An Integrated Solver for Optimization Problems

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June 6 2006

I'd Like To Thank the Program Committee for...

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  - A "foolish" model depends on your vocabulary
  - Larger vocabulary $\Rightarrow$ more natural models
  - Some things we once worried about are now automatic

Outline

- Introduction and Motivation
- SIMPL Concepts
- 3 Modeling Examples
- Computational Experiments
- Future Work and Conclusion

Why Integrate?

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  - Planning and scheduling (jobs, crews, sports, etc.)
  - Routing and transportation
  - Engineering and network design
  - Manufacturing
  - Inventory management
  - Etc.

Integration Continued

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- It combines complementary strengths of different optimization techniques
- Problem structure is more easily captured and exploited

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- Concerned with Constraint Satisfaction Problems (CSPs):
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- Main Ideas:
  - Constraints eliminate infeasible values: domain reduction
  - Local inferences are shared: constraint propagation

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  - city $\in$ { Atlanta, Boston, Miami, San Francisco }
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  - etc.
  - These are called global constraints

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- ECLiPSe, Rodošek, Wallace and Rajian 99
- OPL, Van Hentenryck, Lustig, Michel and Puget 99
- Mosel, Colombani and Heipcke 02
- SCIP, Achterberg 04

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- Generation of cutting planes as a form of logical inference: Bockmayr & Kasper 98, Bockmayr & Eisenbrand 00

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  - Natural specification of integrated models
  - Allow user to reveal problem structure to the solver
- Low-level integration
  - Increased effectiveness when underlying technologies interact at a micro level during the search
- Modularity, flexibility, extensibility, efficiency
  - Make it easy to add new types of constraints, relaxations, solvers and search strategies

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- Search $=$ enumeration of problem restrictions

The Ubiquity of Search, Inference, Relaxation

Solution Method	Restriction	Inference	Relaxation

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Solution Method	Restriction	Inference	Relaxation
MILP	Branch on fractional vars.	Cutting planes, preprocessing	LP relaxation
CP	Split variable domains	Domain reduction, propagation	Current domains
CGO	Split intervals	Interv. propag., lagr. mult.	LP or NLP relaxation
Benders	Subproblem	Benders cuts (nogoods)	Master problem
DPL	Branching	Resolution and confl. clauses	Processed confl. clauses
Tabu Search	Current neighborhood	Tabu list	Same as restriction

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- Infer: constraints drive the inference
  - Each constraint has a filtering/inference module
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- Relax: constraints create the relaxations
  - Each constraint has a relaxation module
  - This module reformulates the constraint according to different relaxations (LP, MILP, CP, etc.)

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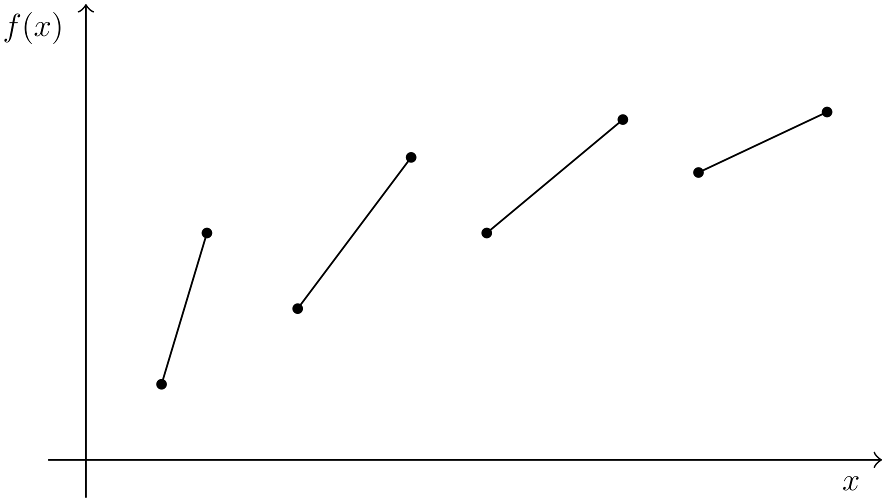
- Previous work: Refalo (1999), Ottosson, Thorsteinsson and Hooker (1999, 2002)

