MIP 2008

Workshop on Mixed Integer Programming

August 4-7, 2008

Columbia University, New York City

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Conference Dinner

The conference dinner will be held Tuesday 8/5 at 7pm at Savann Restaurant, located between West 79th and 80th streets on Amsterdam Avenue. The menu consists of selected appetizers and main courses from their regular menu (plus a couple of choices for dessert and tea or coffee and up to 3 glasses of wine). The price is $50 per person, and must be paid in cash on Monday 8/4 to Oktay Günlük. Guests of conference attendees are welcome to attend. Space is limited, so *please* sign up for the dinner on Monday 8/4.

Conference Schedule

Monday, August 4

10:30-10:45		Welcome Session
10:45-11:30	Rekha Thomas	Small Chvátal Rank
11:30-12:15	Ruriko Yoshida	A Generalization of the Integer Linear Infeasibility Problem
12:15-12:45	Bala Krishnamoorthy	Cascade Knapsack Problems
		Lunch Break
2:30- 3:15	John Hooker	A Principled Approach to MILP Modeling
3:15- 4:00	Peter Malkin	Hilbert's Nullstellensatz and an Algorithm for Proving Combinatorial Infeasibility
		Beverage Break
4:30- 6:30		Poster Session

Tuesday, August 5

9:45-10:30	Kent Andersen	Higher Dimensional Split Closures
10:30-11:00	Daniel Espinoza	Computing with Multi-row Gomory Cuts
		Coffee Break
11:30-12:15	Ronald Rardin	Strong Bounds for Linear Optimization Problems with k-of-m Constraint Systems
12:15-12:45	Hamish Waterer	Mixed Integer Programming Models for Wind Farm Design
		Lunch Break
2:30- 3:15	Ellis Johnson	Seven Things Everyone Should Know about Gomory's Group Problem
3:15- 4:00	Warren Adams	Recent Advances with the Reformulation-Linearization-Technique
		Beverage Break
7:00-	Conference Dinner	Savann Restaurant

Wednesday, August 6

9:45-10:30	Yongpei Guan	Algorithms for Stochastic Lot-Sizing Problems with Backlogging
10:30-11:00	Oktay Günlük	Mingling: Mixed-Integer Rounding with Bounds
		Coffee Break
11:30-12:15	Ignacio Grossmann	Reformulations, Relaxations and Cutting Planes for Linear Generalized Disjunctive Programming
12:15-12:45	Sven Leyffer	Branch-and-Refine for Mixed Integer Nonconvex Optimization
		Lunch Break
2:30-3:15	Alper Atamtürk	Two Submodular Optimization Problems On Risk Aversion
3:15-3:45	Adam Letchford	On Mixed-Integer Quadratic Programming with Box Constraints
3:45-4:15	Christoph Buchheim	Stronger Relaxations for Constrained Quadratic 0-1 Programming
		Beverage Break
4:45- 5:30		Roundtable Discussion

Thursday, August 7

9:45-10:30	Pasquale Avella	Computational Testing of Exact Mixed Knapsack Separation for MIP Problems
10:30-11:00	Emilie Danna	Performance Variability in Mixed Integer Programming
		Coffee Break
11:30-12:15	Mike Trick	Combinatorial Benders' Cuts for Sports Scheduling Optimization
12:15-12:45	Gabriel Tavares	How to Solve Challenging MIP Problems Faster
12:45- 1:00		Concluding Session

Speaker Abstracts

Warren Adams, Clemson University

Recent Advances with the Reformulation-Linearization-Technique

The Reformulation-Linearization-Technique (RLT) is a general methodology for obtaining tight polyhedral outer-approximations of discrete sets. Given a mixed 0-1 linear (or polynomial) program, the RLT conducts two distinct steps of reformulation and linearization. During the reformulation step, "product factors" are formed from the binary variables and their complements. The original problem constraints are multiplied by these factors to generate additional, redundant restrictions. The linearization step then lifts the problem into a higher-dimensional space by substituting a continuous variable for each resulting product term.

Portions of this work were joint efforts with Hanif Sherali and Stephen Henry.

Kent Andersen, University of Copenhagen

Higher Dimensional Split Closures

Split cuts are cutting planes for mixed integer programs whose validity is derived from maximal lattice point free polyhedra of the form $S := \{x : \pi_0 \leq \pi^T x \leq \pi_0 + 1\}$ called split sets. The set obtained by adding all split cuts is called the split closure, and the split closure is known to be a polyhedron. A split set has max-facet-width equal to one in the sense that $\max\{\pi^T x : x \in S\} - \min\{\pi^T x : x \in S\} \leq 1$ for all facets $\pi^T x \geq \pi_0$ of S.

