Computing with multi-row Gomory cuts

Daniel G. Espinoza

Universidad de Chile, FCFM, DII

MIP 2008, Columbia University, New York, USA, August 4, 2008

Outline

1. Introduction and Theory
2. The Experiment Settings
3. The Results

Introduction

- General cutting planes central for practical IP performance.
- Most important family are Gomory cuts (Bixby et al. 2006).
- Much research on extensions, but little practical impact.
- Most attempts focused on cuts derived from single-row systems.

Introduction

- General cutting planes central for practical IP performance.
- Most important family are Gomory cuts (Bixby et al. 2006).
- Much research on extensions, but little practical impact.
- Most attempts focused on cuts derived from single-row systems.

Introduction

- General cutting planes central for practical IP performance.
- Most important family are Gomory cuts (Bixby et al. 2006).
- Much research on extensions, but little practical impact.
- Most attempts focused on cuts derived from single-row systems.

Introduction

- General cutting planes central for practical IP performance.
- Most important family are Gomory cuts (Bixby et al. 2006).
- Much research on extensions, but little practical impact.
- Most attempts focused on cuts derived from single-row systems.

Introduction

- Recent work by Andersen et. al [2005], Cornuejols and Borozan [2007], Gomory [2007], have extended our understanding of multi-row relaxations.
- From the theory side, multi-row relaxations are known to be central in describing convex hull of sets:
  - Cook, Kannan and Schrijver example has infinite CG-rank.
  - Single cut from two rows gives
  - Extensions show need for
- Our goal is to test if these new ideas may have a practical impact.

Introduction

- Recent work by Andersen et. al [2005], Cornuejols and Borozan [2007], Gomory [2007], have extended our understanding of multi-row relaxations.
- From the theory side, multi-row relaxations are known to be central in describing convex hull of sets:
  - Cook, Kannan and Schrijver example has infinite CG-rank.
  - Single cut from two rows gives complete description.
  - Extensions show need for cuts derived from *n* rows.
- Our goal is to test if these new ideas may have a practical impact.

Introduction

- Recent work by Andersen et. al [2005], Cornuejols and Borozan [2007], Gomory [2007], have extended our understanding of multi-row relaxations.
- From the theory side, multi-row relaxations are known to be central in describing convex hull of sets:
  - Cook, Kannan and Schrijver example has infinite CG-rank.
  - Single cut from two rows gives complete description.
  - Extensions show need for cuts derived from n rows.
- Our goal is to test if these new ideas may have a practical impact.

Introduction

- Recent work by Andersen et. al [2005], Cornuejols and Borozan [2007], Gomory [2007], have extended our understanding of multi-row relaxations.
- From the theory side, multi-row relaxations are known to be central in describing convex hull of sets:
  - Cook, Kannan and Schrijver example has infinite CG-rank.
  - Single cut from two rows gives complete description.
  - Extensions show need for cuts derived from n rows.
- Our goal is to test if these new ideas may have a practical impact.

Introduction

- Recent work by Andersen et. al [2005], Cornuejols and Borozan [2007], Gomory [2007], have extended our understanding of multi-row relaxations.
- From the theory side, multi-row relaxations are known to be central in describing convex hull of sets:
  - Cook, Kannan and Schrijver example has infinite CG-rank.
  - Single cut from two rows gives complete description.
  - Extensions show need for cuts derived from n rows.
- Our goal is to test if these new ideas may have a practical impact.