Example 1: Production Planning

Shape of Net Income Function $f(x)$



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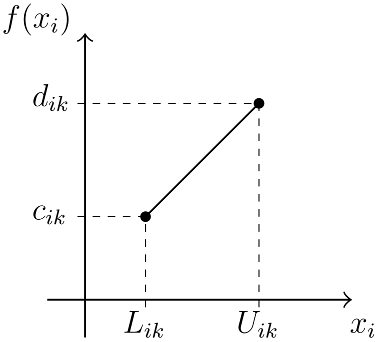

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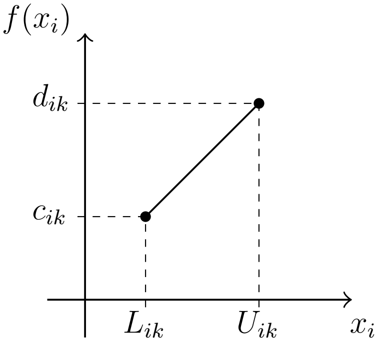

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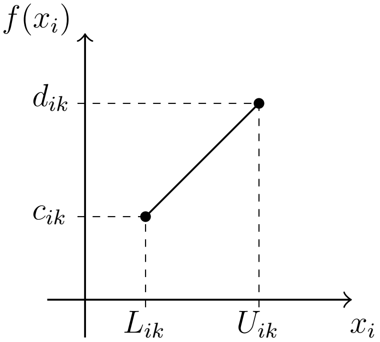

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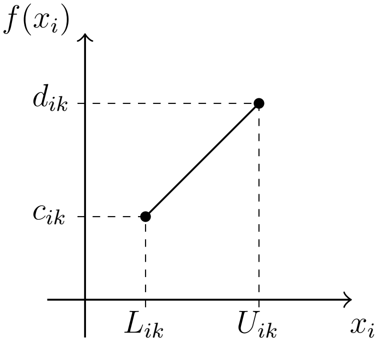

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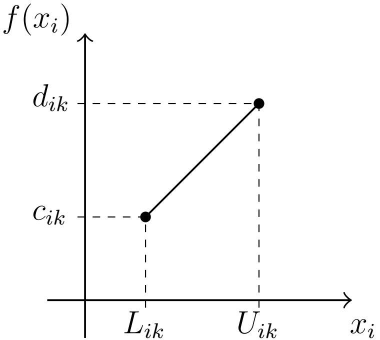

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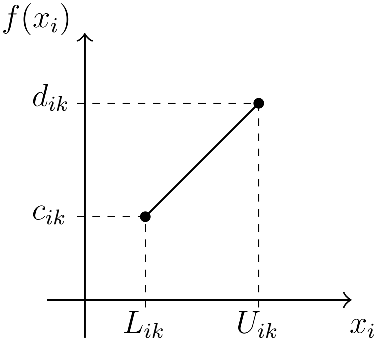

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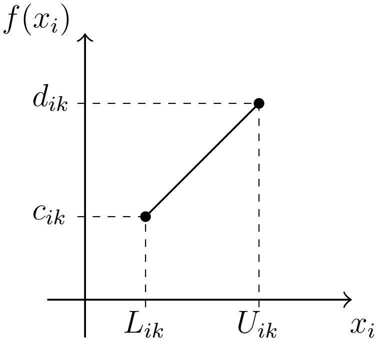

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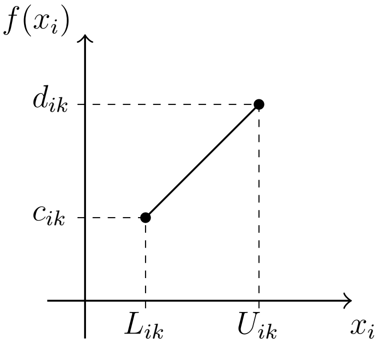
piecewise$(x_i, u_i, L_i, U_i, c_i, d_i), \ \forall \ i$



