminals supported [2], [3]. For example, users transmitting to one base station contribute to inter-cell interference experienced at a neighboring base station and thus impact the number of users that the second base can reliably support. Further, since cellular CDMA capacity is uplink-limited [2], this type of inter-cell interference significantly dictates *overall* network performance. As a result, upgrades of CDMA systems require careful installations of new (interfering) cells as they may significantly alter network capacity and coverage.

The focus of this study is to present basic models and foundational results governing network upgrade design in cellular CDMA systems. Specifically, we study a system with an existing macrocell that is unable to support the traffic demands within its coverage area. We develop an integer programming model to determine an optimal location for deploying a second base station while accounting for both intra-cell interference within and inter-cell interference between the two cells. The solution yields not only an optimal deployment location but also a set of associated base station assignments for each served demand node. The optimization model is structured to accommodate either *optimal* base station assignments or *path gain-based* assignments. *Optimal* base station assignments that minimize overall interference and thus maximize overall system capacity may be obtained by implementing the types of algorithms presented in [4], [5], [6], etc. *Path gain-based* assignments dictate that terminals may only communicate to the base station with which they have the highest path gain. Since path gains are determined in large part by the antenna properties of the base station, an optimal base station location under such path gain-driven assignments will depend on the typical coverage area offered by the newly deployed base, e.g., whether it is a macrocell or microcell base station. Therefore, the optimization model accounts for *tiered* deployments in which the coverage areas of the base stations can vary in their order of magnitude.

The numerical results show that for both optimal and path gain-based assignments, multiple locations for the new base station are able to support the maximum demand when the overall traffic is overloaded for a single-cell system but underloaded for the combined two-cell network. As the traffic demand increases, however, an optimal location for the new base station emerges. Given that there may be multiple solutions to the location problem, we propose an algorithm that selects from among the alternatives a location that requires low total transmit power from user terminals. The numerical results show that the low power locations identified by the algorithm for the optimal and path gain-based assignment schemes are in close proximity to each other. However, the maximum capacity supported at these desirable locations is larger for the optimal assignment scheme than for the path gain-based scheme. Furthermore, in the case of path gain-based assignments, we observe that the optimal maximum capacity is highly dependent on the typical coverage offered by the newly installed base, i.e., on its antenna properties. Overall, the results of this study provide key insights, which will serve as an important building block in developing a general network upgrade design tool for cellular CDMA operators. The extension of this work for larger, multicell systems is the focus of ongoing work.

In Section II, we review related literature on cell design of CDMA networks. We present our optimization model in Section III and discuss solution approaches in Section IV. In Section V, we apply our solution algorithm to upgrade several overloaded network scenarios. The numerical results illustrate several key issues for cellular CDMA operators when they expand their network resources. We conclude with some directions for future research in Section VI.

II. Related Work

While the performance of cellular CDMA systems has been the focus of numerous earlier papers, the network design of these systems has received much less attention. Early work on cellular network design (e.g., [7], [8], [9], [10]) proposed approaches that assumed no intra- or inter-cell interference. More recent work, however, has examined the impact of interference.

For example, in [11], the authors develop an optimization model that incorporates intra-cell interference. In [12], the authors study the problem of where to locate base stations so as to maximize user capacity in a cellular CDMA network. They capture the impact of intra-cell and *average* inter-cell interference using power-control constraints. They find solutions to the difficult nonlinear programming problems using sequential quadratic programming and simulated annealing. In [13], the authors seek optimal base station locations that minimize a cost function, which is a linear combination of total installation cost and total transmitted power, while ensuring that all demand nodes meet signal-to-noise-plus-interference ratio (SINR) requirements. The SINR-based power control constraints give rise to a mixed integer nonlinear programming problem; the authors develop a Tabu Search algorithm to heuristically solve instances. In [14], the authors build on the work of [13] and develop an integer programming model that maximizes service provider net revenues and that considers the SINR constraints as hard constraints. They develop a specialized algorithm and present computational results. In [12], [13] and [14], the authors focus on the design on new CDMA networks, i.e., they assume no interaction with any existing base stations. Furthermore, they consider only systems in which user terminals select base stations according to path gains or received signal powers.

Our work differs from previous work in several important ways. First, we focus on the problem of where to locate a new base station within an existing system. Second, we explicitly model both intra-cell and inter-cell interference. Third, we allow for either path gain-based assignments or optimal assignments. Finally, in the case of path gain-based assignments, we consider the effects of tiered base stations. Since incorporating these features into just a single macrocell system gives rise to a difficult optimization problem, the extension to multicell systems will be challenging. We expect, however, that insights derived from the single cell system will be useful in developing a practical approach to solving larger, multicell systems.

III. Problem Formulation

A. System Model

We study a two-cell system where the location of one base station is known and an optimal location for the second base station is desired. For ease of discussion, we refer to the existing base station as a *macrocell* base station and to the newly deployed base station as a *microcell* base station. We consider a region, $\mathcal{R}$, which contains a macrocell base station located at the origin. Within the region, we model traffic requests using a set of demand nodes, I. Since user demand typically is a continuous function over the network's geographic region, the demand nodes represent a discretized version of the demand within the region. For example, the entire geographic region can be subdivided into L smaller subregions and a demand node may represent the average demand (or some percentile demand) of a subregion, located at its center of gravity. For each demand node $i \in I$, we let (x_i, y_i) denote the location of demand node i, u_i denote the number of active connections from demand node i and g_i denote the path gain from demand node i to the macrocell base. For a microcell deployed at location (x, y), the path gain from demand node i to the microcell base is $h_i(x, y)$.

We assume that user terminals in both cells use single-rate CDMA over the same set of frequencies, thereby creating inter-cell interference. The objective of the *microcell location problem* is to determine a location (x, y) for the microcell base station such that the total number of user terminals supported is maximized subject to meeting quality of service (QoS) requirements. These QoS requirements are expressed in terms of minimum SINR constraints on the uplink.[1]

Ideally, we would like to determine an optimal location for the new microcell over the continuous range of points in $\mathcal{R}$. The difficulty with this goal, however, lies in expressing $h_i(x, y)$ over a continuous range of x and y. Although $h_i(x, y)$ depends (in part) on the distance between demand node i and the microcell at location (x, y), it is generally difficult to characterize $h_i(x, y)$ as a simple continuous function of (x, y).[2] More realistically, field measurements of the path gain for demand node i to the microcell, $h_i(x, y)$, are available to network designers for a *discrete set of candidate microcell locations*. Therefore, instead of solving the continuous location problem, we focus on finding an optimal microcell location from among a discrete set of points.

