Facility Location-Routing-Scheduling Problem: LRS

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① Definition of LRS
② Branch and Price Algorithm
③ Mater and Pricing Problem
④ Solution of Pricing Problem
⑤ What We Have Done
⑥ What We Will Do

LRS

Location Routing and Scheduling Problem:

3 dependent problems:

① locate facilities

② construct routes for vehicles

③ assign routes to vehicles

➜ capacitated facilities

➜ capacitated vehicles

➜ time restriction for the vehicles

Location-Routing and Scheduling Problem:

- ➜ In literature, heuristic solution for LRS problem (no IP formulation)
- ➜ Exact solutions for RS and LR
- ➜ We choose, Branch and Price Algorithm:
  - IP formulation includes many constraints (s.t. sub tour elimination constraints)
  - Can be written in set partitioning problem easily
  - Easy to think routes in terms of columns
  - With set partitioning formulation, many possible columns
- ➜ Other methods to solve:
  - Lagrangian Relaxation
  - Branch and bound and cut
  - Heuristic design
  - ?

Problems in Literature

- Facility Location -too many
- Vehicle Routing -too many
- Routing Scheduling
- Location Routing
- Location Routing and Scheduling

⇒ Location routing:	⇒Location scheduling:
one-to-one relation btw routes and vehicles.	not necessarily one-to-one relation assignment of one vehicle to many paths.

IP Formulation

Objective: Minimize total cost.

$$\text{TotalCost} = \text{Fixed cost of Facility and Vehicle} + \text{Operating cost of Vehicles}$$

Constraints:

① Each demand node should be served once

② # of a vehicle entering a node must be equal to # of the vehicle leaves this node