In this talk we consider using general maximal lattice point free polyhedra to derive valid inequalities for mixed integer sets. We call maximal lattice point free polyhedra with max-facet-width equal to $w > 0$ for split sets with rank w. A split cut of rank w is then a valid inequality whose validity follows from a split set of rank at most w. The w^{th} split closure is the set obtained by adding all valid inequalities of split rank at most w.

Our main result is that the w^{th} split closure is a polyhedron. We also characterize which split rank w^* is required to design a finite cutting plane proof for the validity of an inequality. Specifically, for this value w^*, there is a finite cutting plane proof that only uses split sets of split rank at most w^*, and no finite cutting plane proof that only uses split sets of rank smaller than w^*.

Joint work with Q. Louveaux and R. Weismantel.

Alper Atamtürk, University of California, Berkeley

Two Submodular Optimization Problems On Risk Aversion

Given a finite ground set N and two vectors $a, b \in \mathbb{R}^N$, we consider optimization problems involving a submodular utility function of the form $u(S) = f(a(S)) + b(S)$, where S is a subset of N, f is a strictly concave, increasing, differentiable function, and $a(S)$, $b(S)$ denote the sum of the components of a and b over S.

Both minimization and maximization of u arise frequently in modeling risk aversion in probabilistic combinatorial optimization problems. We will give applications for both types. Whereas minimization of u is solvable by a greedy algorithm, maximization of u is NP-hard.

Our goal in this talk is to present structural results that help us to formulate problems with such utility functions effectively so that they can be solved faster using standard mixed-integer programming solvers. We will present computational results on solving risk-averse capital budgeting problems.

This talk is based on the following recent papers:

1. Maximizing a Class of Submodular Utility Functions (with S. Ahmed)
   `http://www.ieor.berkeley.edu/~atamturk/pubs/submodular-utility.pdf`
2. Polymatroids and Mean-Risk Minimization in Discrete Optimization (with V. Narayanan)
   `http://www.ieor.berkeley.edu/~atamturk/pubs/conicobj.pdf`

Pasquale Avella, Universita del Sannio

Computational testing of exact mixed knapsack separation for MIP problems

We study an exact separation algorithm for mixed knapsack sets with the aim of evaluating its effectiveness in a cutting plane algorithm for Mixed-Integer Programming. First proposed by Boyd in the 90's, exact knapsack separation has recently found a renewed interest. We present a "lightweight" exact separation procedure for mixed knapsack sets and perform a computational experience on a wide set of mixed-integer programming instances from MIPLIB 2003 and "Mittelman" sets. Computational experiments confirm that MIR inequalities are the most important class of valid inequalities from a computational viewpoint. Nevertheless there are several difficult instances where exact separation is able to further raise lower bounds.

Joint work with Igor Vasilyev, Institute of System Dynamics and Control Theory, Siberian Branch of Russian Academy of Sciences.

Christoph Buchheim, University of Cologne

Stronger Relaxations for Constrained Quadratic 0-1 Programming

We consider quadratic 0-1 programs containing linear constraints. In practice, such problems are hard to solve even if the corresponding linear problem is easy, such as, e.g., the assignment problem. The straightforward approach is to linearize the quadratic objective function by introducing new variables and to simply add the given linear constraints; a slightly stronger relaxation can often be obtained by a quadratic reformulation of these constraints.

However, the performance of such an approach usually turns out to be poor, as the size of the problem increases considerably by the new variables and at the same time the standard linearization yields a very weak relaxation. In the talk, we present general methods for strengthening this relaxation.

Moreover, to illustrate the transition from a linear to a quadratic objective function, we discuss the quadratic linear ordering problem. In this example, we show that the polytope of the linearized quadratic problem is a face of the corresponding unconstrained quadratic 0-1 problem. In other words, in order to obtain a full description of the former polytope, it "suffices" to perfectly model the connection between basic and product variables, while most polyhedral knowledge about the linear problem is useless for the quadratic version.

Emilie Danna, ILOG

Performance variability in mixed integer programming

The solving time for a given MIP model sometimes varies dramatically if the branch-and-cut algorithm used to solve it is modified slightly, even if the algorithmic change is theoretically neutral with respect to performance. We will present evidence of this performance variability, examine its causes, describe its consequences, propose a few ideas to address it, and raise several related questions.

Daniel Espinoza, Universidad de Chile

Computing with multi-row Gomory cuts

Cutting planes for mixed integer problems (MIP) are nowadays an integral part of all general purpose software to solve MIP. The most prominent, and computationally significant, class of general cutting planes are Gomory mixed integer cuts (CG). Unfortunately, finding other classes of general cuts for MIP that work well in practice has been elusive.

Recent advances on the understanding of valid inequalities derived from the infinite relaxation introduced by Gomory and Johnson for mixed integer problems, has opened a new possibility of finding such an extension, and moreover, a simple geometrical interpretations for cuts derived from multiple tableau rows.