To formulate the discrete microcell location problem, we define four sets:

- First, we let $J = \{1, 2, \cdots\}$ denote the set of candidate microcell locations; each point $j \in J$ has coordinates $(x_j, y_j) \in \mathcal{R}$. We let $j = 0$ denote the macrocell base station located at the origin.

[1] This is because uplink requirements tend to be more stringent than downlink SINR requirements [2].

[2] $h_i(x, y)$, for example, may also depend on attenuation due to shadowing, which in turn is related to the nature of the obstacles between a user at demand node i and a microcell antenna at location (x, y). These characteristics are difficult to generalize as simple functions of (x, y).

- Second, for each demand node $i \in I$, we define a set J_i that specifies the subset of candidate microcell locations that can serve demand node i. When we determine an optimal microcell location using optimal base station assignments, we set $J_i = J$; for path gain-based assignments, we restrict $J_i = \{j \in J : h_i(x_j, y_j) > g_i\}$.
- Third, for notational convenience, we define $\mathcal{J}_i$ to denote the inclusive set of base station locations that can serve demand node i. That is, $\mathcal{J}_i$ is the set J_i plus the macrocell location if it is feasible to assign demand node i to the macrocell in some scenario. For the case of optimal assignments, $\mathcal{J}_i = J_i \cup \{0\}$; for path gain-based assignments, $\mathcal{J}_i = J_i \cup \{0\}$ *if* there is at least one candidate location $j \in J$ for which $h_i(x_j, y_j) \leq g_i$; otherwise, $\mathcal{J}_i = J_i$.
- Finally, we define I_j as the set of demand nodes that can be served by the base station at location j. For optimal base station assignments, we assume $I_j = I$ for all $j \in J$ and for path gain-based assignments, we assume $I_j = \{i \in I : g_i \geq h_i(x_j, y_j)\}$. For the macrocell, we define $I_0 = I$ for optimal assignments, and we define $I_0 = \{i \in I : g_i \geq h_i(x_j, y_j)$ for some $j \in J\}$ for path gain-based assignments.

B. SINR Constraints

In *single-rate* CDMA systems, the near-far problem necessitates that the received powers of all in-cell users must be equal [15]. We let S_M and S_μ be the received powers at the macrocell and microcell base stations, respectively. To formulate the uplink SINR constraints, we define the following assignment variables:

$$z_{ij} = \begin{cases} 1 & \text{if demand node } i \in I \text{ is served by a base station at location } j \in \mathcal{J}_i \\ 0 & \text{otherwise} \end{cases}$$

Assuming the microcell base station is located at some particular location $j \in J$, the SINR requirement at the macrocell base for a user i can be written as

$$\frac{\frac{W}{R} S_M z_{i0}}{S_M(u_i - 1) + S_M \sum_{l \in I_0, l \neq i} u_l z_{l0} + S_\mu \sum_{l \in I_j, l \neq i} \frac{g_l u_l z_{lj}}{h_l(x_j, y_j)} + \eta W} \geq \Gamma z_{i0}, \tag{1}$$

where W is the system bandwidth; R is the data rate of each user; $\frac{W}{R}$ is its processing gain; ηW is the power of the additive white Gaussian noise at the receiver; and Γ is the minimum SINR requirement. Note that the first term in the denominator captures the same-cell interference due to other transmitting users at demand node i; the second term captures the intra-cell interference due to other demand nodes assigned to the macrocell; and the third term captures the inter-cell interference from demand nodes assigned to the microcell at (x_j, y_j).

Similarly, for the microcell base at (x_j, y_j), we write the SINR requirement for a user k as

$$\frac{\frac{W}{R} S_\mu z_{kj}}{S_\mu (u_k - 1) + S_\mu \sum_{l \in I_j, l \neq k} u_l z_{lj} + S_M \sum_{l \in I_0, l \neq k} \frac{h_l(x_j,y_j) u_l z_{l0}}{g_l} + \eta W} \geq \Gamma z_{kj}. \tag{2}$$

If we let $K = \frac{W}{R\Gamma} + 1$ denote the "single-cell pole capacity" [16], i.e., the maximum number of users that can be supported in the macrocell in the absence of any other cell in the system, then our assumption that the macrocellular coverage is insufficient implies that $K < \sum_{i \in I} u_i$. Rearranging terms and substituting K, we can rewrite the SINR requirements (1) and (2) as

$$S_M z_{i0} \left(K - u_i - \sum_{l \in I_0, l \neq i} u_l z_{l0} \right) - S_\mu z_{i0} \sum_{l \in I_j, l \neq i} \frac{g_l u_l z_{lj}}{h_l(x_j, y_j)} - \eta W z_{i0} \geq 0, \tag{3}$$

and

$$S_\mu z_{kj} \left(K - u_k - \sum_{l \in I_j, l \neq k} u_l z_{lj} \right) - S_M z_{kj} \sum_{l \in I_0, l \neq k} \frac{h_l(x_j, y_j) u_l z_{l0}}{g_l} - \eta W z_{kj} \geq 0. \tag{4}$$

These inequalities can be written in matrix form as

$$\mathbf{A} \begin{bmatrix} S_M \\ S_\mu \end{bmatrix} \geq \begin{bmatrix} \eta W z_{i0} \\ \eta W z_{kj} \end{bmatrix} \tag{5}$$

where

$$\mathbf{A} = \begin{bmatrix} z_{i0} \left(K - u_i - \sum_{l \in I_0, l \neq i} u_l z_{l0} \right) & -z_{i0} \sum_{l \in I_j, l \neq i} \frac{g_l u_l z_{lj}}{h_l(x_j, y_j)} \\ -z_{kj} \sum_{l \in I_0, l \neq k} \frac{h_l(x_j,y_j) u_l z_{l0}}{g_l} & z_{kj} \left(K - u_k - \sum_{l \in I_j, l \neq k} u_l z_{lj} \right) \end{bmatrix}$$

