Mixed Integer Programming Models for Wind Farm Design

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Outline

1. Wind Farm Design
2. Wind Farm Layout Optimisation
    - Wind flow modelling
    - Mixed integer programming model
    - Model validation
3. Future Work
    - Wind farm layout optimisation
    - Wind flow modelling
4. Summary

Wind Farm Design in New Zealand

Complex design problem that includes

- Wind farm layout
- External transmission and access
- Ecological and visual impacts
- Internal reticulation layout

Difficult economic conditions

- Inflated turbine prices due to excess international demand
- Depressed national electricity prices due to gas subsidies
- Lack of carbon pricing mechanisms

Developers driven to seek comparatively large-scale high-wind sites

Huge scope for the application of optimisation techniques

Wind Farm Layout Optimisation

Determine an optimal location of turbines that maximises the net power produced, subject to constraints on the number of turbines, turbine proximity, and turbine wake

Industry use “off-the-shelf” commercial software packages

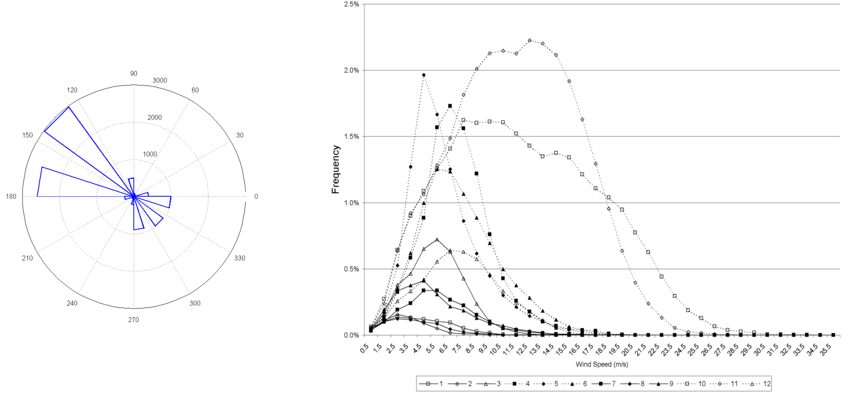
- WindFarm, WindFarmer, WindPRO, ...
- Heuristic optimisation algorithms
  - User specifies an initial feasible layout
  - Perform local neighbourhood search with random restarts
- Limited in the types of constraints that can be modelled

Previous work in the literature

- Genetic and evolutionary algorithms
- Greedy improvement heuristics

Is a MIP approach viable?

Wind Speed Frequency Distribution



Wind speed frequency distribution for six years of data from a site in the lower North Island, partitioned into 30° sectors and 1ms^{-1} ranges

Wind Flow Models

Industry standard models use a linear flow model

- WAsP, MS-Micro, ...

Model inputs

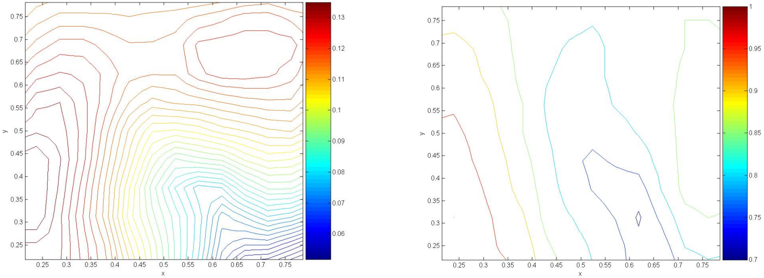
- Wind speed frequency distribution
- Digital terrain model

Two-parameter Weibull distributions are used to continuously approximate the empirical wind speed frequency distributions

Model output

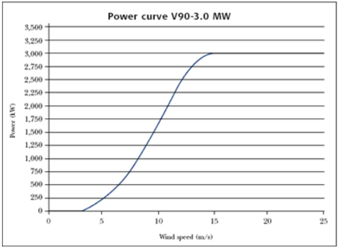
- Spatial wind speed distribution called a wind resource grid

Digital Terrain Map and Wind Resource Data



Digital terrain map contours and wind resource grid mean wind speed distribution at a height of 80m