③ Capacity restriction for facility

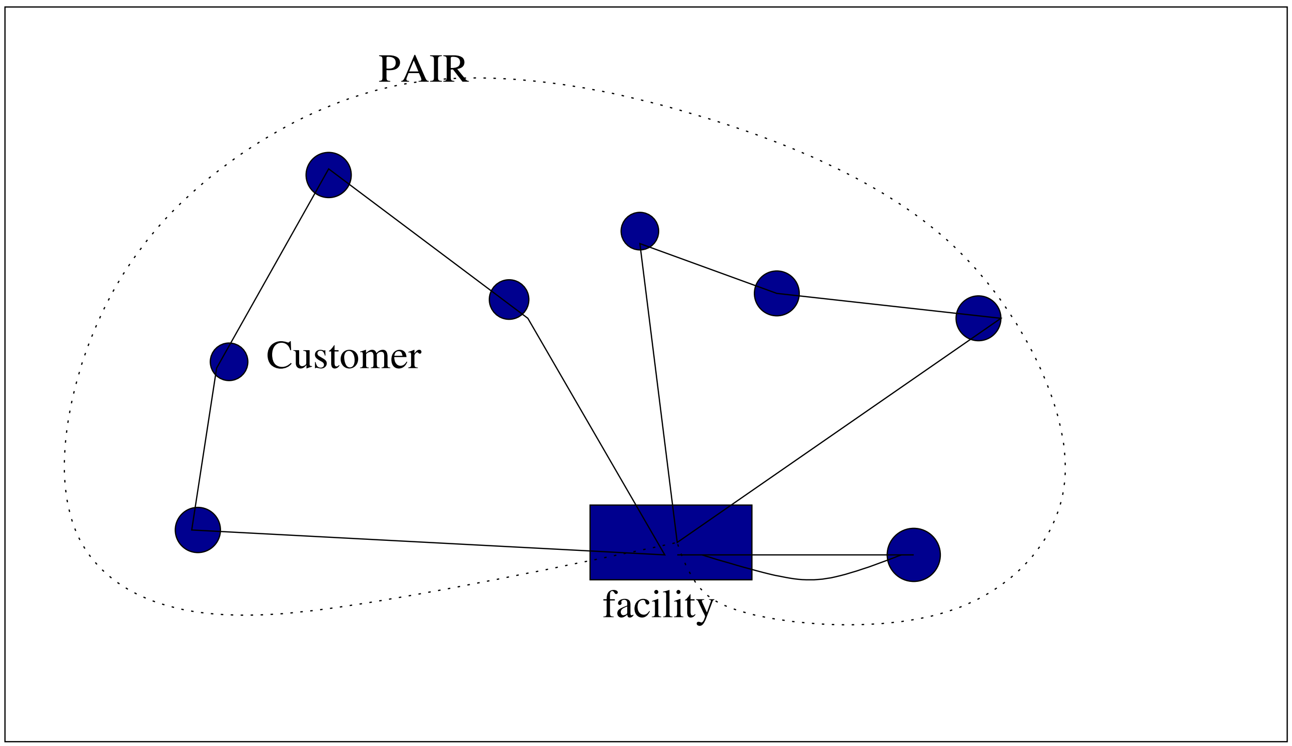
④ Capacity restriction for vehicles

⑤ Flow balance equations (to satisfy demand and eliminate the subtours)

⑥ Time restriction to the routes

Set Partitioning Model:

- Pairing Concept:
  Set of routes assigned to a vehicle and can be served within the given time limit.



Set Partitioning Model:

- Variables for set partitioning based on pairing concept:

$$Z_{jp} = \begin{cases} 1 & \text{if pairing } p \text{ is chosen for facility } j, \forall p \in P_j, \forall j \in M \\ 0 & \text{otherwise} \end{cases}$$

P_j:set of feasible pairs of facility j

$$T_j = \begin{cases} 1 & \text{if facility j is open}, \forall j \in M \\ 0 & \text{otherwise} \end{cases}$$

N:set of customers; M:set of facilities; I=$N \cup M$

Set Partitioning Model:

$$Min \sum_{j \in M} T_j.FixCost + \sum_{j \in M} \sum_{p \in P_j} C_{jp}.Z_{jp} \tag{1}$$

s.t.

$$\sum_{j \in M} \sum_{p \in P_j} a_{ipj}.Z_{jp} = 1 \ \ \forall i \in N \tag{2}$$

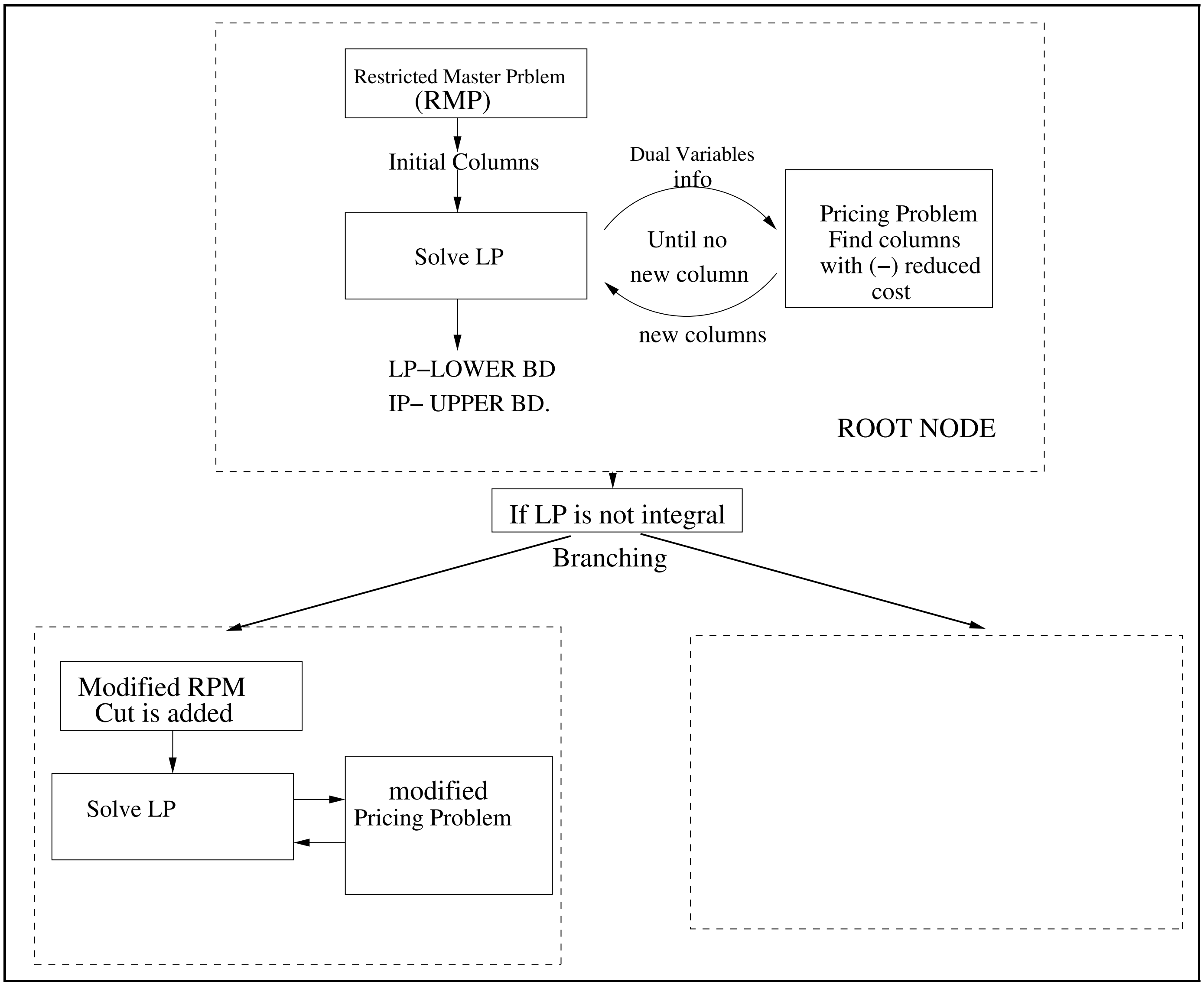
$$\sum_{i \in N} \sum_{p \in P_j} a_{ipj}.Demand_i.Z_{jp} \leq \ Cap_j.T_j \ \ \forall j \in M \tag{3}$$