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   branching = { income:most }
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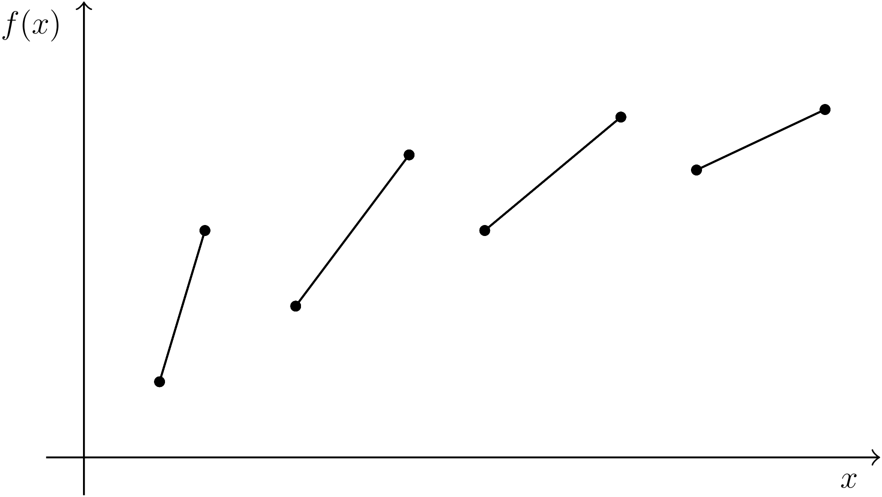
Relaxation and Branching for `piecewise`



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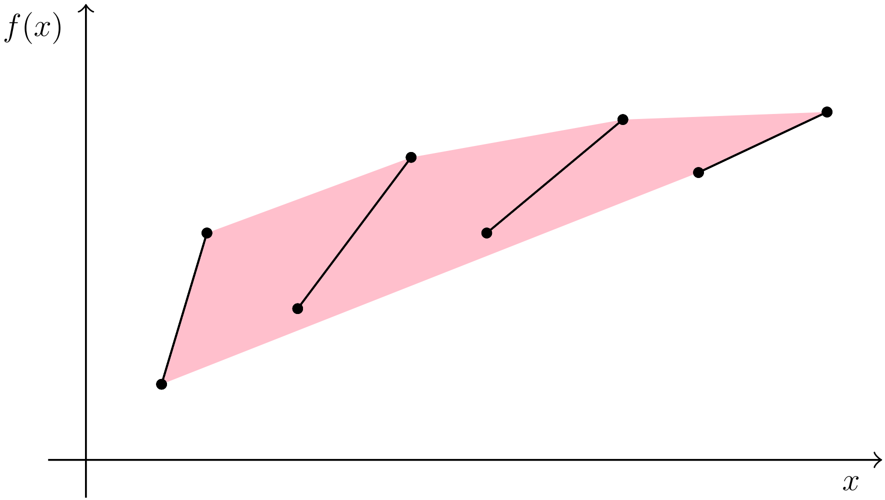

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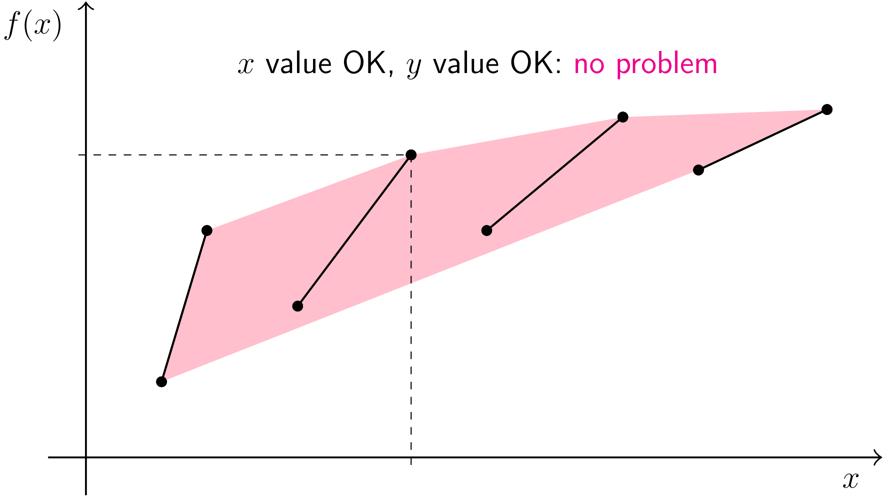