In this talk, we present some known results related to valid inequalities derived from the infinite group relaxation of groups of tableau rows, and investigate the computational impact of using several subclasses of minimal valid inequalities for this relaxation.

We also propose some new ground sets to generate inequalities, and possible ways to find custom-made ground sets for a given set of tableau rows. We test these ideas on a set of MIPs, including MIPLIB 3.0 and MIPLIB 2003.

Ignacio Grossmann, Carnegie Mellon University

Reformulations, Relaxations and Cutting Planes for Linear Generalized Disjunctive Programming

Generalized disjunctive programming (GDP) is an extension of the well-known disjunctive programming paradigm developed by Balas. The GDP formulation involves Boolean and continuous variables that are specified in algebraic constraints, disjunctions and logic propositions, which is an alternative representation to the traditional mixed integer programming (MIP) formulation. Our research in this class of problems, which has been motivated by its potential for improved modeling and solution methods, has led to the development of customized algorithms that exploit the underlying logical structure of the problem in both the linear and nonlinear cases.

However, an outstanding question that has remained is the exact relationship between GDP and disjunctive programming. In this work, we establish for the linear case connections between disjunctive programming and generalized disjunctive programming, which provide new theoretical and computational insights that allow us to exploit the rich theory developed Balas.

In particular, we propose a novel family of MIP reformulations corresponding to the original GDP model that result in tighter relaxations and stronger cutting planes than reported in previous work. We illustrate this theory on the strip-packing problem for which computational results are presented. We also describe the application of these ideas to the global optimization of bilinear GDP problems.

This talk is based on the PhD work by Nick Sawaya and Juan Pablo Ruiz.

Yongpei Guan, University of Oklahoma

Algorithms for Stochastic Lot-Sizing Problems with Backlogging

As a traditional model in the operations research and management science domain, lot-sizing problem is embedded in many application problems such as production and inventory planning and has been consistently drawing attentions from researchers. There is significant research progress on polynomial time algorithm developments for deterministic uncapacitated lot-sizing problems based on Wagner-and-Whitin property. However, in practice, problem parameters are seldom known in advance. For most cases, even the distribution of the problem parameters is not known.

In this talk we consider basic versions of deterministic lot-sizing models in which problem parameters (e.g., demand) are stochastic and develop corresponding scenario tree based stochastic lot-sizing models. For these models, a backward dynamic programming recursion framework is developed based on production path properties. This framework allows us to show that the optimal value function is piecewise linear and continuous, which we can further use to define polynomial time algorithms for several different problems, including those with backlogging and varying capacities under certain scenario tree structure.

Moreover, we develop polynomial time algorithms with complexities $O(n^2)$ and $O(n^2 T \log n)$ respectively for stochastic uncapacitated and constant capacitated lot-sizing problems, both with backlogging and regardless of scenario tree structure.

Oktay Günlük, IBM Research

Mingling: Mixed-Integer Rounding with Bounds

Mixed-integer rounding (MIR) is a simple, yet powerful procedure for generating valid inequalities for mixed-integer programs. When used as cutting planes, MIR inequalities are very effective for problems with unbounded integer variables. For problems with bounded integer variables, however, cutting planes based on lifting techniques appear to be more effective. This is not surprising as lifting techniques make explicit use of the bounds on variables, whereas the MIR procedure does not.

In this paper we describe a simple procedure, which we call mingling, for incorporating variable bound information into mixed-integer rounding. By explicitly using the variable bounds, the mingling procedure leads to strong inequalities for mixed-integer sets with bounded variables. We show that facets of the mixed-integer knapsack sets derived earlier by superadditive lifting techniques are mingling inequalities. In particular, the mingling inequalities developed in this paper subsume the continuous cover and reverse continuous cover inequalities of Marchand and Wolsey as well as the continuous integer knapsack cover and pack inequalities of Atamtürk. In addition, mingling inequalities give a generalization of the two-step MIR inequalities of Dash and Günlük under some conditions.

Joint work with Alper Atamtürk.

John Hooker, Carnegie Mellon University

A Principled Approach to MILP Modeling

A problem can be formulated as an MILP if and only if the projection of its feasible set onto continuous space is a finite union of polyhedra with the same recession cone. This provides a general approach to MILP modeling, but it can lead to large or impractical formulations because it relies wholly on a disjunctive analysis. In this talk I present a alternative method for constructing MILP models that analyzes a problem in terms of disjunctive, knapsack, and logical constraints. I show by a series of examples that this approach generates, in a principled way, models that have come to be regarded through folklore or practical experience as "good" models for a wide variety of problems.