$$Z_{jp} \ \leq \ T_j \ \ \forall j \in M, p \in P_j \tag{4}$$

$$Z_{jp}, T_j \in \{0, 1\}, \forall j \in M, p \in P_j \tag{5}$$

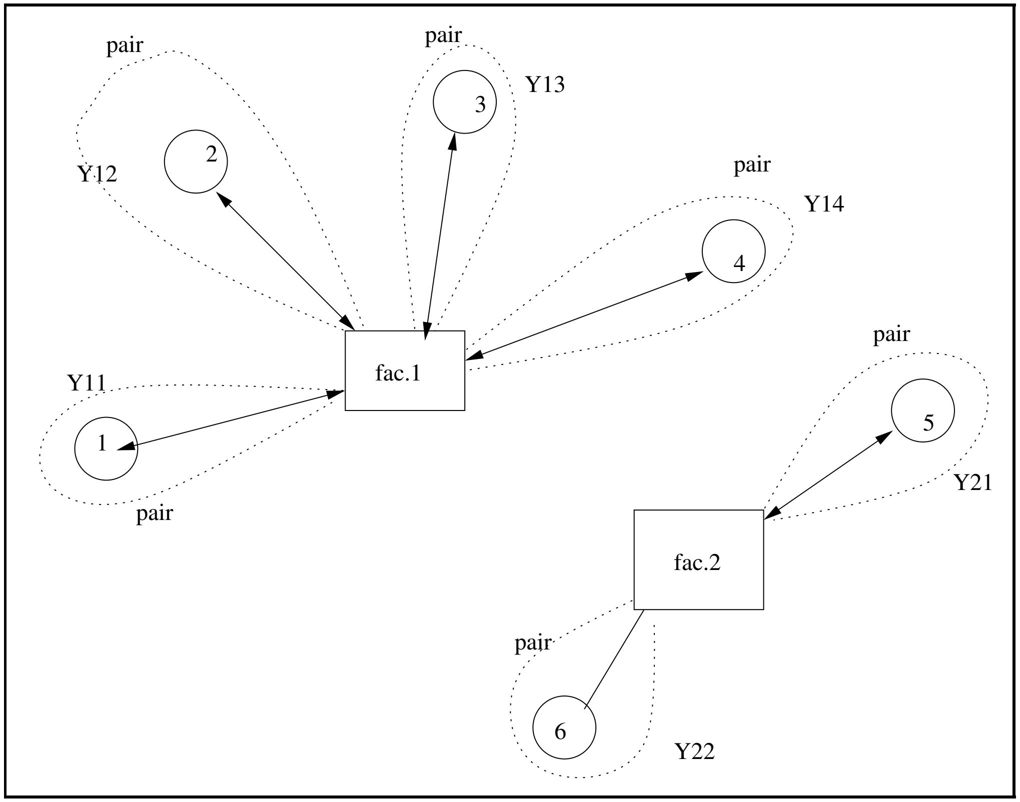
a_{ipj}=1 if node i is in pairing p of facility j.

