Mixed Integer Programming Models for Wind Farm Design

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Outline

1. Wind Farm Design

2. Wind Farm Layout Optimisation
   - Wind flow modelling
   - Mixed integer programming model
   - Model validation

3. Future Work
   - Wind farm layout optimisation
   - Wind flow modelling

4. Summary
Wind Farm Design in New Zealand

Complex design problem that includes
- Wind farm layout
- External transmission and access
- Ecological and visual impacts
- Internal reticulation layout

Difficult economic conditions
- Inflated turbine prices due to excess international demand
- Depressed national electricity prices due to gas subsidies
- Lack of carbon pricing mechanisms

Developers driven to seek comparatively large-scale high-wind sites

Huge scope for the application of optimisation techniques
Wind Farm Layout Optimisation

Determine an optimal location of turbines that maximises the net power produced, subject to constraints on the number of turbines, turbine proximity, and turbine wake.

Industry use “off-the-shelf” commercial software packages
- WindFarm, WindFarmer, WindPRO, ...
- Heuristic optimisation algorithms
  - User specifies an initial feasible layout
  - Perform local neighbourhood search with random restarts
- Limited in the types of constraints that can be modelled

Previous work in the literature
- Genetic and evolutionary algorithms
- Greedy improvement heuristics

Is a MIP approach viable?
Wind speed frequency distribution for six years of data from a site in the lower North Island, partitioned into 30° sectors and 1 ms$^{-1}$ ranges.
Wind Flow Models

Industry standard models use a linear flow model

- WAsP, MS-Micro, ...

Model inputs

- Wind speed frequency distribution
- Digital terrain model

Two-parameter Weibull distributions are used to continuously approximate the empirical wind speed frequency distributions

Model output

- Spatial wind speed distribution called a wind resource grid
Digital terrain map contours and wind resource grid mean wind speed distribution at a height of 80m.
Turbine Power Output

Power curve for a Vestas V90-3.0MW turbine

Ideal power

- Total expected power output by a turbine in isolation
- Ideal power \( P = \int P(u) \cdot F(u) \, du \)
  - \( P(u) \) is the turbine power function
  - \( F(u) \) is the wind speed frequency cumulative distribution function
Turbine Proximity and Wake Interference

Turbine proximity
- Minimum turbine separation requirements prescribe an exclusion zone around any turbine location

Turbine wake interference
- Turbine wakes create an energy deficit downstream
- The interference $I$ between a pair of turbines is the net expected power loss that results from the superposition of the energy deficits at one turbine due to the wake of the other turbine
- Interference is calculated using the PARK turbine wake model (Jensen, 1983; Katic, Hojstrup and Jensen, 1986)
Consider a graph $G = (V, E)$ where

- Set of nodes $V$ denote potential turbine locations
- Set of edges $E$ are partitioned into mutually exclusive sets
  - $E_P$ denote proximity edges
  - $E_I$ denote interference edges
- An edge $(u, v) \in E$ corresponds to a pair of turbine locations
  - If $(u, v) \in E_P$ then they violate the minimum separation distance
  - If $(u, v) \in E_I$ then the turbine wakes impact one another

Let

- $P_v$ denote the ideal power of a turbine located at $v \in V$
- $I_{uv}$ denote the power loss due to interference between turbines located at $u$ and $v$ where $(u, v) \in E_I$
- $Q$ denote the set of all maximal complete subgraphs of the graph $G_P = (V, E_P)$
Mixed Integer Programming Model

\[
\begin{align*}
\text{maximise} & \quad \sum_{v \in V} P_v x_v - \sum_{(u,v) \in E_I} I_{uv} y_{uv} \quad \text{(Net power)} \\
\text{subject to} & \quad \sum_{v \in Q} x_v \leq 1, \quad Q \in Q \quad \text{(Proximity)} \\
& \quad x_u + x_v \leq 1 + y_{uv}, \quad (u, v) \in E_I \quad \text{(Interference)} \\
& \quad \sum_{v \in V} x_v \leq k \quad \text{(Turbine limit)} \\
& \quad x_v \in \{0, 1\}, \quad y_{uv} \geq 0, \quad v \in V, \ (u, v) \in E_I
\end{align*}
\]