Ellis Johnson, Georgia Institute of Technology

Seven Things Everyone Should Know about Gomory's Group Problem

1. The asymptotic theorem
2. The binary group problem
3. The generality of the subadditive characterization of facets
4. The subadditive dual
5. Subadditive functions on the unit interval
6. Periodic subadditive functions on $\mathbb{R}^m$ with directional derivatives
7. LP problems with multiple rhs

Bala Krishnamoorthy, Washington State University

Cascade Knapsack Problems

There are many ways to design difficult integer programs. Difficult knapsack problems that are provably hard for branch-and-bound have been proposed by Jeroslow in 1974, Chvátal and Todd, Chvátal and Avis in 1980, and later on by Cornuejols et al., Nemhauser et al., Hunsaker et al., and Aardal and Lenstra. Another notable class of hard IPs is the marketshare problems, proposed by Cornuejols and Dawande.

These instances either

1. are theoretically hard for branch-and-bound, but easily solved by preprocessing; or
2. have large coefficients; or
3. have unbounded variables, or
4. have several constraints.

We propose a family of 0-1 knapsack problems, in which one generates one node, when branching on a vector, say p_1; after this, *no* node, when branching on another vector, say p_2. Thus, the right branching sequence has a "cascade" effect. The coefficients are of modest size, but the resulting instances seem as hard for commercial solvers as the marketshare problems. In a typical instance with $n = 30$, the largest coefficient is 8933; however, CPLEX 9.0 takes 56 million nodes to prove infeasibility. The cascade problems illustrate another phenomenon studied by Cook and Kannan, namely the geometric width not being a good predictor of the integer width.

Joint work with Gabor Pataki at UNC Chapel Hill.

Adam Letchford, Lancaster University

On Mixed-Integer Quadratic Programming with Box Constraints

We consider a new, yet fundamental, non-linear mixed-integer programming problem, that we call Mixed-Integer Quadratic Programming with Box Constraints and denote by MIQPB. By this, we mean minimising a quadratic function of a set of variables, subject to lower and upper bounds on the variables, together with integrality constraints on a subset of the variables. (The objective function need not be convex or concave.) MIQPB includes as special cases two classical NP-hard problems: Unconstrained Boolean Quadratic Programming and Non-Convex Quadratic Programming with Box Constraints. As a result, one needs concepts from both combinatorial optimisation and global optimisation to tackle MIQPBs adequately.

We study the convex hull of feasible solutions to MIQPBs. (Note that they are not polyhedral unless all variables are integer-constrained.) We prove several results concerned with the structure of these convex sets and then present several classes of valid inequalities and facets. Our results generalise results of Yajima and Fujie and complement some recent results of Anstreicher and Burer.

Sven Leyffer, Argonne National Laboratory

Branch-and-Refine for Mixed Integer Nonconvex Optimization

The efficient management of energy is one of the most important challenges of this century. We consider an application arising in the efficient management of energy, namely the problem of optimal power flow. This problem is pivotal to the management of transmission systems. Especially, we focus on the tertiary voltage control (TVC) problem, whose aim it is to predict the effects of an expansion of the network, or to determine the optimal operating conditions of the existing network. This application gives rise to a nonconvex mixed-integer nonlinear programming (MINLP) problem.

Nonconvex MINLPs are among the most difficult optimization problem: they combine the difficulty of optimizing over discrete variable sets with the challenges of handling nonconvex functions. We propose a new global optimization method for solving nonconvex MINLPs. Our method decomposes the nonlinear functions into one- and two-dimensional components for which piecewise linear envelopes are constructed using ideas similar to special ordered sets. The resulting relaxation is then successively refined by branching on integer or continuous variables. We prove many interesting results, and present some preliminary numerical experience.

Joint work with Annick Sartenaer, University of Namur; Emilie Wanufelle, University of Namur.

Peter Malkin, University of California-Davis

Hilbert's Nullstellensatz and an Algorithm for Proving Combinatorial Infeasibility

Systems of polynomial equations over an algebraically-closed field K can be used to concisely model many combinatorial problems. In this way, a combinatorial problem is feasible (e.g., a graph is 3-colorable, hamiltonian, etc.) if and only if a related system of polynomial equations has a solution over K. In this paper, we investigate an algorithm aimed at proving combinatorial infeasibility based on the observed low degree of Hilbert's Nullstellensatz certificates for polynomial systems arising in combinatorics and on large-scale linear-algebra computations over K. We report on experiments based on the problem of proving the non-3-colorability of graphs. We successfully solved graph problem instances having thousands of nodes and tens of thousands of edges.

