Non-Cyclic Train Timetabling and Comparability Graphs

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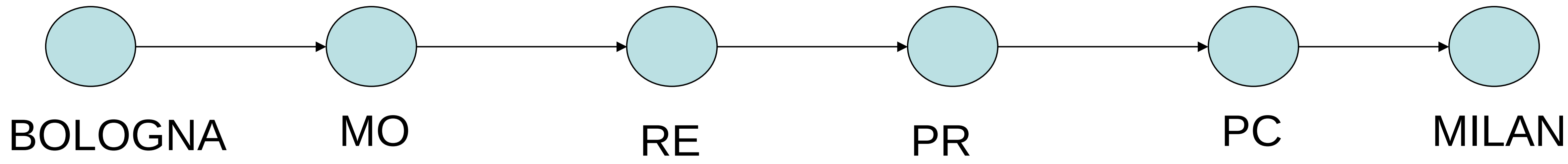
Non-Cyclic Train Timetabling Problem

<u>INPUT</u> :

- **Single Line with a one-way track** (approach easy to extend to railway network)



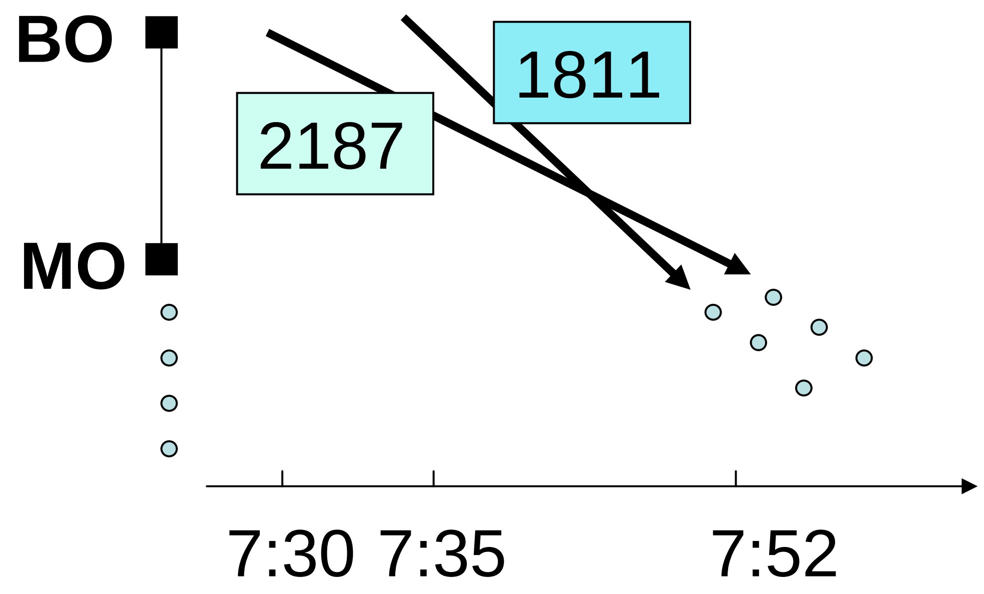
- **List T of Trains with "ideal timetables"**

EUROSTAR 1811: BO 7:35 - MI 9:10

REGIONAL 2187:

BO 7:30	-	MO 7:52	
MO 7:54	-	RE 8:12	
RE 8:14	-	PR 8:26	
PR 8:28	-	PC 8:55	

Ideal Timetables are CONFLICTING!!!!



No-Conflict Constraints (in the basic version of the problem) :

- **no overtaking** between stations (allowed only within stations)
- **min time** between consecutive **departures** from each station
- **min time** between consecutive **arrivals** at each station

OUTPUT :

- **"Adjusted" non-conflicting timetables with maximum total profit:**

Train Adjustments:

- **shift** departure time from initial station
- **stretch** increase stop time in intermediate stations

Train Profit:

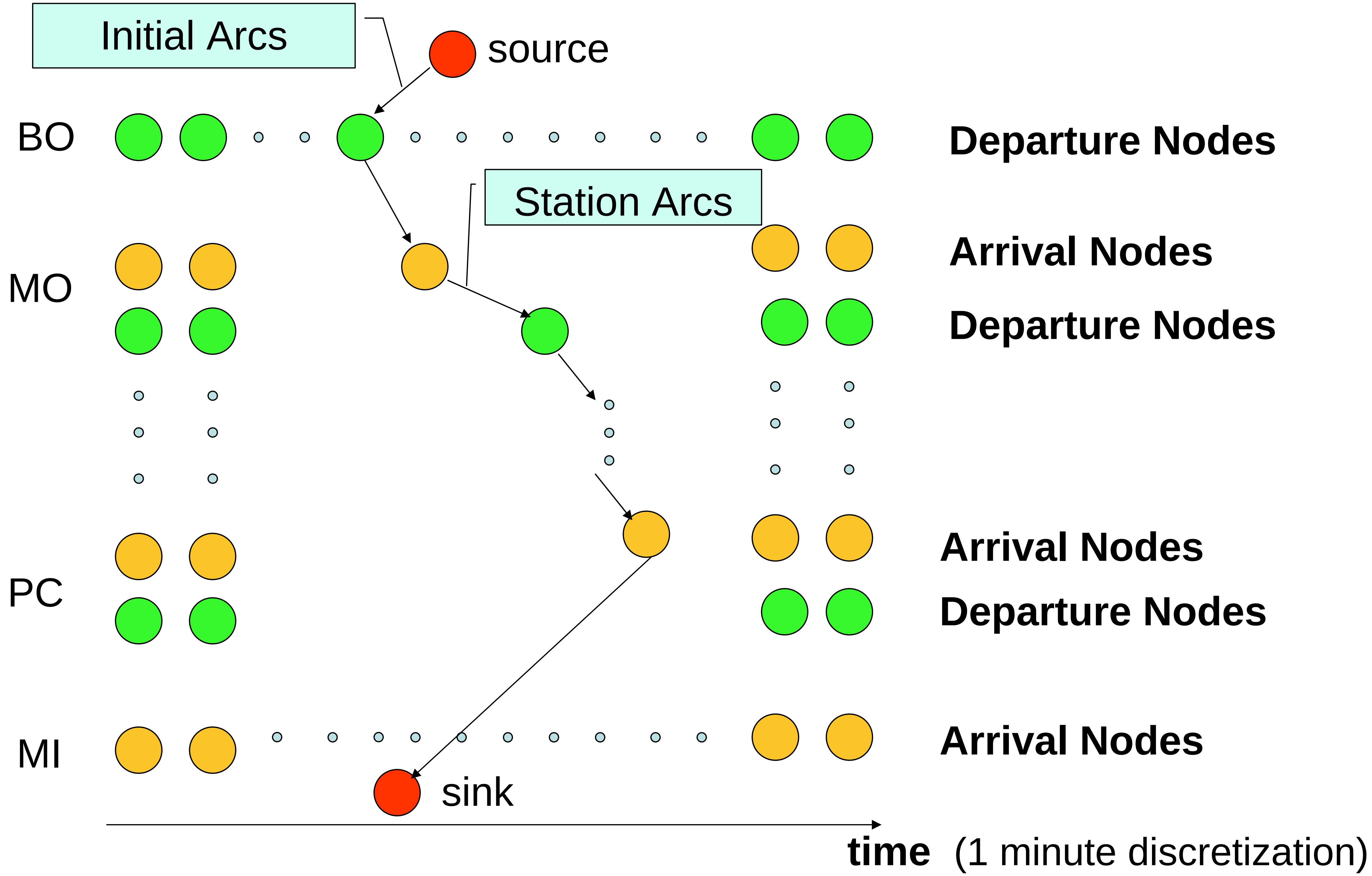
$$\pi_j - \phi_j(shift_j) - \sum_i \varphi_{ij}(stretch_{ij})$$