Case study
- 1.7km x 2.0km domain overlaid by a 48m regularly spaced grid
- 1069 nodes, 2388 proximity cliques, 66903 interference edges
- 67972 variables, 69292 constraints, 226313 non-zeros
Results

- Priority branching
- 1800 second time limit

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<th>Turbines</th>
<th>Relaxations</th>
<th>Best Bound</th>
<th>Best Integer</th>
<th>% Nodes</th>
<th>Gap (%)</th>
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CPLEX 10.2 with default options running on a 3.2GHz Pentium 4 with 2GB RAM
Cuts = Clique cuts / Implied bound cuts, % Nodes = when best integer was found, + = heuristic
Results

- Interference constraints placed in a cut pool
- Priority branching
- 1800 second time limit

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CPLEX 10.2 with default options running on a 3.2GHz Pentium 4 with 2GB RAM

Cuts = Clique cuts / Lazy constraints, % Nodes = when best integer was found, + = heuristic
Comparison with WindFarmer

GH WindFarmer (Garrad Hassan Ltd)
- Widely used in industry
- Proprietary improvements to ideal power calculations and the PARK turbine wake model give better estimates of net power

Error
- Relative difference between the net power calculated by WindFarmer and the MIP for the same turbine locations

Improvement
- Relative improvement of a WindFarmer solution found using the MIP solution as a starting solution
Comparison using solutions for problems with 10–20 turbines

Observations

- Strong positive correlation between the error and improvement
- Suggests improved techniques are needed for calculating model data rather than there being a weakness with the MIP model
Transitive Interference Constraints

Empirical evidence suggests the interference a turbine experiences is predominantly due to the wake of a single upwind turbine.

Let

- $D$ denote the set of wind directions
- $U_d^v$ denote the set of locations upwind of $v \in V$ with respect to wind direction $d \in D$
- $I_{uv}^d$ denote the power loss experienced by a turbine located at $u \in V$ due to interference from an upwind turbine located at $v \in U_u^d$ with respect to wind direction $d \in D$

\[
x_u + x_v - \sum_{w \in U_u^d} I_{uw}^d y_{uv}^d \leq 1 + y_{uv}^d, \quad u \in V, \ d \in D, \ v \in U_u^d
\]
Additional Design Parameters

- Capital budget constraints
  - Generalisation of turbine limit constraint

- Line of sight constraints
  - Eliminate potential turbine locations in MIP model

- NZS sound level restrictions
  - Budget-type constraint

- Reticulation layout
  - Costs negligible next to turbine capital costs

- Sensitivity to empirical input data
  - Robust solutions
Linear vs Non-Linear Flow Models

Linear flow models

- Industry standard
- WAsP and MS-Micro
- Non-linear effects induced by complex terrain and high wind speeds

Non-linear CFD flow models
Qualitative Wind Statistics

Important at sites with complex terrain and high wind speeds

Background and induced turbulence
- Variation in wind velocity over short lengths of time
- Cyclical loading patterns on turbine structures

Maximum gusts
- Maximum wind speed at a particular location within time frame
- Difficult to predict the spatial variation over the domain

Inflow angles
- Large vertical velocity components increase the inflow angle
- Negative impact on turbine operation and maintenance

Eliminate potential turbine locations in MIP model
Conclusions

Wind farm design in New Zealand
- Complex design problem under difficult economic conditions
- Huge scope for the application of optimisation techniques

Wind flow modelling
- CFD has many advantages compared to linear flow models
- Non-linear turbulent wind flow in New Zealand’s complex terrain

Wind farm layout optimisation
- MIP models appear to be viable
- More amenable to modelling site specific conditions
- Will require state-of-the-art MIP techniques
- Adapt the extensive literature for similar MIP problems
Thank you for your attention
Questions?

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