The talk is based on a paper of the same name with Jesus De Loera, Jon Lee and Susan Margulies, which is available on the arxiv at http://arxiv.org/abs/0801.3788

Ronald Rardin, University of Arkansas

Strong Bounds for Linear Optimization Problems with k-of-m Constraint Systems

Optimization problems arise in a variety of applications that would be linear programs except for one or more constraint systems in which up to a specified number k of the m constraints in the system are allowed to be violated. Our own work has focused on such systems in the context of "dose-volume limits" in radiation therapy planning wherein constraints correspond to implied dose at distinct points in some tissue, and the requirement is that at least a given percent of the points must satisfy a maximum dose restriction while other points may receive heavier doses. In finance, a similar structure exists in portfolio optimization problems where the constraints correspond to investment return under different scenarios and the requirement is to permit investments where no more than some specified percent of scenarios fall short of a specified return limit.

Such problems are NP-hard yet can easily be modeled as linear integer programs with 0-1 variables. However, such mixed-integer-programming models often exhibit massive gaps between optimal and LP-relaxation solution values. In this paper, we investigate families of cutting planes derived from viewing the underlying problem as a disjunctive program. In addition, we consider novel families of one-row-at-a-time restrictions of the given model that combine to produce a valid bound for the full problem. Motivations behind these ideas are explained, and computational results are provided.

Joint work with Ali Tuncel, Purdue University; Mark Langer, M.D., Indiana School of Medicine; and Jean-Philippe P. Richard, Purdue University.

Gabriel Tavares, Dash Optimization

How to solve challenging MIP problems faster

We are often asked by our customers to solve MIP problems never before solved and to solve MIP problems that can be solved faster than ever. These two objectives, while seemingly unrelated, has led our development team to come up with a new way to meet both objectives and to put this power in the hands of the Xpress-MP User. This talk will focus on our new product, Xpress-Tuner, and how this tool allows Xpress-MP users to (i) automatically fine tune the Xpress-Optimizer to solve MIP problems that are unsolvable with any out-of-the box optimization software and to (ii) automatically tune the Xpress-Optimizer to solve MIP problems faster, frequently by a factor of 5 to 10 times.

Rekha Thomas, University of Washington

Small Chvátal Rank

We introduce a new measure of complexity of the integer hull of a rational polyhedron called its small Chvátal rank. While the Chvátal rank counts the number of rounds of cuts that need to be added to a polyhedron to obtain its integer hull, the small Chvátal rank is the number of iterations of a Hilbert basis procedure needed to obtain just the facet normals of the integer hull. In many situations, the small Chvátal rank is very small. For instance, in the plane, this rank is always at most one while the Chvátal rank can be arbitrarily high.

If G is a perfect graph on n vertices, the small Chvátal rank of the standard relaxation of the stable set polytope of G is at most two while the Chvátal rank is $O(\log n)$. We present bounds for polytopes in the unit cube and characterize the rank 0 case. In general, the two ranks have the same asymptotic growth.

Joint work with Tristram Bogart, Annie Raymond, Andreas Schulz and Sebastian Pokutta.

Mike Trick, Carnegie Mellon University

Combinatorial Benders' Cuts for Sports Scheduling Optimization

Combinatorial, or logical, Benders' cuts are a generalization of Benders' original linear programming dual based cuts, where information from the subproblem is passed back to the master problem in the form of cuts that embed the information "Any better (or feasible) solution must satisfy this constraint". Such cuts have proven particularly useful in sports scheduling due to the nested layers of decisions that must be made. An example of this nesting is the decision on the home versus away choice, followed by the decision of who to play compatible with that choice. We illustrate the use of combinatorial Benders' cuts in a variety of sports scheduling related applications, including a heuristic guided by Benders' cuts for umpire scheduling.

Hamish Waterer, University of Auckland, New Zealand

Mixed integer programming models for wind farm design

There is significant potential for optimizing the design of a wind farm in New Zealand. The complex nature of the wind resource and the larger size of the wind farms being built increase the complexity of the decisions that need to be made, while tight economic margins create a drive for greater efficiency. Current industry practice utilises commercial packages that are heuristic in nature and limited in the types of constraints that can be modelled.

A mixed integer programming model for optimizing the layout of a wind farm has been developed that is capable of determining the optimal locations of turbines subject to constraints on the number of turbines, turbine proximity, and turbine wake. Results have shown that this model produces layouts that are comparable to those from a commercial package. Moreover, this model can be extended to include capital budget constraints, noise and line of sight restrictions, constraints relating to wind quality such as maximum gusts, inflow angles and turbulence, as well as modelling reticulation and different mixes of turbines.

Joint work with Stuart Donovan, Gary Nates, and Rosalind Archer.

Ruriko Yoshida, University of Kentucky

A generalization of the integer linear infeasibility problem

Does a given system of linear equations $Ax = b$ have a nonnegative integer solution? This is a fundamental question in many areas, such as operations research, number theory, and statistics. In terms of optimization, this is called an integer feasibility problem. A generalized integer feasibility problem is to find a finite representation of all b such that there does not exist a nonnegative integral solution in the system with a given A. One such problem is the well-known Frobenius problem.