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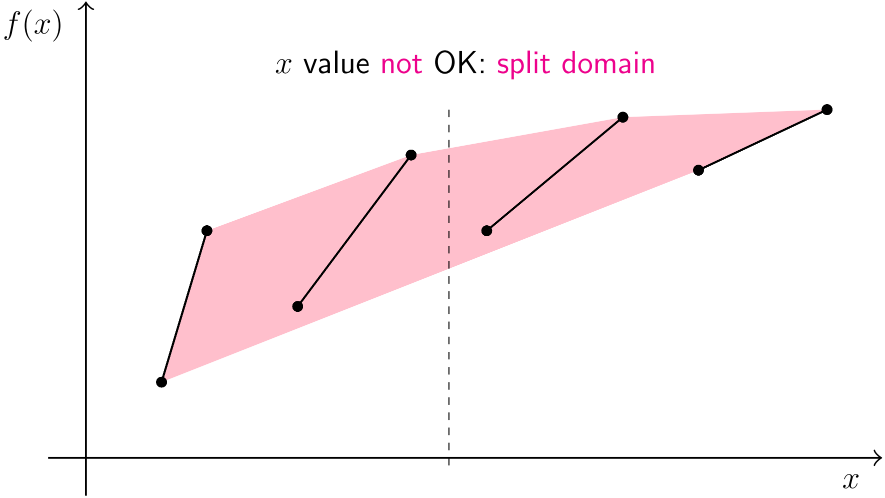

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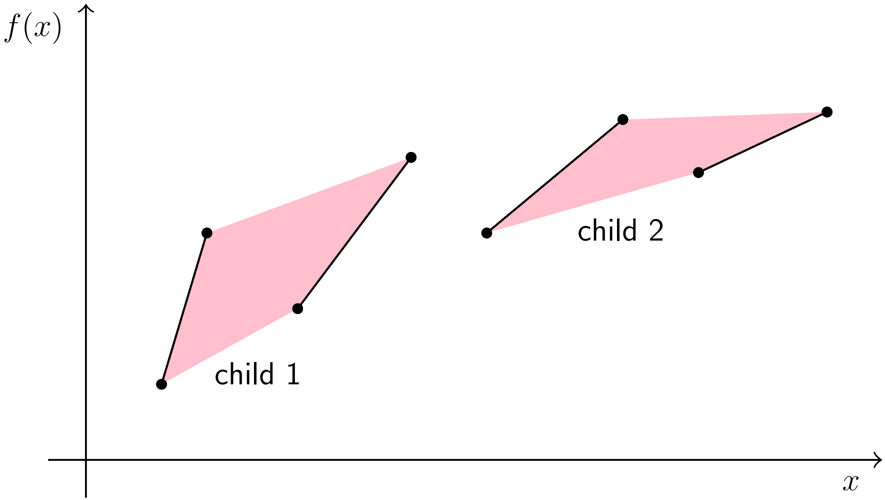

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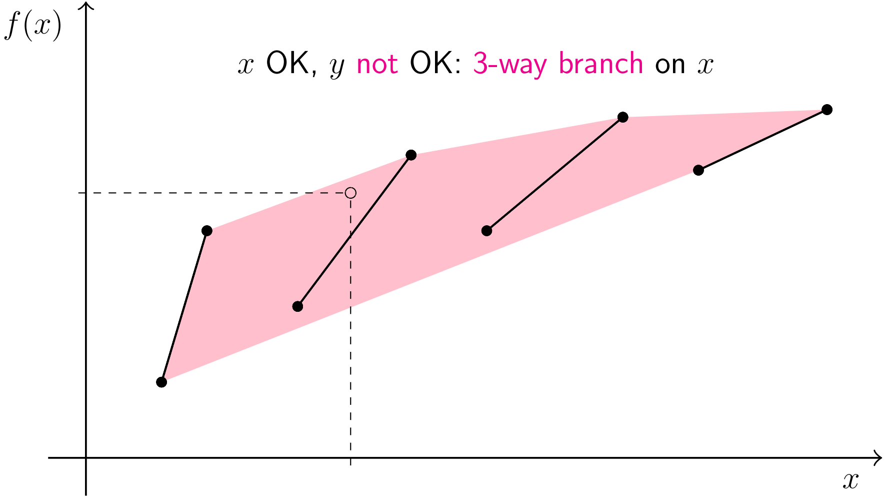