The SINR requirements (1) and (2) are satisfied if and only if there exist positive received power levels such that (5) can be met with equality for all pairs of users i and k. This further implies that the SINR conditions can be met if and only if $det(\mathbf{A}) \geq 0$ for all $i \in I$ and $k \in I$, or equivalently, if

$$\begin{aligned} & z_{i0} z_{kj} \left(K - u_i - \sum_{l \in I_0, l \neq i} u_l z_{l0} \right) \left(K - u_k - \sum_{l \in I_j, l \neq k} u_l z_{lj} \right) \\ - \quad & z_{i0} z_{kj} \sum_{l \in I_j, l \neq i} \frac{u_l z_{lj}}{f_l(x_j, y_j)} \sum_{l \in I_0, l \neq k} f_l(x_m, y_m) u_l z_{l0} \geq 0, \end{aligned} \tag{6}$$

where

$$f_i(x_j, y_j) = \frac{h_i(x_j, y_j)}{g_i}. \tag{7}$$

Because of the zero-one property of each variable z_{i0} and z_{kj}, the product $z_{i0}z_{kj}$ is always greater than or equal to 0. If either z_{i0} or z_{kj} equals 0, the constraint in (6) is automatically met; the full constraint (6) therefore is active only when both z_{i0} and z_{kj} equal one. In that case, we observe

$$\sum_{l\in I_0, l\neq i} u_l z_{l0} = \sum_{l\in I_0} u_l z_{l0} - u_i \tag{8}$$

$$\sum_{l\in I_j, l\neq k} u_l z_{lj} = \sum_{l\in I_j} u_l z_{lj} - u_k \tag{9}$$

$$\sum_{l\in I_j, l\neq i} \frac{u_l z_{lj}}{f_l(x_j, y_j)} = \sum_{l\in I_j} \frac{u_l z_{lj}}{f_l(x_j, y_j)} \tag{10}$$

$$\sum_{l\in I_0, l\neq k} f_l(x_j, y_j) u_l z_{l0} = \sum_{l\in I_0} f_l(x_j, y_j) u_l z_{l0} \tag{11}$$

Equations (10) and (11) are valid since we assume that a user can be assigned either to the microcell or to the macrocell. That is, since $z_{i0} = 1$, we require $z_{ij} = 0$; and since $z_{kj} = 1$, we require $z_{k0} = 0$.[3] Substituting equations (8) through (11) in constraint (6) yields a *feasibility condition* that is independent of i and k:

$$\left(K - \sum_{l\in I_0} u_l z_{l0}\right)\left(K - \sum_{l\in I_j} u_l z_{lj}\right) - \sum_{l\in I_j} \frac{u_l z_{lj}}{f_l(x_j, y_j)} \cdot \sum_{l\in I_0} f_l(x_j, y_j) u_l z_{l0} \geq 0. \tag{12}$$

Therefore, for a microcell located at a particular location $j \in J$, the single nonlinear constraint (12) expresses the uplink SINR requirements for all users, both those assigned to the macrocell base located at the origin and those assigned to the microcell base located at candidate point j.

C. The Microcell Location Problem

The objective of the *microcell location problem* is to select one of the candidate locations for the microcell and to determine the assignment of demand nodes to base stations in such a way as to maximize the total number of user terminals supported subject to meeting uplink SINR requirements. To formulate the problem, we define a class of decision variables for the candidate locations:

$$q_j = \begin{cases} 1 & \text{if candidate location } j \in J \text{ is selected for microcell deployment} \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

[3] Thus, we design the system so that high demand locations are not in soft-handoff, i.e., user terminals at demand nodes do not communicate to *both* base stations simultaneously.

We formulate the microcell location problem as the following optimization problem.

$$\min -\sum_{i\in I}\sum_{j\in \mathcal{J}_i} u_i z_{ij} \tag{14}$$

subject to

$$q_j\left[\left(K-\sum_{k\in I_0}u_k z_{k0}\right)\left(K-\sum_{l\in I_j}u_l z_{lj}\right)-\sum_{k\in I_j}\sum_{l\in I_0}\frac{f_l(x_j,y_j)}{f_k(x_j,y_j)}u_k u_l z_{kj} z_{l0}\right]\geq 0 \quad \forall j\in J \tag{15}$$

$$\sum_{j\in \mathcal{J}_i} z_{ij} \;\leq\; 1 \quad \forall i\in I \tag{16}$$

$$\sum_{j\in J} q_j \;=\; 1 \tag{17}$$

$$z_{ij}-q_j \;\leq\; 0 \quad \forall i\in I, \forall j\in J_i \tag{18}$$

$$z_{ij} \;\in\; \{0,1\} \quad \forall i\in I, \forall j\in \mathcal{J}_i \tag{19}$$

$$q_j \;\in\; \{0,1\} \quad \forall j\in J \tag{20}$$

The objective (14) of maximizing the total number of users supported is expressed equivalently as minimizing the negative value. Constraints (15) ensure that the SINR requirements are met if location j is selected for the microcell. Constraints (16) allow each demand node to be assigned to at most one base station. Constraint (17) restricts the microcell to be located at one candidate location. Constraints (18) link the assignment and location variables; a demand node i cannot be assigned to the microcell at location j unless location j has been selected. Constraints (19) and (20) restrict the assignment and location variables to binary values. The formulation accommodates optimal assignments or path gain-based assignments via the set definitions specified earlier.

As written, the microcell location problem is a $0-1$ integer program with a set of nonlinear constraints (15). Although we have a feasibility condition for each candidate microcell location, only the feasibility condition corresponding to a selected location is active in any solution. Since exactly one location will be selected, we can combine all of the constraints (15) into one by summing over the set of candidate locations:

$$\sum_{j\in J} q_j\left(K-\sum_{k\in I_0}u_k z_{k0}\right)\left(K-\sum_{l\in I_j}u_l z_{jl}\right)-\sum_{j\in J} q_j\sum_{k\in I_j}\sum_{l\in I_0}\frac{f_l(x_j,y_j)}{f_k(x_j,y_j)}u_k u_l z_{kj} z_{l0}\geq 0 \tag{21}$$

Multiplying out the product terms in the first term of (21), we obtain the following:

$$\sum_{j\in J} q_j K^2-\sum_{j\in J} q_j K\sum_{k\in I_0}u_k z_{k0}-\sum_{j\in J} q_j K\sum_{l\in I_j}u_l z_{lj}+\sum_{j\in J} q_j\sum_{k\in I_0}u_k z_{k0}\sum_{l\in I_j}u_l z_{lj} \tag{22}$$