BRANCH AND PRICE ALGORITHM



Restricted Master Problem:

- Initial pairs are formed



- Each pair represent a column in set partitioning formulation
- Restricted-since includes set of columns, not all columns

Pricing problem:

- Create 'pair': a column for Y_{jp}
- If 3rd const changed to:

$$\sum_{j \in P_j} a_{ip} Z_{jp} \leq T_j \ \ \forall j \in M \text{ and } i \in N \quad (6)$$

We have: π_i, μ_j, γ_{ji} dual variables

- **Reduced Cost** for Y_{jp}

$$\hat{C_{jp}} = C_{jp} - \sum_{i \in N} a_{ipj}.\pi_i + \sum_{i \in N} a_{ipj}.Demand_i.\mu_j + \sum_{i \in N} a_{ipj}.\sigma_{ji} \quad (7)$$

C_{jp} = Operating cost of the vehicle ($\propto$ travel time) + Fixed Cost of a vehicle

$\Rightarrow$ Independent pricing problem for each facility

Elementary Shortest Path with resource constraint

⇒ Pricing Problem = ESPRC If:

- ➜ Set up a network, including all customers and a source and sink nodes
- ➜ Arc costs:

$$c_{kl} = OperCost.d_{kl} - \pi_l + Demand_i.\mu_j + \sigma_{jl} \tag{8}$$

- ➜ find minimum cost path to the sink
- ➜ in our case allow visits more than once to sink
- ➜ If Total cost of path + Vehicle fixed Cost ≤ 0,
  add the column to restricted master problem
- ➜ stop when the shorthest path does not give negative cost column

Elementary Shorthest Path with resource constraint

- What is an elementary path?
  Each node can be visited at most once.
- Why elementary instead of walks?
  Trade of between more difficult pricing problem and more depth in branch and bound tree
- In our case: # of visits to sink ≥ 1
- In each visit to sink, current truck load is set to zero
- Adapt the Labelling Algorithm for ESPRC by Feillet, Dejax, Gendreau, Geuguen.

ESPRC

- Problem: too many feasible paths
- Keep resource consumptions, visited nodes, and cost
- Keep unreachable nodes for each label
- A node may be unreachable from other if not enough resource or is already visited.
- Eliminate dominated labels with respect to resource consumption and unreachable nodes.

CURRENTLY

What we have done

- ➜ Design Master Problem and Pricing Problem
- ➜ Adapted ESPRC algorithm to solve Pricing Problem
- ➜ Do the column generation
- ➜ Solve the root node

MINTO

- **MINTO:**Mixed INTeger Optimizer
- MINTO uses LP solver and do branch and bound algorithm
- MINTO can do many applications such as preprocessing, constraint generation, primal heurisitcs
- MINTO allows user to write own algorithm (for column generation, constraint generation, heuristics, ..) specific to the problem

Prof. Linderoth supports MINTO in our University

WHAT ELSE?

- **COIN-BCP:**(Common Optimization INterface) and **SYMPHONY**
  - Open source
  - allows parallellization in branch and bound tree
  - supported by Prof. Ralphs

NEXT

What we will do

- ➜ More implementation
  - Test problems
  - Determine the right number of columns to be generated in each time
  - Different LP algorithms, to find better reduced costs
  - See how well the root node solution
  - Create column pool
  - Branching strategies
  - parallelization
- ➜ Alternate solution: 2-sub problems approach
- ➜ IP formulation
- ➜ Focus on the pricing problem

2-SUB PROBLEM APPROACH

➜ Master problem includes 3 set of variables:

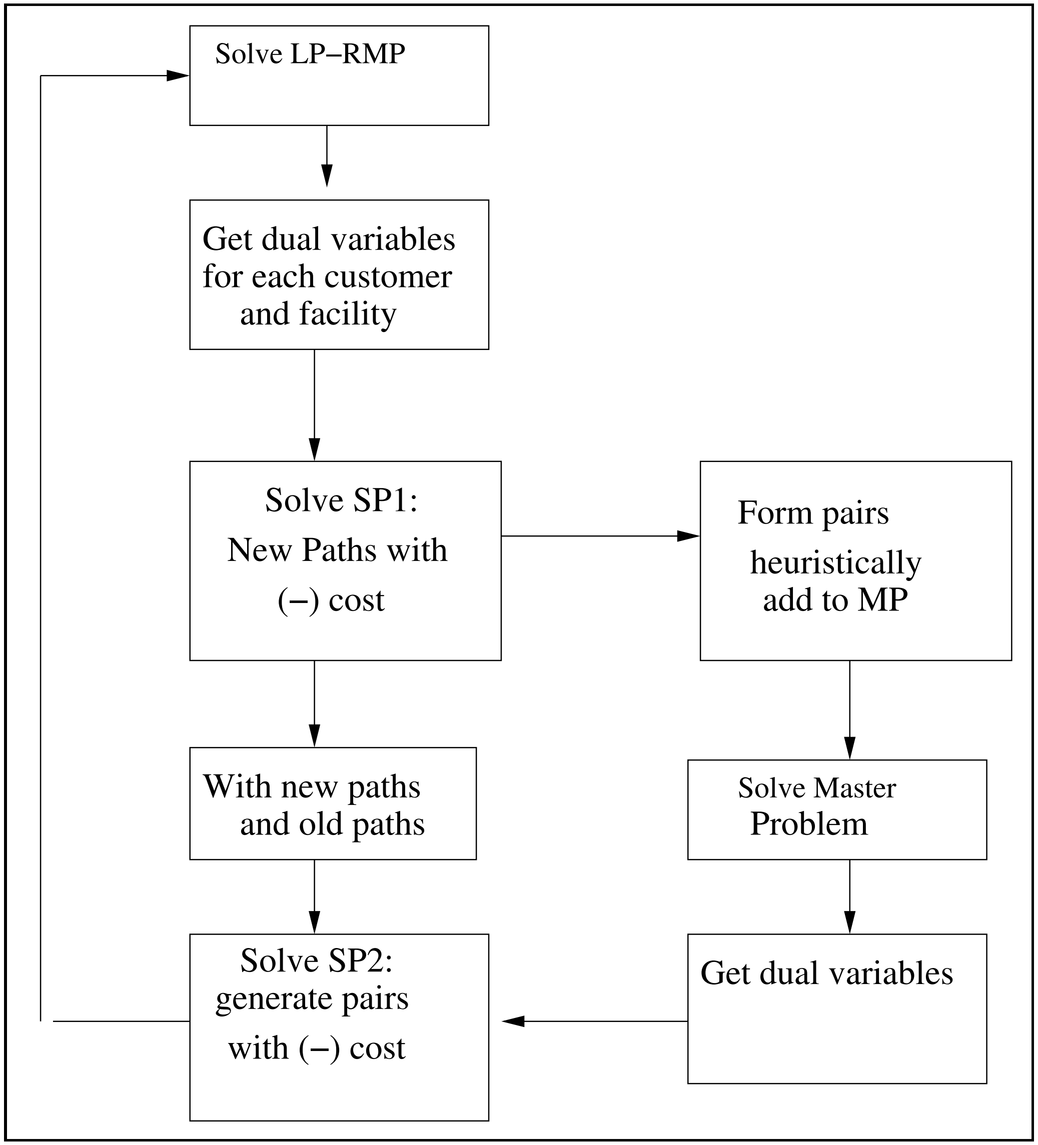
- Location variables
- $$Z_{jp} = \begin{cases} 1 & \text{if pairing } p \text{ is chosen for facility } j, \forall p \in P_j, \forall j \in M \\ 0 & \text{otherwise} \end{cases}$$
- $$X_{jk} = \begin{cases} 1 & \text{if path } k \text{ is chosen for facility } j, \forall k \in S_j, \forall j \in M \\ 0 & \text{otherwise} \end{cases}$$

➜ 2 nested sub problems: generating paths,and combining these paths as pairs.

➜ SP1: Generating paths: vehicle routing problem or elementary shortest path with 2 resources

➜ SP2: Combining paths: knapsack problem

2-SUB PROBLEM ALGORITHM



THANKS...

Any Questions?