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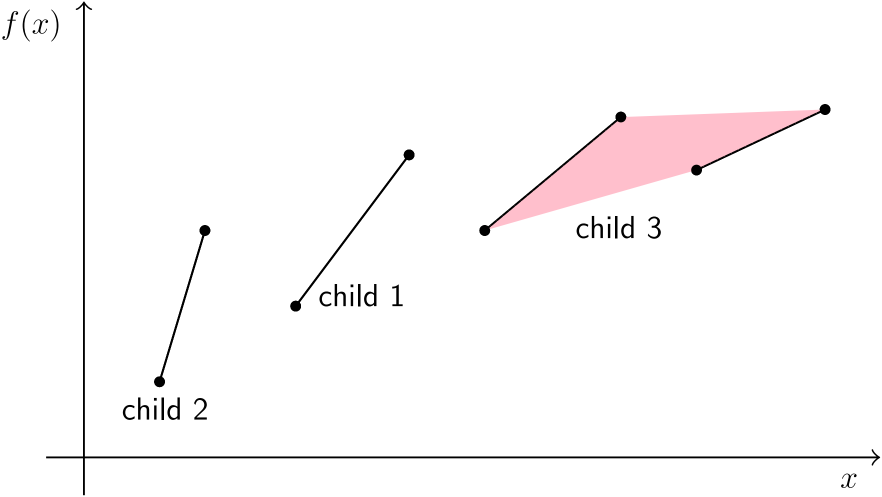

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Computational Results: Number of Search Nodes

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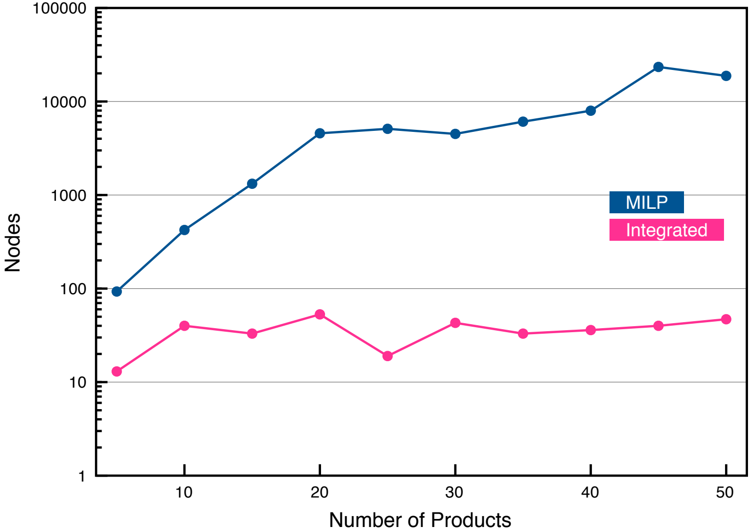

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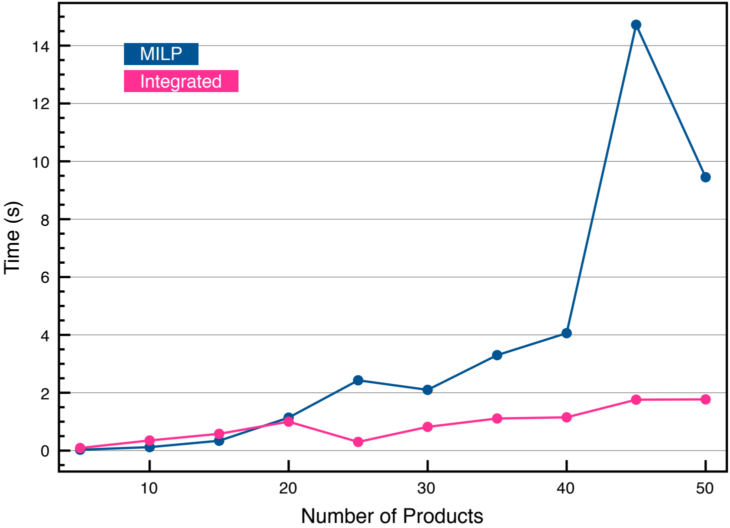
Computational Results: CPU Time (s)