From constraint (17), $\sum_{j\in J} q_j = 1$, we can simplify the first two terms of (22). From constraints (18), $z_{ij} - q_j \leq 0$, we get that $z_{lj}q_j = z_{lj}$, which we can use to simplify the last two terms of (22). Then, we can write (22) as

$$K^2 - K\sum_{k\in I_0} u_k z_{k0} - K\sum_{j\in J}\sum_{l\in I_j} u_l z_{lj} + \sum_{j\in J}\sum_{k\in I_0}\sum_{l\in I_j} u_k u_l z_{k0} z_{lj} \tag{23}$$

In the second term of (21), we again use the fact that $z_{lj}q_j = z_{lj}$ to simplify the expression as

$$\sum_{j\in J}\sum_{k\in I_j}\sum_{l\in I_0} \frac{f_l(x_j, y_j)}{f_k(x_j, y_j)} u_k u_l z_{kj} z_{l0} \tag{24}$$

Therefore, we can replace the set of constraints (15) with the following single SINR constraint on the system

$$\begin{aligned} K^2 - K\sum_{k\in I_0} u_k z_{k0} - K\sum_{j\in J}\sum_{l\in I_j} u_l z_{lj} + \sum_{j\in J}\sum_{k\in I_0}\sum_{l\in I_j} u_k u_l z_{k0} z_{lj} &\quad - \\ \sum_{j\in J}\sum_{k\in I_j}\sum_{l\in I_0} \frac{f_l(x_j, y_j)}{f_k(x_j, y_j)} u_k u_l z_{kj} z_{l0} &\quad \geq \quad 0 \end{aligned} \tag{25}$$

Even after all of the simplifications, however, constraint (25) is still nonlinear, and the microcell location problem is still a nonlinearly constrained $0-1$ integer program. The field of integer nonlinear programming, unlike the field of integer *linear* programming, is still relatively new and general solution approaches that guarantee global optimality are not yet available. Recent research (e.g., [17], [18], [19]) indicates that applying deterministic branch-and-bound algorithms to the global optimization of mixed integer nonlinear programs is promising. However, designing such an algorithm requires significant customization in terms of developing appropriate relaxations, range reduction procedures and branching strategies [20]. Instead of pursuing that approach, we apply a standard technique for linearizing the nonlinear constraint to obtain an integer *linear* program.

The nonlinearity in the SINR constraint (25) arises from bilinear terms involving the product of assignment variables, e.g., $z_{kj}z_{l0}$. We introduce a new variable y_{kjl} to represent the product $z_{kj}z_{l0}$ for all j in J, k in I_j and l in I_0. Substituting y_{kjl} for $z_{kj}z_{l0}$ in (25), we obtain a SINR constraint that is linear in the y_{kjl} terms:

$$\begin{aligned} K^2 - K\sum_{k\in I_0} u_k z_{k0} - K\sum_{j\in J}\sum_{l\in I_j} u_l z_{lj} + \sum_{j\in J}\sum_{k\in I_0}\sum_{l\in I_j} u_k u_l y_{ljk} &\quad - \\ \sum_{j\in J}\sum_{k\in I_j}\sum_{l\in I_0} \frac{f_l(x_j, y_j)}{f_k(x_j, y_j)} u_k u_l y_{kjl} &\quad \geq \quad 0 \end{aligned} \tag{26}$$

To enforce the behavior that the variable y_{kjl} takes on the value one only when both z_{kj} and z_{l0} are one and the value zero otherwise, we add the following constraints:

$$z_{kj} - (1 - z_{l0}) \leq y_{kjl} \leq z_{kj} \quad (27)$$

$$0 \leq y_{kjl} \leq z_{l0} \quad (28)$$

Then, we reformulate the microcell location problem as the following integer linear program:

$$\min - \sum_{i \in I} \sum_{j \in \mathcal{J}_i} u_i z_{ij}$$

subject to

$$(16), (17), (18), (19), (20), (26), (27), (28)$$

Note that the linearization process increases the size of the formulation. We add $|J||I|^2$ y_{kjl} variables in the case of optimal assignments and $(\sum_{j \in J} |I_j|)|I_0|$ y_{kjl} variables in the case of path gain-based assignments. In either case, we add four constraints for each new variable. Eventually, the size of the expanded formulation may become too large to solve using standard integer programming techniques, especially if the linear programming relaxation bound is weak. At that point, we may seek to strengthen the formulation by identifying and adding valid inequalities or to investigate alternative solution approaches. Throughout the rest of the paper, references to the microcell location problem refer to the linear formulation.

IV. Solution Algorithm

The microcell location problem is a $0-1$ integer programming problem that can be solved using a standard branch-and-bound algorithm. The solution to a particular instance specifies a location for the microcell and a set of feasible assignments, and the corresponding objective function value specifies the maximum capacity that the system can support. Depending on the scenario, there may be multiple candidate locations (and possibly multiple corresponding assignments) that can support the maximum capacity. In that case, we would like to identify a location and a set of assignments that minimizes the total transmit power. For a microcell at location j, the total transmit power is calculated as

$$\text{Ptotal}_j = \sum_{i \in I_0} \frac{S_M}{g_i} z_{i0} + \sum_{i \in I_j} \frac{S_\mu}{h_i(x_j, y_j)} z_{ij} \quad (29)$$

Using this expression, we can write the following formulation for the problem of determining the location and the set of terminal-to-base assignments that minimizes the total transmit power:

$$\min \sum_{j \in J} \text{Ptotal}_j q_j \tag{30}$$

subject to

$$(16), (17), (18), (19), (20), (26), (27), (28)$$

There are two main issues to consider in solving instances of this formulation. First, we are only interested in solutions that can support the maximum capacity. If we know the maximum capacity value, say z^*, we can solve the problem with the additional constraint

$$\sum_{i \in I} \sum_{j \in \mathcal{J}_i} z_{ij} \geq z^* \tag{31}$$

Alternatively, we can solve a series of problems for decreasing values of z^* (starting from $|I|$ for example) until we find a feasible solution. Second, even for a fixed location and a known maximum capacity value, the problem is still a $0-1$ integer programming problem but now with a nonlinear objective function. Although solving this problem exactly to find the minimum transmit power solution is difficult, finding a low transmit power solution may be possible. An assignment vector **z** that maximizes the difference between the value of the left-hand side of (26) and zero, i.e., a vector **z** that maximizes $det(\mathbf{A})$, should correspond to a low transmit power solution.[4] To find a solution that maximizes $det(\mathbf{A})$, we define a variable $\epsilon \geq 0$ to measure the difference between the value of the left-hand side of (26) and zero. Then, we propose the following algorithm for identifying a low transmit power (but not necessarily minimum transmit power) solution.