In this paper we study the generalized integer feasibility problem and also the multi-dimensional Frobenius problem. To study a family of systems with no nonnegative integer solution, we focus on a commutative semigroup generated by a finite subset of $\mathbb{Z}^d$ and its saturation. An element in the difference of the semigroup and its saturation is called a "hole". We show the necessary and sufficient conditions for the finiteness of the set of holes. Also we define fundamental holes and saturation points of a commutative semigroup.

Then, we show the simultaneous finiteness of the set of holes, the set of non-saturation points, and the set of generators for saturation points. As examples we apply our results to some three- and four-way contingency tables and two-way contingency tables under the common effect diagonal model from statistics. Then we will discuss the time complexities of our algorithms using generating functions.

Joint work with A. Takemura.

Poster Abstracts

Kerem Akartunali, University of Melbourne

Two-period Relaxations on Big-bucket Production Planning Problems

We investigate multi-level, multi-item production planning problems with big bucket capacities, i.e., multiple items competing for the same resources, which frequently occur in practice yet remain daunting in their difficulty to solve. We propose a methodology to generate all valid inequalities based on the convex hull closure of the two-period subproblems, which may be the simplest model that captures the basis of the difficulty of these problems. We will present computational results and discuss possible extensions.

Joint work with Andrew Miller.

I. Esra Büyüktahtakin, University of Florida

Dynamic programming based inequalities for the capacitated lot-sizing problem

Iterative solutions of dynamic programming formulations for the capacitated lot-sizing problem are used to generate valid inequalities for an equivalent mixed integer programming formulation. The cuts are shown to be quite effective in solving instances when compared to solving the MIP formulation directly with CPLEX and against other cutting plane approaches. The method may also show promise in more general applications.

Joint work with Joseph C. Hartman and J. Cole Smith.

Valentina Cacchiani, University of Bologna

A Hybrid Approach to Beam Angle Optimization in Intensity-Modulated Radiation Therapy

Intensity-Modulated Radiation Therapy is the technique of delivering radiation to cancer patients by using intensity-modulated beams, with the aim of reducing the intensity of the beams that go through critical structures and reaching the dose prescription in the target volume. Two decisions are of fundamental importance: to select the beam angles and to compute the intensity of the beams used to deliver the radiation to the patient. Usually, these two decisions are made separately: firstly, the treatment planners, on the basis of experience and intuition, decide the orientation of the beams and then the intensities of the beams are optimized by using an automated software tool.

Automatic beam angle selection is an important problem and is generally based on human experience by now. In this context, we face the problem of optimizing both the decisions, developing an algorithm which automatically selects the beam angles and computes the beam intensities. This problem is called Beam Angle Optimization. It appears to be highly non-convex and non-linear, and with many local minima. Hence, we propose a Hybrid Heuristic method, which combines a Simulated Annealing procedure

with the knowledge of the gradient of the objective function. The beam intensities are optimized by solving a Linear Programming model, with the objective of minimizing the overall radiation delivered to the patient. Experimental results are performed on simulated data, showing the advantages that come from our approach.

Joint work with Dimitris Bertsimas, Thomas Bortfeld, David Craft, and Omid Nohadani.

Silvia Canale, University of Rome

A Cutting Plane Approach to Cardinality and 0-norm Optimization

Cardinality constraints appear in a large number of applications, as diverse as finance, bioinformatics, and data mining. While such applications reside in the core of our economy and welfare, optimization problems with cardinality constraints are among the hardest ones in the field of operations research. We present a polyhedral study of linear optimization with cardinality constraints. In addition, we present a branch-and-cut and a Lagrangian relaxation algorithm to tackle large-scale optimization problems with cardinality constraints. Finally, we apply the algorithm to the problem of maximizing sparsity of large-margin classifiers used in supervised machine learning.

Joint work with I.R. de Farias Jr., and M. Zhao.

Claudia D'Ambrosio, University of Bologna

Non-Linear Programming based heuristic for Mixed-Integer Linear Programming

From both a theoretical and a practical viewpoint, finding feasible solutions for MILP problems is, in general, difficult. In recent years, some effective heuristics, based on exploiting informations obtained solving relaxations of the original MILP, were presented (e.g., [1]).

In this work we start observing that the integrality requirement, the difficult part of an MILP problem, could be interpreted as a non-convex part of the model. Then, finding a feasible solution to a MILP problem is formulated as a Non-Convex Non-Linear Programming problem: we want to satisfy the linear constraints by minimizing a non-linear (non-convex) objective function modeling the violation of the integrality requirements. Thus, any NLP solution with 0 objective function is MILP feasible.