Example 1: Production Planning

Computational Results: CPU Time (s)



Example 2: Product Configuration

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- Previous work: Thorsteinsson and Ottosson (2001)

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$z_{ij} = a_{ijt_i} q_i$ and element$(t_i, (a_{ij1}q_i, \ldots, a_{ijn}q_i), z_{ij})$

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Computational Results: Number of Search Nodes

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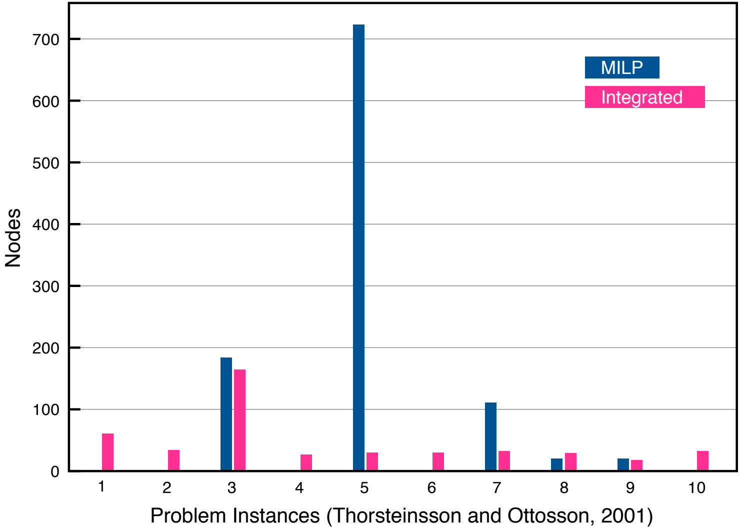

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Computational Results: CPU Time (s)

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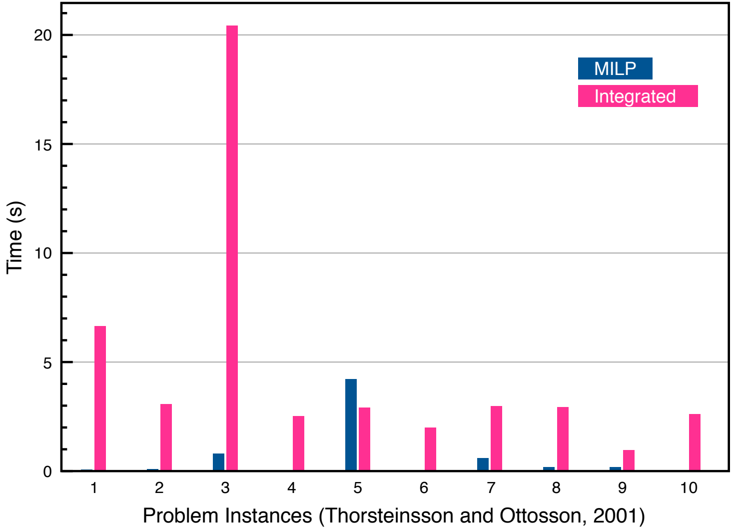

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- In SIMPL we can get the same results with very little effort