ALGORITHM *Microcell Location*

1. Solve the microcell location problem to determine the maximum system capacity. Let z^* denote the maximum capacity value (the objective function value).
2. For each candidate location $j \in J$,

[4]The largest $det(\mathbf{A})$ value corresponds to small (but not necessarily smallest) values of S_M and S_μ since $det(\mathbf{A})$ forms the (common) denominator of both S_M and S_μ. Small values of S_M and S_μ yield low (but not necessarily minimum) total transmit power.

a. Fix q_j to 1 and fix q to 0 for all $l \in J, l \neq j$.

b. Solve the following modified version of the microcell location problem.

$$\min -\epsilon \tag{32}$$

subject to

$$K^2 - K \sum_{k \in I_0} u_k z_{k0} - K \sum_{j \in J} \sum_{l \in I_j} u_l z_{lj} + \sum_{j \in J} \sum_{k \in I_0} \sum_{l \in I_j} u_k u_l y_{ljk} \quad - \\ \sum_{j \in J} \sum_{k \in I_j} \sum_{l \in I_0} \frac{f_l(x_j, y_j)}{f_k(x_j, y_j)} u_k u_l y_{kjl} \quad \geq \quad \epsilon \tag{33}$$

$$(16), (17), (18), (19), (20), (27), (28), (31)$$

c. If there is a feasible solution, compute the total transmit power of the solution and store the value as Ptotal_j.

3. From among all feasible solutions, select the candidate location with the smallest value of Ptotal.

END OF *Microcell Location*

V. Numerical Results

In this section, we present the results of our numerical experiments. To test our algorithm, we created 10 network scenarios for a CDMA system with processing gain (W/R) of 128 and a minimum required SINR of 5, i.e., $K = W/\Gamma R + 1 = 26.6$. For each network scenario, we generated the locations of the demand nodes within a square region $\mathcal{R}$ with sides of length 1 km. Specifically, we assume that the existing macrocell base station is located at the origin and that the square region extends from -0.5 km to 0.5 km in both the x and y directions. For simplicity, we assume unit demand nodes, i.e., $u_i = 1$ for all $i \in I$.

For each network scenario, demand node locations are determined using some combination of the following three location distributions:

(1) A uniform distribution over [-0.5 km, 0.5 km] $\times$ [-0.5 km, 0.5 km];

(2) A uniform distribution over [0.2 km, 0.4 km] $\times$ [-0.1 km, 0.1 km]; or

(3) A uniform distribution over [-0.1 km, 0.1 km] $\times$ [0.2 km, 0.4 km].

Options (2) and (3), when used, create hotspot regions centered at (0.3 km, 0 km) and (0 km, 0.3 km), respectively. Table I summarizes the demand node characteristics of the 10 network scenarios. Scenario 1, for example, contains 30 demand nodes uniformly distributed over $\mathcal{R}$, i.e., generated using distribution (1). Scenario 3 contains the 30 nodes from Scenario 1 plus 8 additional nodes obtained using distribution (2), i.e., with a hotspot centered at (0.3 km, 0 km). Overall, we can classify the 10 network scenarios as examples of four possible *node configurations*: uniformly distributed over $\mathcal{R}$ (Scenarios 1, 2, and 8); uniformly distributed over $\mathcal{R}$ with a hotspot centered around (0.3 km, 0 km) (Scenarios 3, 5, 9); uniformly distributed over $\mathcal{R}$ with a hotspot centered around (0 km, 0.3 km) (Scenarios 4, 6, 10); and uniformly distributed over $\mathcal{R}$ with two hotspots centered around (0.3 km, 0 km) and (0 km, 0.3 km) (Scenario 7).

For the candidate locations for the microcell, any discrete set of locations could be used. In the results presented here, we consider a set of regular grid points. In the case of a grid, we note that as the number of points increases, an optimal solution of the discrete location problem approaches an optimal solution of the continuous location problem. We generated 80 points that are spaced 0.1 km apart in both the x and y coordinates over the range [-0.4 km, 0.4 km], which results in a grid of nine points by nine points. Since we assume that the macrocell is located at the origin, we exclude the origin as a candidate location for the microcell. We number the grid points as shown in Figure 1.

In practical cases, the path gains g_i and $h_i(x_j, y_j)$ would be determined using field measurements. For demonstration purposes, we can generate g_i and $h_i(x_j, y_j)$ as some set of numbers, e.g., as samples of well-established statistical propagation models. To simplify matters, we choose a simple distance-based path gain model:

$$\begin{aligned} g_i &= \frac{H_M}{(x_i^2 + y_i^2)^2} \\ h_i(x_j, y_j) &= \frac{H_\mu}{((x_i - x_j)^2 + (y_i - y_j)^2)^2} \end{aligned}$$

where the path-loss exponent is 4 and H_M and H_μ capture the effects of the antenna heights and gains at the macrocell and microcell bases, respectively.[5] In the case of tiered base stations, e.g., when the newly installed base is in fact a microcell (that is, it has a smaller coverage area than the existing macrocell base), we require that $H \equiv H_M/H_\mu > 1$. For propagation models where the impact of tiered architecture is captured by multiplicative factors similar to H_M and H_μ, we see that $f_i(x_j, y_j)$ is inversely proportional to H. Furthermore, examining condition (12), we

[5] This model is a simplification of the propagation model presented in [21].

observe that under *optimal* assignments when $I_j = I_0 = I$, the effect of H cancels out in the feasibility constraint. Thus, in such cases, the tiered nature of the base stations has *no impact* on the microcell location when optimal base assignments are permitted. On the other hand, as we demonstrate later, the value of H greatly affects the network solution when path gain-based assignments are used.