Ideal profit

Arbitrary monotone functions

If profit is negative cancel the train

Representation on Time-Space GRAPH



Train Timetable ⟺ **Path**

Train Profit ⟺ **Path Profit**

Ideal profit and **Shift** cost on initial arcs

Stretch cost on station arcs

ILP Formulation

x_p Variables associated with feasible paths of the Time-Space Graph

P_j Collection of paths for train *j*

p_p Profit of path $p \in P_j$

$\mathcal{S} = \mathcal{S}_1 \cup \mathcal{S}_2 \cup \ldots$

$\mathcal{S}_l$ **Maximal Stable Set** of G_l

$$\max \sum_{j \in T} \sum_{p \in P_j} p_p x_p$$

$$\sum_{p \in S} x_p \leq 1, \qquad S \in \mathcal{S}$$

$$x_p \in \{0,1\}, \qquad p \in P_j,\ j \in T$$

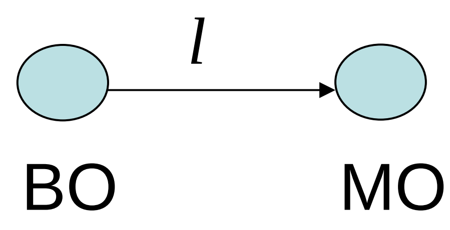
Path Compatibility Graph G_l

compatible paths:

- associated with different trains
- contain compatible arcs on line segment l

Satisfy no-conflict constraints ⇒ Transitive relation

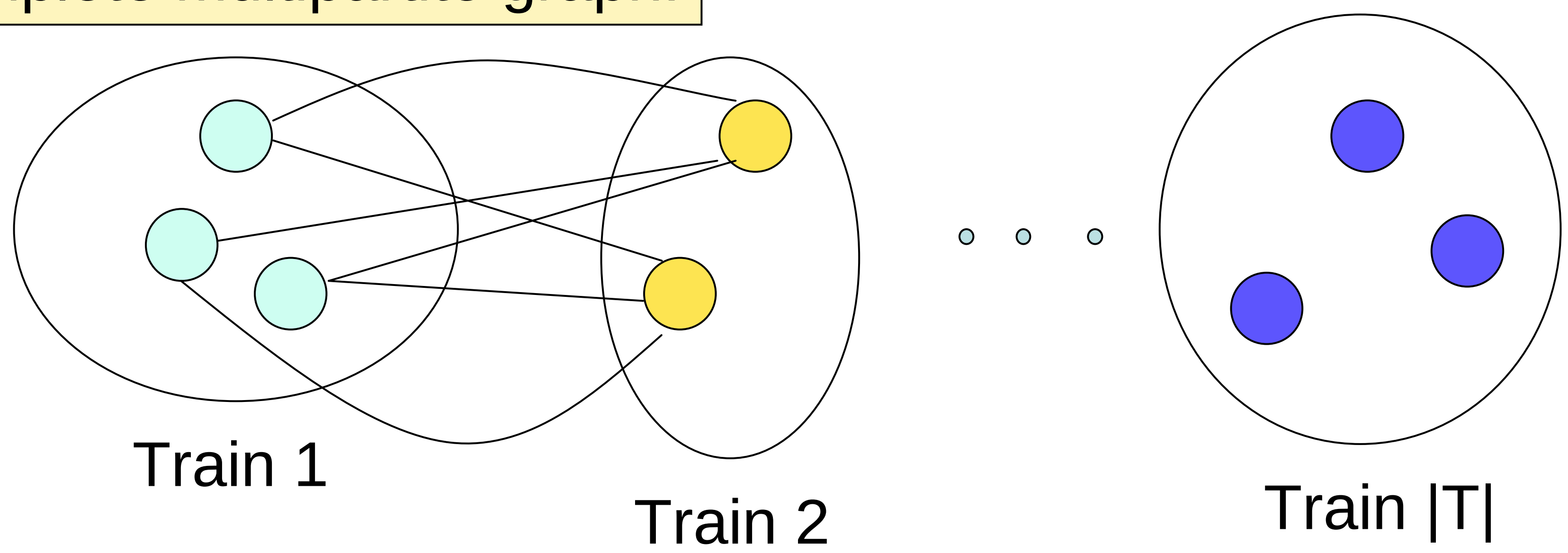
Line segment l between two consecutive stations on the line:



G_l has **vertex set**: $P_1 \cup P_2 \cup \ldots \cup P_{|T|}$

and **is the edge intersection** of:

- complete multipartite graph:



- comparability graph associated with the transitive relation

Separation Problem for $\sum_{p \in S} x_p \leq 1, \quad S \in \mathcal{S}$

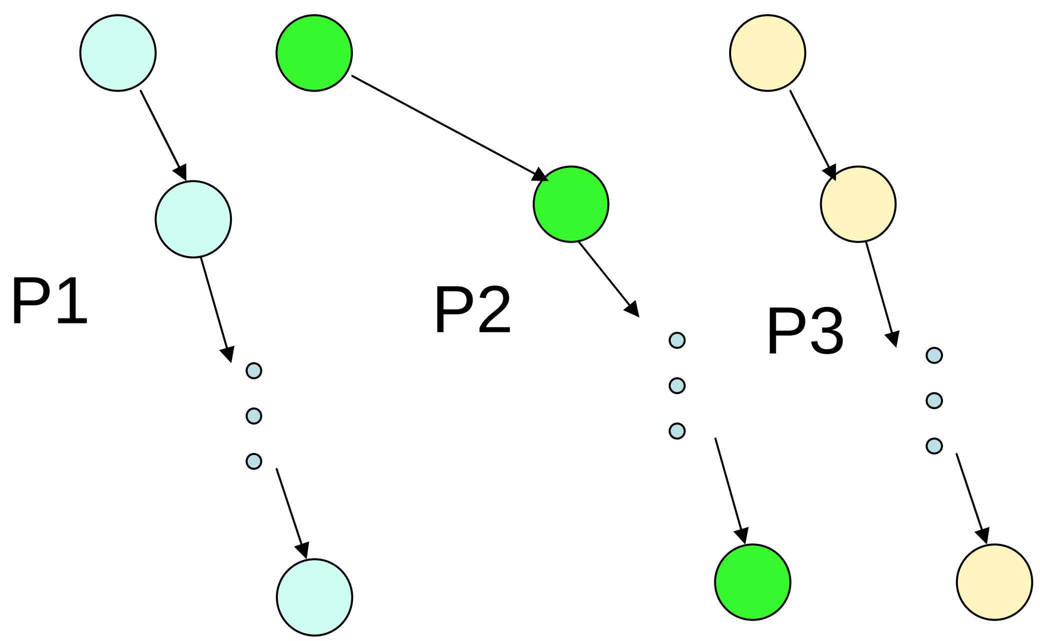
⇕

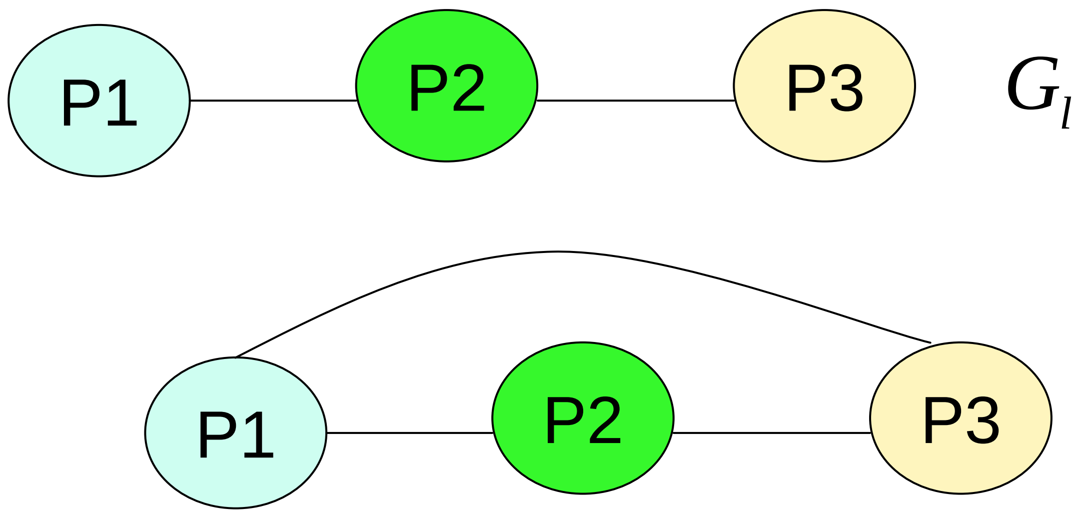
Max-weight Stable Set on G_l

First Solution Approach (heuristic method): "transitivization" of G_l

Replace the graph with its transitive closure ⇨ comparability graph

Compute the max-weight stable set on the comparability graph (heuristic method)



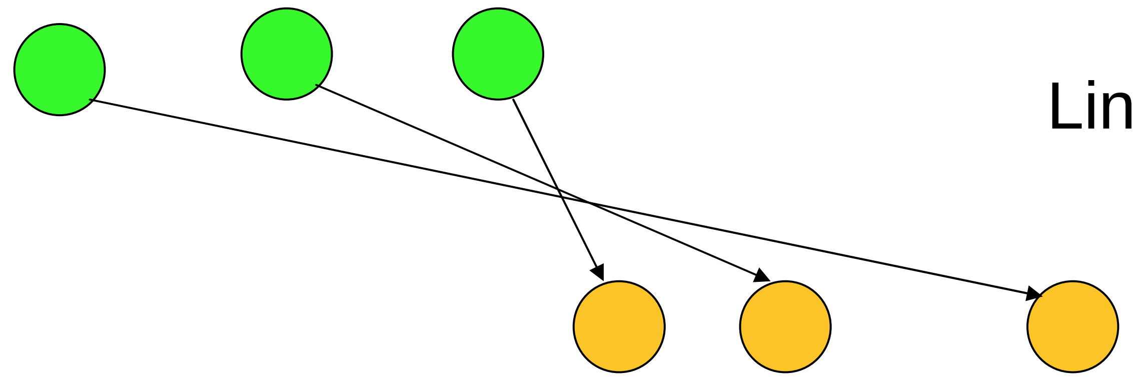


Second Solution Approach (exact method): dynamic programming