From this viewpoint, we can integrate effective methods studied for both NLP and MILP problems. In particular, classical methods to solve NLP problems are applied (e.g., [2]), obtaining then a local solution: if such a solution has strictly positive value it fractional. In this case general purpose cutting planes for MILPs are added. It is well-known that NLP problems are hard to solve, but the recent improvements in the solvers and the exploitation of some structure allow us to obtain interesting results.

References:

1. M. Fischetti, F. Glover, A. Lodi, The Feasibility Pump, Mathematical Programming, 104(1), pages 91-104, 2005.
2. A. Waechter and L. T. Biegler, On the Implementation of a Primal-Dual Interior Point Filter Line Search Algorithm for Large-Scale Nonlinear Programming, Mathematical Programming, 106(1), pages 25-57, 2006.

Joint work with Matteo Fischetti, Andrea Lodi, and Andreas Waechter.

F. Kilinc-Karzan, Georgia Institute of Technology

Approximating the Stability Region for Binary Mixed-Integer Programs

We consider optimization problems with some binary variables, where the objective function is linear in these variables. The stability region of a given solution of such a problem is the polyhedral set of objective coefficients for which the solution is optimal. A priori knowledge of this set provides valuable information for sensitivity analysis and re-optimization when there is objective coefficient uncertainty. An exact description of the stability region typically requires an exponential number of inequalities. We develop useful polyhedral inner and outer approximations of the stability region using only a linear number of inequalities. Furthermore, when a new objective function is not in the stability region, we produce a list of good solutions that can be used as a quick heuristic or as a warm start for future solves.

Joint work with A. Toriello, S. Ahmed, G. Nemhauser and M. Savelsbergh.

Edward D. Kim, University of California-Davis

Multi-index Transportation Polytopes and Applications to Linear and Integer Programming

Transportation polytopes are classical objects in operations research and mathematical programming. In this poster, we discuss the graphs of multi-index transportation polytopes. Bounding the diameter of polytopes is particularly interesting since the diameter is a lower bound on the number of iterations required by the simplex method using any pivot rule. We present a quadratic upper bound on the diameter of the graph of 3-way axial transportation polytopes. Bounding the diameter of 3-way transportation polytopes is particularly interesting since any convex polytope is the face of an axial 3-way transportation polytope (De Loera, Onn). Moreover, any convex rational polytope is isomorphically representable as a 3-way planar transportation polytope (De Loera, Onn).

Joint work with Jesus A. De Loera, Shmuel Onn and Francisco Santos.

Christian Raack, ZIB

Capacitated Multi-Commodity-Flow Cuts

Many general mixed integer programs in practice contain a block-structure coming from multi-commodity-flow (MCF) formulations. There might be as many network matrices as there are commodities. These network matrices all represent the same graph and they are usually coupled by some “capacity”-constraints, restricting the “flow” on arcs (or edges) of this graph.

It is known that for certain network-dimensioning problems it is possible to dramatically reduce computation times and gaps when generating cut-inequalities (and similar network based inequalities) within the branch & cut procedure. There are mainly two reasons for this. First, these inequalities are strong in the sense that they might define high dimensional faces. And second, state of the art MIP solvers (such as CPLEX or SCIP) are not able to detect network structure within general mixed integer programming formulations.

Our work focuses on automatically detecting such MCF structures together with the coupling of the commodities by capacity constraints in general MIP instances. We identify the underlying network and derive cutting planes based on the network structure. We currently concentrate on network cuts. Given a cut in the network, we aggregate flow conservation constraints for one of the two shores and capacity constraints corresponding to cut arcs. This yields base inequalities which can then be used (following the c-MIR approach (Marchand & Wolsey 1998)) to generate cut-set inequalities (Atamtürk 2002), flow-cover inequalities (Padberg 1985, Van Roy & Wolsey 1986, Louveaux & Wolsey 2003), and the like, depending on the type of capacity constraints. These inequalities then correspond to a chosen network cut. Our implementation uses SCIP (scip.zib.de) and already shows promising results on pure network-design instances.

Domenico Salvagnini, University of Bologna

A Local Dominance Procedure for Mixed-Integer Linear Programming

Among the hardest Mixed-Integer Linear Programming (MILP) problems, the ones that exhibit a symmetric nature are particularly important in practice, as they arise in both theoretical and practical combinatorial optimization problems. A theoretical concept that generalizes the notion of symmetry is that of dominance. This concept, although known since a long time, is typically not used in general-purpose MILP codes, due to the intrinsic difficulties arising when using the classical definitions in a completely general context.

We study a general-purpose dominance procedure proposed in the 80’s by Fischetti and Toth, that overcomes some of the main drawbacks of the classical dominance criteria. Both theoretical and practical issues concerning this procedure are considered, and important improvements are proposed. Computational results on a test-bed made of hard knapsack and network loading problems are reported, showing that the proposed method can lead to considerable speedup when embedded in a general-purpose MILP solver.