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$$
\begin{aligned}
&\min \textstyle\sum_{ij} c_{ij} x_{ij} \\
&r_j \le t_j \le d_j - \textstyle\sum_i p_{ij} x_{ij},\ \forall\, j \\
&\textstyle\sum_i x_{ij} = 1,\ \forall\, j \\
&y_{jk} + y_{kj} \le 1,\ \forall\, k > j \\
&y_{jk} + y_{kj} \ge x_{ij} + x_{ik} - 1,\ \forall\, k > j,\ i \\
&y_{jk} + y_{kj} + x_{ij} + x_{i'k} \le 2,\ \forall\, k > j,\ i' \ne i \\
&t_k \ge t_j + \textstyle\sum_i p_{ij} x_{ij} - M(1 - y_{jk}),\ \forall\, k \ne j \\
&\textstyle\sum_j p_{ij} x_{ij} \le \max_j\{d_j\} - \min_j\{r_j\},\ \forall\, i
\end{aligned}
$$

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Benders Decomposition Approach

- Master Problem
  - Assign jobs to machines at minimum cost
  - “Ignore” release dates and due dates
  - $x_{ij} = 1$ if job i assigned to machine j

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$$\sum_{i \in I_j} x_{ij} \leq |I_j| - 1$$

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$$r_j \leq t_j \leq d_j - p_{y_j j}, \ \forall \ j$$

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$$\text{cumulative}((t_j, p_{ij}, 1 \,|\, x_{ij} = 1), 1),\ \forall\ i$$

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Need to tell the solver how to decompose the model

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 machinecap means {
   cumulative({t[j],p[i][j],1} forall j | x[i][j]=1, 1) forall i
   relaxation = { ip:master, cp:sub:decomp } }
   inference = { feasibility } }
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   r[j] <= t[j] <= d[j] - p[y[j]][j] forall j
   relaxation = { ip:master, cp:sub } }
 machinecap means {
   cumulative({t[j],p[i][j],1} forall j | x[i][j]=1, 1) forall i
   relaxation = { ip:master, cp:sub:decomp } }
   inference = { feasibility } }
SEARCH
 type = { benders }
```

Example 3: Job Scheduling on Parallel Machines
Computational Results

Instances from Jain and Grossmann (2001)

Long Processing Times

		MILP		Integrated Benders		
Jobs	Machines	Nodes	Time (s)	Iterations	Cuts	Time (s)
3	2					
7	3					
12	3					
15	5					
20	5					
22	5					

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Jobs	Machines	Nodes	Time (s)	Iterations	Cuts	Time (s)
3	2	1	0.00			
7	3	1	0.02			
12	3	11060	16.50			
15	5	3674	14.30			
20	5	159400	3123.34			
22	5	$>$ 5.0M	$>$ 48h			

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3	2	1	0.00	2	1	0.00
7	3	1	0.02	12	14	0.09
12	3	11060	16.50	26	37	0.58
15	5	3674	14.30	22	31	0.96
20	5	159400	3123.34	30	52	3.21
22	5	$>$ 5.0M	$>$ 48h	38	59	6.70

Example 3: Job Scheduling on Parallel Machines
Computational Results (continued)

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Shorter processing times make the problem easier to solve

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Computational Results (continued)

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Short Processing Times

		MILP		Integrated Benders		
Jobs	Machines	Nodes	Time (s)	Iterations	Cuts	Time (s)
3	2	1	0.00			
7	3	1	0.01			
12	3	4950	1.98			
15	5	14000	19.80			
20	5	140	5.73			
22	5	$>$ 16.9M	$>$ 48h			
25	5	$>$ 4.5M	$>$ 48h			

Example 3: Job Scheduling on Parallel Machines
Computational Results (continued)

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7	3	1	0.01	1	0	0.01
12	3	4950	1.98	1	0	0.01
15	5	14000	19.80	1	0	0.03
20	5	140	5.73	3	3	0.12
22	5	$>$ 16.9M	$>$ 48h	5	4	0.38
25	5	$>$ 4.5M	$>$ 48h	16	22	0.86

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  - etc.

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  - Whichever way we go, we still need to clean it up a bit...

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- SIMPL is
  - a step toward making integrated methods more accessible to a larger group of users
  - a very useful research tool
- We still have a long way to go, but important steps have been taken and initial results are encouraging

That's All Folks!

Thank you!

Any Questions?

Constraint Programming Example

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$x \in \{1, 3\}$
$y \in \{1, 2, 3\}$
$z \in \{1, 2, 3\}$
$w \in \{1, 2, 4\}$

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