Turbine Power Output



Power curve for a Vestas V90-3.0MW turbine

Ideal power

- Total expected power output by a turbine in isolation
- Ideal power $P = \int P(u) \cdot F(u) \, du$
    - $P(u)$ is the turbine power function
    - $F(u)$ is the wind speed frequency cumulative distribution function

Turbine Proximity and Wake Interference

Turbine proximity

- Minimum turbine separation requirements prescribe an exclusion zone around any turbine location

Turbine wake interference

- Turbine wakes create an energy deficit downstream
- The interference I between a pair of turbines is the net expected power loss that results from the superposition of the energy deficits at one turbine due to the wake of the other turbine
- Interference is calculated using the PARK turbine wake model (Jensen,1983; Katic, Hojstrup and Jensen, 1986)

Notation

Consider a graph $G = (V, E)$ where

- Set of nodes V denote potential turbine locations
- Set of edges E are partitioned into mutually exclusive sets
  - E_P denote proximity edges
  - E_I denote interference edges
- An edge $(u, v) \in E$ corresponds to a pair of turbine locations
  - If $(u, v) \in E_P$ then they violate the minimum separation distance
  - If $(u, v) \in E_I$ then the turbine wakes impact one another

Let

- P_v denote the ideal power of a turbine located at $v \in V$
- I_{uv} denote the power loss due to interference between turbines located at u and v where $(u, v) \in E_I$
- $\mathcal{Q}$ denote the set of all maximal complete subgraphs of the graph $G_P = (V, E_P)$

Mixed Integer Programming Model

$$\begin{array}{lll} \text{maximise} \sum_{v \in V} P_v x_v - \sum_{(u,v) \in E_I} I_{uv} y_{uv} & & \text{(Net power)} \\ \text{subject to} \sum_{v \in Q} x_v \leq 1, & Q \in \mathcal{Q} & \text{(Proximity)} \\ x_u + x_v \leq 1 + y_{uv}, & (u,v) \in E_I & \text{(Interference)} \\ \sum_{v \in V} x_v \leq k & & \text{(Turbine limit)} \\ x_v \in \{0,1\},\ y_{uv} \geq 0, & v \in V,\ (u,v) \in E_I & \end{array}$$

Case study

- 1.7km x 2.0km domain overlaid by a 48m regularly spaced grid
- 1069 nodes, 2388 proximity cliques, 66903 interference edges
- 67972 variables, 69292 constraints, 226313 non-zeros

Results

- Priority branching
- 1800 second time limit

	Relaxations				Best	Best	%	Gap
Turbines	LP	Root	Cuts	Nodes	Bound	Integer	Nodes	(%)
10	38054	37985	62 / 311	8192	37652	36832	90.3+	2.2
15	56622	56494	142 / 90	2679	56135	53974	93.3+	4.0
20	75007	74791	194 / 38	1705	74326	70598	91.8	5.3
30	111117	110743	293 / 0	311	110271	102868	0.0+	7.2
50	181306	180549	495 / 0	33	180512	161819	0.0+	11.6
75	264933	263369	620 / 6	1	263370	216620	0.0+	21.6
100	295027	279088	498 / 2	13	279047	216620	0.0	28.8

CPLEX 10.2 with default options running on a 3.2GHz Pentium 4 with 2GB RAM

Cuts = Clique cuts / Implied bound cuts, % Nodes = when best integer was found, + = heuristic

Results

- Interference constraints placed in a cut pool
- Priority branching
- 1800 second time limit

	Relaxations				Best	Best	%	Gap
Turbines	LP	Root	Cuts	Nodes	Bound	Integer	Nodes	(%)
10	38054	37988	65 / 11279	46101	37486	36837	83.0	1.7
15	56622	56489	148 / 21194	14775	56053	53986	67.1	3.8
20	75007	74792	175 / 24071	9101	74409	70580	85.3	5.4
30	111117	110738	348 / 26010	5922	110381	102716	16.2	7.5
50	181306	180541	499 / 14911	2641	180212	161913	94.3	11.3
75	264958	263393	627 / 7931	1481	262834	217670	13.5+	20.7
100	295036	279417	474 / 8466	2121	276012	222220	35.8+	24.2

