2001k:90001 90-00 90C22

★**Handbook of semidefinite programming.**

Theory, algorithms, and applications.

Edited by Henry Wolkowicz, Romesh Saigal and Lieven Vandenberghe.

International Series in Operations Research & Management Science, 27.

Kluwer Academic Publishers, *Boston*, *MA*, 2000. *xxviii*+654 *pp.* $185.00. *ISBN* 0-7923-7771-0

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SDP (Semidefinite Programming) is the newest branch of mathematical programming sweeping up the attention of the worldwide mathematical research community.

Even though isolated papers related to SDP under various names began to appear as far back as the 1940s, the atmosphere for studying SDP as a unified subject was not ripe until the late 1980s when IPM (interior point methods) were fully developed in the context of LP (linear programming). The well attended International Symposium on Mathematical Programming held in Ann Arbor in 1994 was the first conference at which there were a significant number of talks on SDP. SDP has remained the most active area of research in mathematics since then, with well over 1000 research publications so far.

SDP refers to optimization problems that can be expressed in the form: find a real square symmetric matrix $X = (x_{ij})$ of order n to minimize $C{\bullet}X$ subject to $A_i{\bullet}X = b_i$, $i = 1, \dots, m$, and $X \succeq 0$, where $C = (C_{ij}), A_1, \dots, A_m$ are given real square symmetric matrices of order n $X \succeq 0$ means that X must be positive semidefinite, and $C{\bullet}X$ is the trace of the matrix CX, i.e., it is $\sum_{i=1}^n \sum_{j=1}^n C_{ij}X_{ij}$.

There are three powerful beacons attracting researchers to the SDP area.

The first is the ease with which duality theory and IPM developed for LP can be extended to the SDP, leading to very elegant algorithmic and computational complexity results. Now the primal-only (dual-only) IPM, the long-step and short-step path following primal dual IPM, the predictor-corrector algorithms of LP, and the potential reduction (feasible and infeasible) algorithms have all been extended to the SDP. Unified convergence analysis to show that these algorithms have polynomial worst case complexity to reduce the duality gap by a factor of at least $2^{-O(L)}$, where L is the size of the initial data, has been carried out. Many of the principal researchers who contributed to the development of IPM for LP, and others, have taken part in this development. Some of these same researchers have contributed well-written chapters on these algorithms and their analysis in this handbook.

The second is a spectacular result obtained by Goemans and Williamson on the maximum cut problem using SDP. Given a graph (V, E) with nonnegative edge weights, the maximum cut problem is to find a cut of maximum weight. It is an NP-hard problem for which Goemans and Williamson developed a randomized approximation algorithm based on SDP with a performance guarantee of 0.87856!

The third is the versatility of the SDP model. Following the success of Goemans and Williamson, a variety of nonlinear convex relaxations based on the SDP for several NP-hard combinatorial optimization problems, and Boolean and nonconvex quadratic optimization problems, are being investigated. Researchers are finding that in several of these problems, SDP relaxations are provably closer to the true optimum objective value than the linear relaxations. So, the development of SDP relaxations is an important advance in discrete and nonconvex optimization.

SDP has also emerged as an important tool in engineering applications for the analysis of nonlinear and time-varying systems and for controller synthesis, computer-aided control system design and structural optimization. SDP relaxations are also being used in stochastic optimization models involving multiclass queueing networks representing stochastic and dynamic generalizations of a job shop, in numerically constructing optimal designs in statistics, in the study of matrix completion problems (positive semidefinite matrix completion problem, and Euclidean distance matrix completion problem), and min-max eigenvalue problems. This handbook has survey chapters on each of these applications.

This handbook offers a broad advanced level overview of the current state of the art in SDP. The area is so new that most of the work reported here has been carried out in the last 12 years. It includes 19 chapters (each providing an up-to-date summary of one aspect of SDP written by a different set of authors) divided into three parts titled: theory, algorithms, applications and extensions. All the chapters are written using consistent notation.

I believe that the editors have been quite successful in their goal of bringing out an excellent research monograph covering the various aspects of SDP. The handbook belongs on the shelf of every researcher in optimization. *K. G. Murty* (1-MI-I)