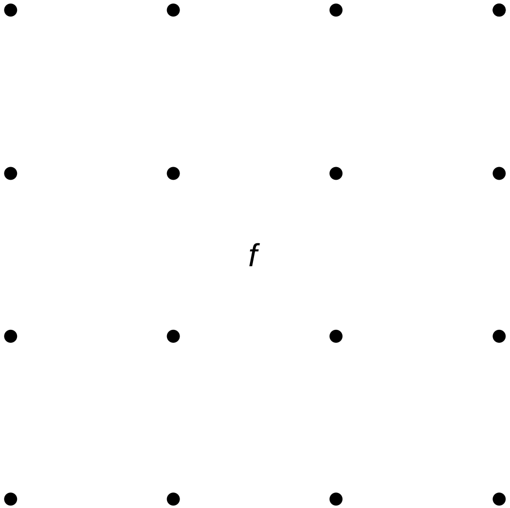
Introduction

- Recent work by Andersen et. al [2005], Cornuejols and Borozan [2007], Gomory [2007], have extended our understanding of multi-row relaxations.
- From the theory side, multi-row relaxations are known to be central in describing convex hull of sets:
  - Cook, Kannan and Schrijver example has infinite CG-rank.
  - Single cut from two rows gives complete description.
  - Extensions show need for cuts derived from n rows.
- Our goal is to test if these new ideas may have a practical impact.

How we derive cuts from two row systems?

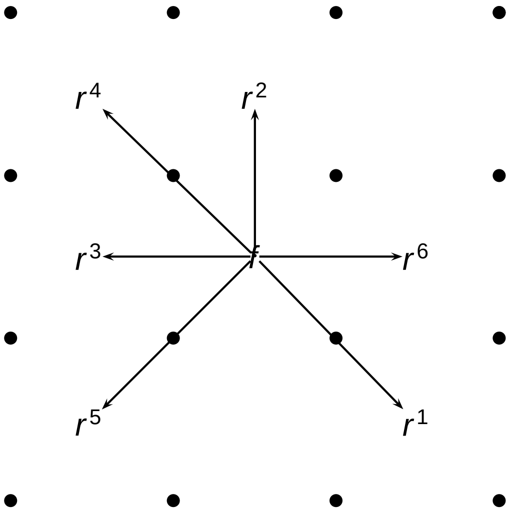
- We consider $x, s \in \mathbb{Z}^2 \times \mathbb{R}^J_+$. .

How we derive cuts from two row systems?



- We consider $x, s \in \mathbb{Z}^2 \times \mathbb{R}^J_+$.
- Assume $f, r^j \in \mathbb{Q}^2$, $f \notin \mathbb{Z}^2$. .

How we derive cuts from two row systems?

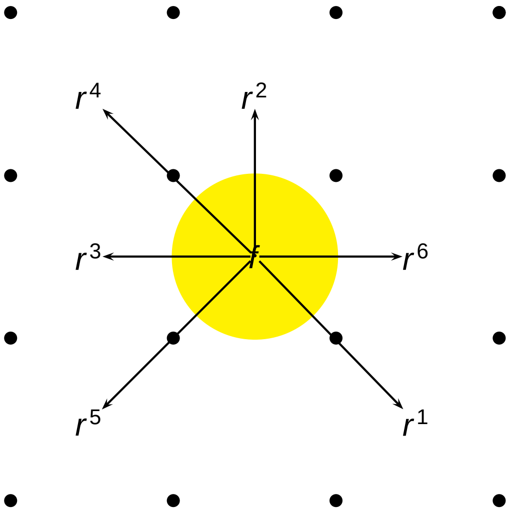


- We consider $x, s \in \mathbb{Z}^2 \times \mathbb{R}^J_+$.
- Assume

$$x = f + \sum_{j \in J} r^j s_j$$

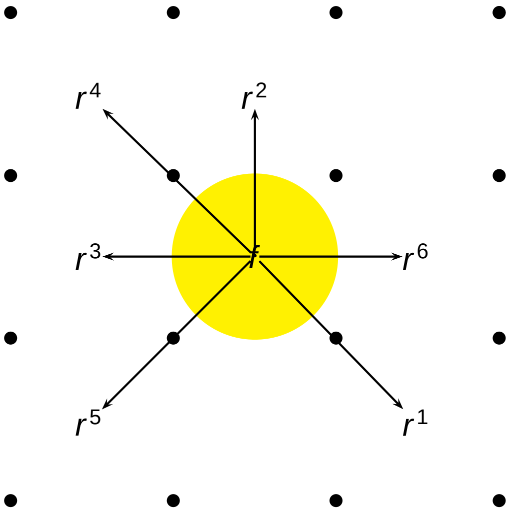
. .