Table II summarizes the results of the experiments for the optimal assignment scheme. In the table, Max Cap refers to the maximum number of users that can be supported, # Locations indicates the number of grid locations that can support this maximum, Low Ptotal j^* specifies the index of the grid point that yields the lowest total transmit power (obtained via the Microcell Location Algorithm, Section IV) and N_M and N_μ indicate the number of users assigned to the macrocell and the microcell, respectively, in the low total transmit power solution. These results can be summarized as follows:

- For scenarios 1 through 7 (i.e., $|I| \leq 42$), the maximum capacity that can be supported is equal to the number of demand nodes, and there are multiple microcell locations that can support this maximum capacity. For scenarios 8 through 10, the maximum capacity that can be supported is less than the number of nodes and only one or two locations can support the maximum. Thus, we note that when the system is over-loaded for the single-cell scenario but lightly-loaded in the two-cell system, multiple optimal microcell locations exist. However, as the sytem load increases, an optimal location emerges.
- For a given type of node configuration, the candidate location with the low transmit power remains within a small area as the number of demand nodes increases. As the number of demand nodes varies, the selected location tends to move to a horizontally or vertically adjacent grid point. For scenarios 1, 2 and 8 in which the demand nodes are uniformly distributed across the entire region, the selected location moves from point 47 to 46, a horizontally adjacent point. For scenarios 3, 5 and 9 in which there is a hotspot centered at (0.3 km, 0 km), the selected location moves from 52 to 51 to 43; 52 and 51 are horizontally adjacent and 52 and 43 are vertically adjacent. For scenarios 4, 6 and 10 in which there is a hotspot centered at (0 km, 0.3 km), the selected location moves from 65 to 66 to 67; these points all are horizontally adjacent.
- In a lightly-loaded system, the total number of users supported by the macrocell (N_M) can be different than the number of users supported by the optimally-located microcell (N_μ).

Under lightly-loaded conditions, since there are multiple candidate locations that support the maximum capacity, the assignments and thus the numbers of users supported in each cell can vary significantly, even though the total user capacity is the same. However, as the system load increases, there are only a few (e.g., for scenarios 7-10, only 1 or 2) locations that support the maximum capacity, and the corresponding values of N_M amd N_μ are less disparate.

Tables III and IV summarize the results of the experiments for the path gain-based assignment scheme. For each scenario, the results are given for increasing values of H, which captures the effect of higher antenna height and gain of the macrocell base. The path gain-based results yield the following observations:

- The maximum capacity that can be supported decreases as H increases, as does the number of grid points that can support the maximum. Only for small values of H in scenarios 1 through 7 does the maximum capacity equal the number of users. For scenarios 8 through 10, the maximum capacity is less than the number of users. Again, we note that when the two-cell system is under-loaded, multiple candidate locations are feasible; however, as the system becomes heavily loaded, one optimal location emerges.
- Within a particular scenario, as H increases, the candidate location that yields the low transmit power solution remains within a small area, moving horizontally, vertically or diagonally to an adjacent point. Across scenarios with the same demand node configuration and same H value, the selected location also remains relatively consistent (as $|I|$ increases).
- As expected, the number of users assigned to the macrocell increases while the number of users assigned to the microcell decreases as H increases. This is because the microcell coverage shrinks relative to the macrocell coverage as H increases, restricting the total number of users that may be admitted to the overall network.
- For the hotspot configurations, the (low power) assignments of demand nodes to bases stay relatively consistent as H increases. For the hotspot configurations, as H increases and capacity decreases, nodes are simply dropped at the microcell in most instances; in a few instances, nodes are swapped between the two base stations. For the uniform configurations, however, there is much more variation and swapping of demand nodes as H increases.[6]

[6]This observation is not presented in the tables here but was noted in the experimental results.

In comparing the optimal assignment scheme and the path gain-based assignment scheme, we observe the following:

- Overall, more grid points support the maximum capacity in the case of optimal assignments than path gain-based assignments.
- The maximum capacity values are comparable for the two assignment schemes in the case of $H = 1$ for all scenarios and $H = 5$ for some scenarios. For higher values of H, the optimal assignment scheme results in higher maximum capacity values.
- The locations selected by the low power solution (j^*) are similar for the two assignment schemes in most scenarios; the selected locations in scenarios 1, 2 and 8 (with uniform configuration) are most disparate.
- The assignments of nodes to base stations in the low power solutions are fairly consistent between the two assignment schemes when $H = 1$ and the node configuration contains one or more hotspots. For example, Figures 2 and 3 show for scenario 9 how the demand nodes are assigned to the base stations under the optimal assignment scheme and the path gain-based scheme, respectively. When $H = 1$ and the nodes are uniformly distributed, there is a larger variation in the low power assignments for the two assignment methodologies.[7]

VI. CONCLUSION

In this paper, we present an optimization model to determine an optimal location for deploying a new base station in an overloaded, single-macrocell CDMA system. We incorporate explicitly the effect of both inter- and intra-cell interference and consider both optimal and path gain-based assignment schemes. Incorporating these features into the simple two-cell system alone gives rise to a difficult optimization problem. We develop a computationally efficient method to find locations for microcell deployment that maximize user capacity with low total transmit power.

Our numerical results show that, for both assignment schemes, several base station locations may support the maximum capacity when the two-cell network is underloaded. As the traffic demand increases, however, an optimal location for the new base station emerges. Although the two assignment schemes identify low power base station schemes that are in close proximity to each other, the maximum capacity supported at those locations is larger for the optimal

[7]This observation is not presented here in the tables but was noted in the experimental results.

assignment scheme than for the path gain-based one. Furthermore, in the case of path gain-based assignments, we observe that the maximum capacity is highly dependent on the typical coverage offered by the newly installed base.

Given the complexity of the microcell location problem for this two-cell system, we expect that the extension to multicell CDMA cellular systems will be challenging. However, we believe that the insights derived from the study of the two-cell system will play an important role in the development of an effective solution algorithm for larger, multicell systems.

