Non-Cyclic Train Timetabling and Comparability Graphs

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Non-Cyclic Train Timetabling Problem

**INPUT:**

- **Single Line with a one-way track** (approach easy to extend to railway network)

  ![Diagram showing BOLOGNA to MILAN with stops at MO, RE, PR, PC]

- **List T of Trains with “ideal timetables”**

<table>
<thead>
<tr>
<th>EUORSTAR 1811:</th>
<th>REGIONAL 2187:</th>
</tr>
</thead>
<tbody>
<tr>
<td>BO 7:35 - MI 9:10</td>
<td>BO 7:30 - MO 7:52</td>
</tr>
<tr>
<td></td>
<td>MO 7:54 - RE 8:12</td>
</tr>
<tr>
<td></td>
<td>RE 8:14 - PR 8:26</td>
</tr>
<tr>
<td></td>
<td>PR 8:28 - PC 8:55</td>
</tr>
</tbody>
</table>

Ideal Timetables are **CONFLICTING!!!!**
No-Conflict Constraints (in the basic version of the problem):

- **no overtaking** between stations (allowed only within stations)
- **min time** between consecutive **departures** from each station
- **min time** between consecutive **arrivals** at each station

**OUTPUT:**

- “Adjusted” non-conflicting timetables with maximum total profit:

  **Train Adjustments:**
  
  - **shift** departure time from initial station
  - **stretch** increase stop time in intermediate stations

  **Train Profit:**
  
  \[
  \pi_j - \phi_j(shift_j) - \sum_i \phi_{ij}(stretch_{ij})
  \]

  Ideal profit

  If profit is negative cancel the train

  Arbitrary monotone functions
Representation on Time-Space GRAPH

Initial Arcs

BO

MO

PC

MI

source

Departure Nodes

Arrival Nodes

Station Arcs

Arrival Nodes

Departure Nodes

Arrival Nodes

Departure Nodes

Train Timetable Path

Train Profit Path Profit

Ideal profit and Shift cost on initial arcs

Stretch cost on station arcs

time (1 minute discretization)
ILP Formulation

$\mathbf{x}_p$ Variables associated with feasible paths of the Time-Space Graph

$P_j$ Collection of paths for train $j$

$p_p$ Profit of path $p \in P_j$

$S = S_1 \cup S_2 \cup \ldots$

$S_l$ Maximal Stable Set of $G_l$

$max \sum_{j \in T} \sum_{p \in P_j} p_p x_p$

$\sum_{p \in S} x_p \leq 1, \quad S \in S$

$x_p \in \{0, 1\}, \quad p \in P_j, j \in T$

Path Compatibility Graph $G_l$
compatible paths:
- associated with different trains
- contain compatible arcs on line segment \( l \)

Satisfy no-conflict constraints \( \rightarrow \) Transitive relation

Line segment \( l \) between two consecutive stations on the line:

\[ G_l \]

has vertex set:
\[
P_1 \cup P_2 \cup \ldots \cup P_{|T|}
\]

and is the edge intersection of:

- complete multipartite graph:

\[
\begin{align*}
\text{Train 1} & \quad \text{Train 2} \\
\text{BO} & \quad \text{MO}
\end{align*}
\]

- comparability graph associated with the transitive relation
Separation Problem for \( \sum_{p \in S} x_p \leq 1, \quad S \in \mathcal{S} \)

Max-weight Stable Set on \( G_l \)

First Solution Approach (heuristic method): "transitivization" of \( G_l \)

Replace the graph with its transitive closure \( \rightarrow \) comparability graph

Compute the max-weight stable set on the comparability graph (heuristic method)
Second Solution Approach (exact method): dynamic programming

Simplified case: find max-weight set of crossing arcs

- consider arcs by decreasing slope
- for each pair of nodes \((u,v)\) store maximum weighted set of crossing arcs having departure node \(\geq u\) and arrival node \(\leq v\).

Can be extended to \(G_I\)
Computational Results

Tests on real-world instances provided by Italian Railways (Rete Ferroviaria Italiana)

- LP upper bound up to 10% better than Lagrangian upper bounds
- Heuristic solutions improved by up to 5%
- Provably optimal solutions in some cases

Future work

- Extension to the Railway Network
- Local search