How we derive cuts from two row systems?



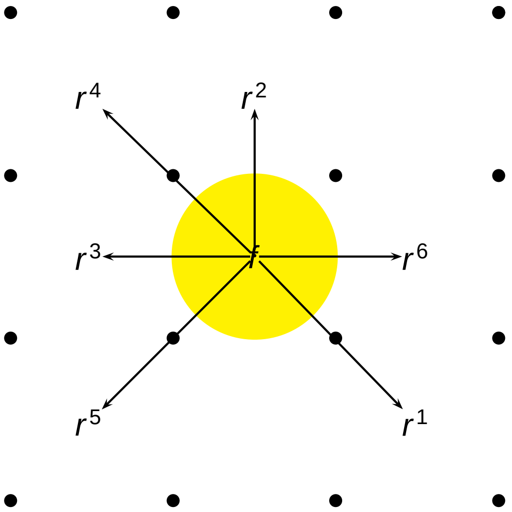
- We consider $x, s \in \mathbb{Z}^2 \times \mathbb{R}^J_+$.
- Let $C \subset \mathbb{R}^2$ s.t. $f \in C$, convex and $\text{int}(C) \cap \mathbb{Z}^2 = \emptyset$.

How we derive cuts from two row systems?



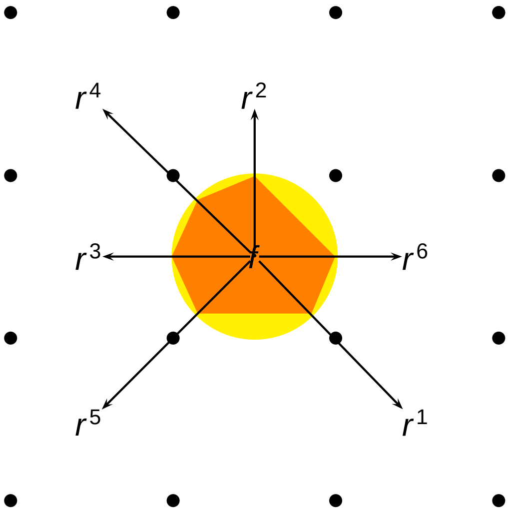
- We consider $x, s \in \mathbb{Z}^2 \times \mathbb{R}^J_+$.
- Let $C \subset \mathbb{R}^2$ s.t. $f \in C$, convex and $\text{int}(C) \cap \mathbb{Z}^2 = \emptyset$
- Let $\alpha_j = \max\{\alpha : f + \alpha r^j \in C\}$.

How we derive cuts from two row systems?



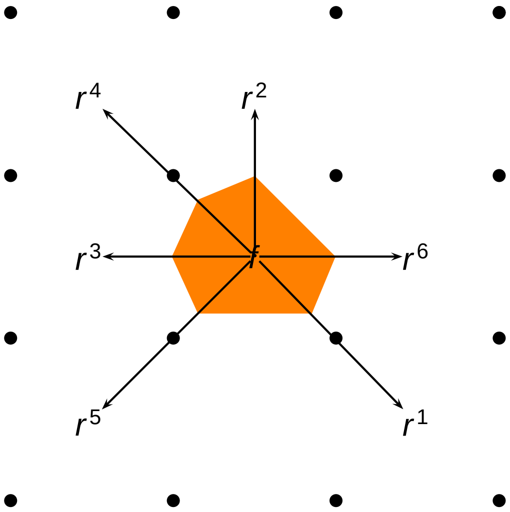
- We consider $x, s \in \mathbb{Z}^2 \times \mathbb{R}^J_+$.
- Let $C \subset \mathbb{R}^2$ s.t. $f \in C$, convex and $\text{int}(C) \cap \mathbb{Z}^2 = \emptyset$
- Let $\alpha_j = \max\{\alpha : f + \alpha r^j \in C\}$
- Let $S = \{s \in \mathbb{R}^J_+ : \sum_{j \in J} \frac{s_j}{\alpha_j} \leq 1\}$.

How we derive cuts from two row systems?