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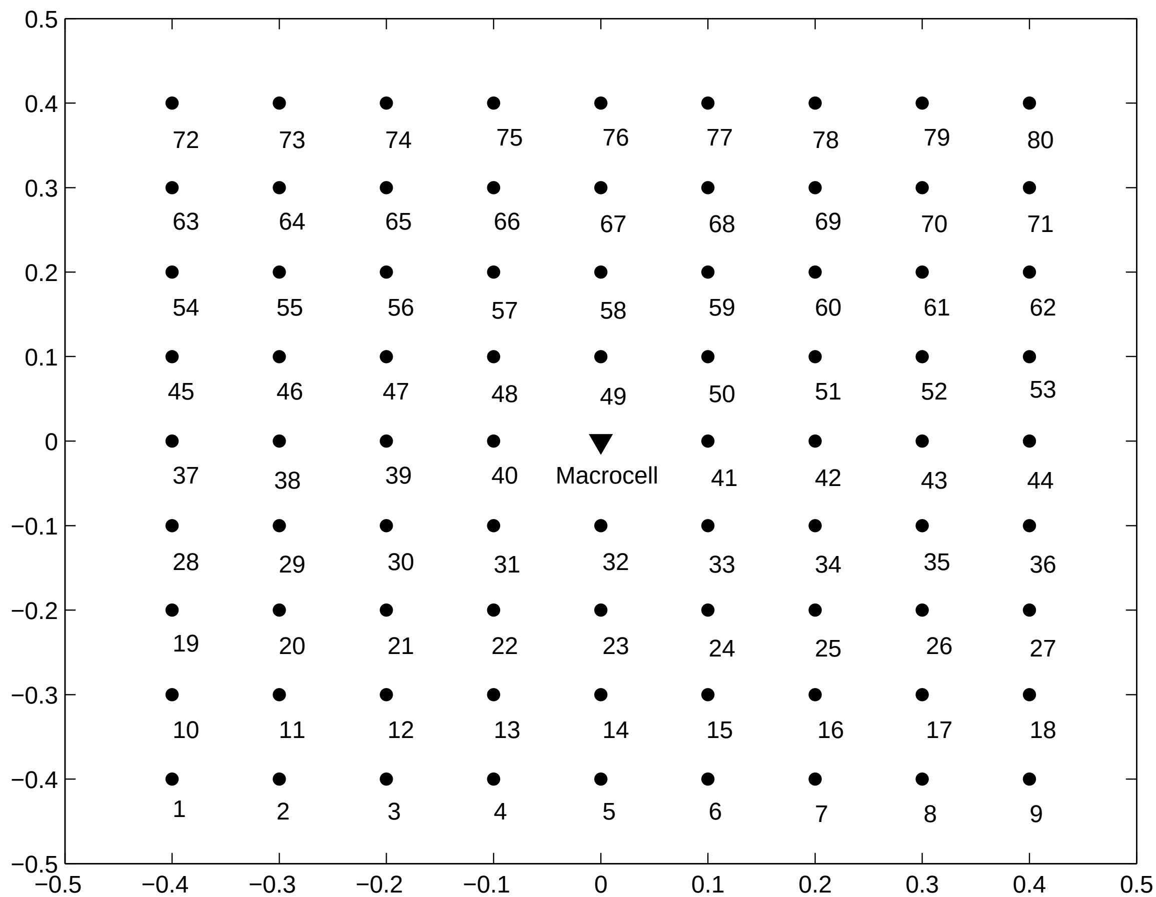

Fig. 1. Square region $\mathcal{R}$ with candidate $|J| = 80$ candidate grid points.