<u>Simplified case</u>: find max-weight set of crossing arcs

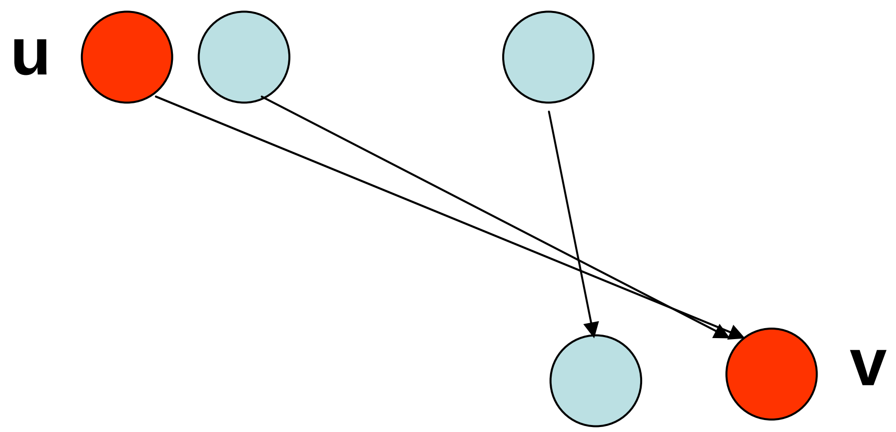
departure nodes

arrival nodes



Line segment $\boldsymbol{l}$

- consider arcs by decreasing slope
- for each pair of nodes (**u**,**v**) store maximum weighted set of crossing arcs having departure node >= **u** and arrival node <= **v**.



<u>Can be extended</u> to G_l

Computational Results

Tests on real-world instances provided by Italian Railways (Rete Ferroviaria Italiana)

- LP upper bound up to 10% better than Lagrangian upper bounds
- Heuristic solutions improved by up to 5%
- Provably optimal solutions in some cases

Future work

- Extension to the Railway Network
- Local search