CPLEX 10.2 with default options running on a 3.2GHz Pentium 4 with 2GB RAM

Cuts = Clique cuts / Lazy constraints, % Nodes = when best integer was found, + = heuristic

Comparison with WindFarmer

GH WindFarmer (Garrad Hassan Ltd)

- Widely used in industry
- Proprietary improvements to ideal power calculations and the PARK turbine wake model give better estimates of net power

Error

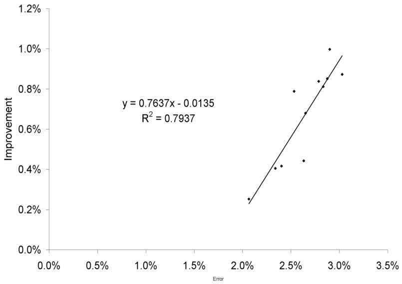
- Relative difference between the net power calculated by WindFarmer and the MIP for the same turbine locations

Improvement

- Relative improvement of a WindFarmer solution found using the MIP solution as a starting solution

Results

Comparison using solutions for problems with 10–20 turbines



Observations

- Strong positive correlation between the error and improvement
- Suggests improved techniques are needed for calculating model data rather than there being a weakness with the MIP model

Transitive Interference Constraints

Empirical evidence suggests the interference a turbine experiences is predominantly due to the wake of a single upwind turbine

Let

- D denote the set of wind directions
- U_v^d denote the set of locations upwind of $v \in V$ with respect to wind direction $d \in D$
- I_{uv}^d denote the power loss experienced by a turbine located at $u \in V$ due to interference from an upwind turbine located at $v \in U_u^d$ with respect to wind direction $d \in D$

$$x_u + x_v - \sum_{\substack{w \in U_u^d \\ I_{uw}^d > I_{uv}^d}} x_w \leq 1 + y_{uv}^d, \quad u \in V,\ d \in D,\ v \in U_u^d$$

Additional Design Parameters

Capital budget constraints

- Generalisation of turbine limit constraint

Line of sight constraints

- Eliminate potential turbine locations in MIP model

NZS sound level restrictions

- Budget-type constraint

Reticulation layout

- Costs negligible next to turbine capital costs

Sensitivity to empirical input data

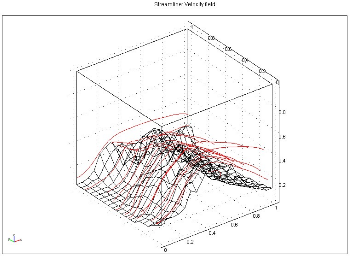
- Robust solutions

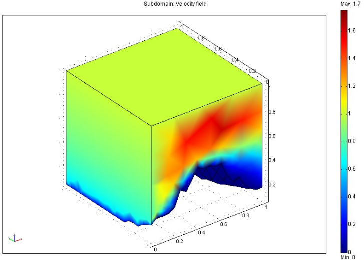
Linear vs Non-Linear Flow Models

Linear flow models

- Industry standard
- WAsP and MS-Micro
- Non-linear effects induced by complex terrain and high wind speeds

Non-linear CFD flow models


Streamline: Velocity field


Subdomain: Velocity field

Qualitative Wind Statistics

Important at sites with complex terrain and high wind speeds

Background and induced turbulence

- Variation in wind velocity over short lengths of time
- Cyclical loading patterns on turbine structures

Maximum gusts

- Maximum wind speed at a particular location within time frame
- Difficult to predict the spatial variation over the domain

Inflow angles

- Large vertical velocity components increase the inflow angle
- Negative impact on turbine operation and maintenance

Eliminate potential turbine locations in MIP model

Conclusions

Wind farm design in New Zealand

- Complex design problem under difficult economic conditions
- Huge scope for the application of optimisation techniques

Wind flow modelling

- CFD has many advantages compared to linear flow models
- Non-linear turbulent wind flow in New Zealand's complex terrain

Wind farm layout optimisation

- MIP models appear to be viable
- More amenable to modelling site specific conditions
- Will require state-of-the-art MIP techniques
- Adapt the extensive literature for similar MIP problems

Thank you for your attention

Questions?

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