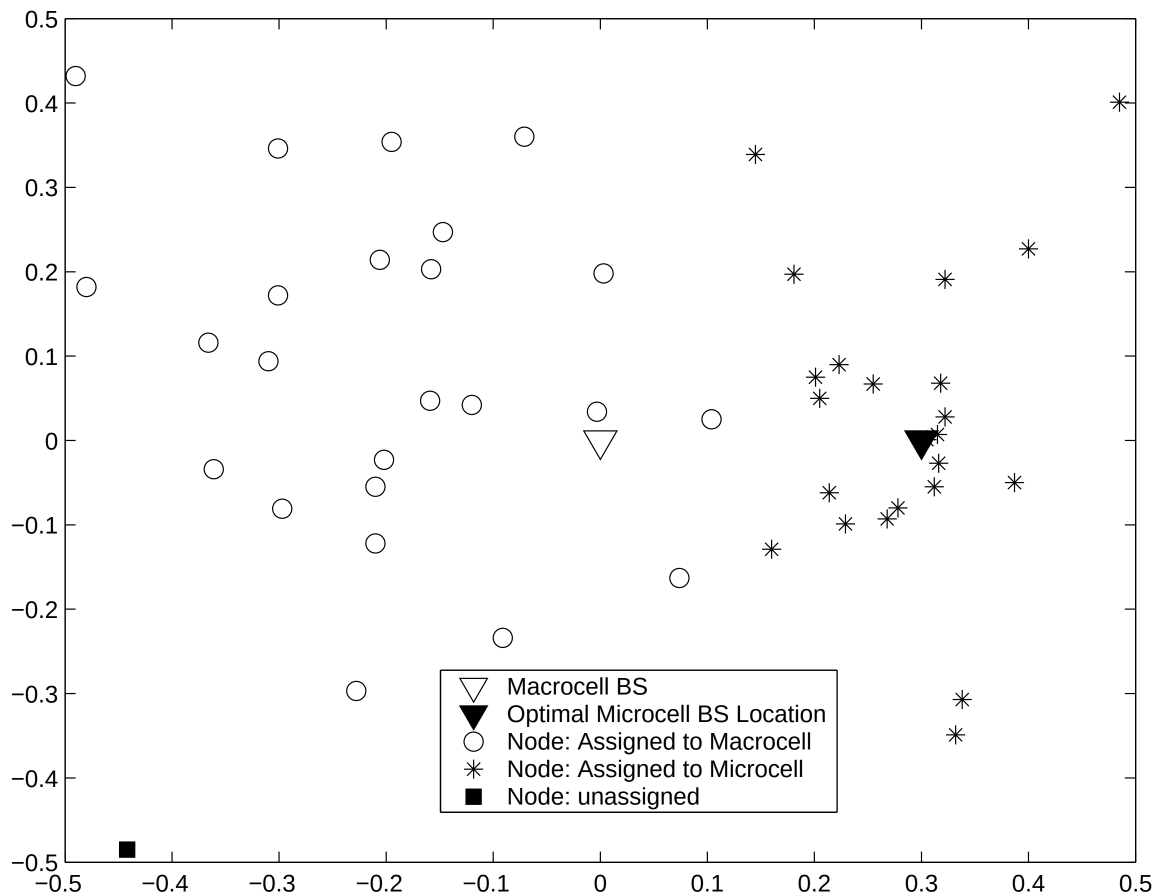

Fig. 2. Low Power Solution for Scenario 9 with Optimal Assignment Scheme

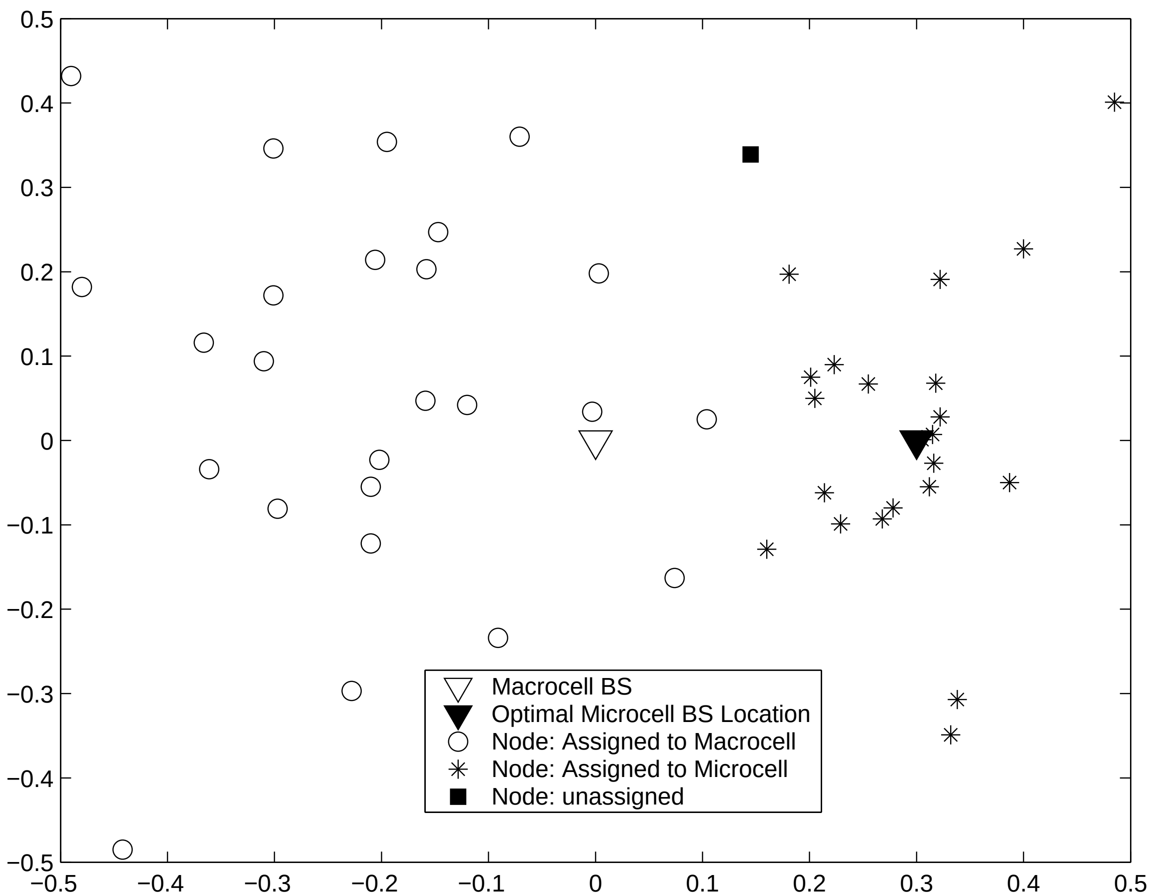

Fig. 3. Low Power Solution for Scenario 9 with Path Gain-Based Assignment Scheme

Scenario	$\|I\|$	Configuration	Scenario	$\|I\|$	Configuration
1	30	30 from (1)	6	42	38 from scenario 4 + 4 from (3)
2	38	38 from (1)	7	42	30 from scenario 1 + 6 from (2) + 6 from (3)
3	38	30 from scenario 1 + 8 from (2)	8	48	38 from scenario 2 + 10 from (1)
4	38	30 from scenario 1 + 8 from (3)	9	48	42 from scenario 5 + 3 from (1) + 3 from (2)
5	42	38 from scenario 3 + 4 from (2)	10	48	42 from scenario 6 + 3 from (1) + 3 from (3)

TABLE I

DEMAND NODE CHARACTERISTICS OF NETWORK SCENARIOS

Scenario	$\|I\|$	Max Cap	# Locations	Low Ptotal j^*	N_M	N_μ
1	30	30	80	47	15	15
2	38	38	69	47	20	18
3	38	38	64	52	21	17
4	38	38	48	65	21	17
5	42	42	14	51	23	19
6	42	42	10	66	21	21
7	42	42	5	67	22	20
8	48	44	1	46	23	21
9	48	47	1	43	24	23
10	48	45	2	67	22	23

TABLE II

SOLUTIONS FOR OPTIMAL ASSIGNMENTS

Scenario	H	$\|I\|$	Max Cap	# Locations	Low Ptotal j^*	N_M	N_μ
1	1	30	30	38	64	20	10
	5	30	30	4	64	21	9
	10	30	29	2	56	21	8
	15	30	28	64	45	24	4
	20	30	28	59	45	24	4
2	1	38	38	16	66	24	14
	5	38	36	3	64	26	10
	10	38	34	3	65	26	8
	15	38	34	1	65	26	8
	20	38	33	1	65	26	7
3	1	38	38	22	44	23	15
	5	38	38	4	52	25	13
	10	38	37	2	52	26	11
	15	38	36	2	52	26	10
	20	38	35	1	42	26	9
4	1	38	38	21	65	22	16
	5	38	38	5	67	24	14
	10	38	37	4	67	25	12
	15	38	37	1	66	26	11
	20	38	37	1	66	26	11
5	1	42	42	8	43	21	21
	5	42	42	1	42	25	17
	10	42	40	3	43	25	15
	15	42	40	1	42	26	14
	20	42	39	1	42	26	13
6	1	42	42	8	66	21	21
	5	42	42	2	67	24	18
	10	42	40	1	67	24	16
	15	42	38	3	66	25	13
	20	42	38	2	66	25	13
7	1	42	42	2	67	22	20
	5	42	38	2	52	26	12
	10	42	36	4	67	26	10
	15	42	35	3	66	26	9
	20	42	35	1	66	26	9

TABLE III

SOLUTIONS FOR PATH GAIN-BASED ASSIGNMENTS

Scenario	H	$\lvert I \rvert$	Max Cap	# Locations	Low Ptotal j^*	N_M	N_μ
8	1	48	43	3	67	23	20
	5	48	38	4	64	26	12
	10	48	36	3	55	26	10
	15	48	34	4	56	26	8
	20	48	34	2	65	26	8
9	1	48	47	1	43	25	22
	5	48	43	2	42	25	18
	10	48	42	1	43	26	16
	15	48	41	1	42	26	15
	20	48	38	1	42	24	14
10	1	48	45	2	67	22	23
	5	48	44	2	67	25	19
	10	48	42	2	67	25	17
	15	48	41	1	58	26	15
	20	48	38	2	58	25	13

TABLE IV

SOLUTIONS FOR PATH GAIN-BASED ASSIGNMENTS