Simon Spoorendonk, University of Copenhagen

Chvátal-Gomory Cuts in Branch-and-cut-and-price Algorithms

This paper focuses on how to apply Chvátal-Gomory cuts in a branch-and-cut-and-price algorithm. A straight forward approach to develop cutting planes in a column generation context is to derive cuts from the original formulation and decompose the generated constraints. However, one can also use cutting planes derived directly from master problem variables. It may not seem obvious how this affects the column generation process as values of dual variables of the generated cuts must be taken into account. A description on how to extend the original formulation such that cut on master problem variables can be expressed in the context of the original solution space is given. Furthermore, the impact of such cuts branch-and-cut-and-price algorithm it is discussed. Especially with regard to how the cutting planes affects the complexity of the pricing problems.

Dan Steffy, Georgia Institute of Technology

Challenges in exact linear and integer programming: exact precision linear algebra

A successful approach to solving linear programming problems exactly has been to solve the problems with increasing levels of fixed precision, checking the final basis in exact arithmetic and then doing additional pivots if necessary. This work is a computational study comparing different techniques for the core element of our exact computation: solving sparse rational systems of linear equations exactly.

Joint work with William Cook and Sanjeeb Dash.

Mustafa Tural, UNC Chapel Hill

Parallel Approximation and Integer Programming Reformulation

We show that in a knapsack feasibility problem an integral vector p, which is short, and near parallel to the constraint vector gives a branching direction with small integer width. We use this result to analyze two computationally efficient reformulation techniques on low density knapsack problems. We do not assume any a priori structure on the weight vector.

Both reformulations have a constraint matrix with columns reduced in the sense of Lenstra, Lenstra, and Lovász. We prove an upper bound on the integer width along the last variable, which becomes 1, when the density is sufficiently small. In the proof we extract from the transformation matrices a vector which is near parallel to the constraint vector a. The near parallel vector is a good branching direction in the original knapsack problem, and this transfers to the last variable in the reformulations.

Joint work with Gabor Pataki at UNC Chapel Hill.

Juan Pablo Vielma, Georgia Institute of Technology

Modeling Disjunctive Constraints with a Logarithmic Number of Binary Variables and Constraints

Many combinatorial constraints over continuous variables such as SOS1 and SOS2 constraints can be interpreted as disjunctive constraints that restrict the variables to lie in the union of m specially structured polyhedra. Known mixed integer binary formulations for these constraints have a number of binary variables and extra constraints that is linear in m. We give sufficient conditions for constructing formulations for these constraints with a number of binary variables and extra constraints that is logarithmic in m.

Using these conditions we introduce the first mixed integer binary formulations for SOS1 and SOS2 constraints that use a number of binary variables and extra constraints that is logarithmic in the number of continuous variables constrained. We also introduce the first mixed integer binary formulations for piecewise linear functions of one and two variables that use a number of binary variables and extra constraints that is logarithmic in the number of linear pieces of the functions. We prove that the new formulations for piecewise linear functions have favorable tightness properties and present computational results showing that they can significantly outperform other mixed integer binary formulations.

Joint work with George L. Nemhauser.

Arrigo Zanette, University of Bologna

Can Pure Cutting Plane Algorithms Work?

We discuss an implementation of the lexicographic version of Gomory's fractional cutting plane method and of two heuristics mimicking the latter. In computational testing on a battery of MIPLIB problems we compare the performance of these variants with that of the standard Gomory algorithm, both in the single-cut and in the multi-cut (rounds of cuts) version, and show that they provide a radical improvement over the standard procedure. In particular, we report the exact solution of ILP instances from MIPLIB such as `stein15, stein27`, and `bm23`, for which the standard Gomory cutting plane algorithm is not able to close more than a tiny fraction of the integrality gap. We also offer an explanation for this surprising phenomenon.

Bo Zeng, University of Oklahoma

Lifted Valid Inequalities for Stochastic Dynamic Knapsack Sets

Polyhedral study on the stochastic dynamic knapsack problem (SDKP) has been proven to be very useful to understand the structure of stochastic lot-sizing problems and other typical stochastic integer program sets. In this paper, we investigate sequence independent lifting and partial sequence independent lifting to generate valid inequalities for SDKP. First, we derive conditions under which lifting is (complete) sequence independent, build strong superadditive approximating lifting functions and describe special cases in which the lifted inequalities are facet-defining.

We then consider a partial sequence independent lifting approach, i.e. sequential lifting from path to path and sequence independent lifting inside paths, and build strong approximating lifting function for such mechanism. We finally compare these two approaches, discuss the connection with the pairing scheme, and derive cutting planes from SDKP for stochastic lot-sizing problems.

Joint work with Yongpei Guan.