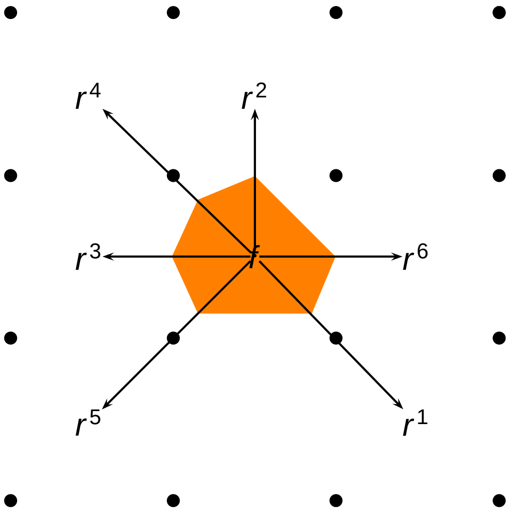
- We consider $x, s \in \mathbb{Z}^2 \times \mathbb{R}^J_+$.
- Let $C \subset \mathbb{R}^2$ s.t. $f \in C$, convex and $\text{int}(C) \cap \mathbb{Z}^2 = \emptyset$
- Let $\alpha_j = \max\{\alpha : f + \alpha r^j \in C\}$
- Let $S = \{s \in \mathbb{R}^J_+ : \sum_{j \in J} \frac{s_j}{\alpha_j} \leq 1\}$.
- $\Rightarrow C' := f + r \cdot S \subseteq C$.

How we derive cuts from two row systems?



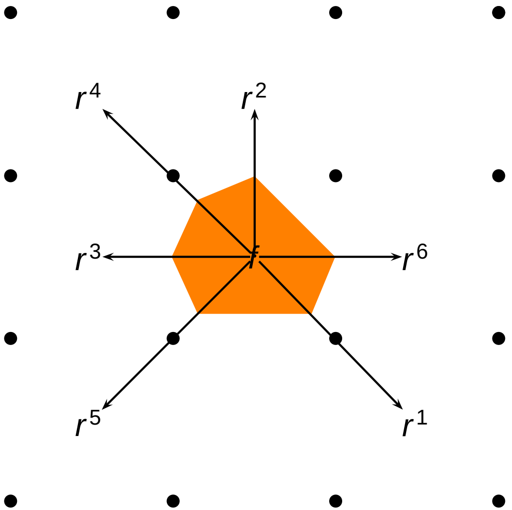
- We consider $x, s \in \mathbb{Z}^2 \times \mathbb{R}^J_+$.
- Let $C \subset \mathbb{R}^2$ s.t. $f \in C$, convex and $\text{int}(C) \cap \mathbb{Z}^2 = \emptyset$
- Let $\alpha_j = \max\{\alpha : f + \alpha r^j \in C\}$
- Let $S = \{s \in \mathbb{R}^J_+ : \sum_{j \in J} \frac{s_j}{\alpha_j} \leq 1\}$.
- $\sum_{j \in J} \frac{s_j}{\alpha_j} \geq 1$ is valid

How we derive cuts from two row systems?



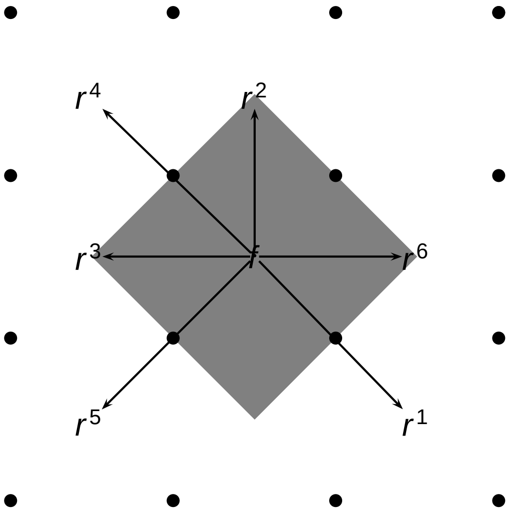
- We consider $x, s \in \mathbb{Z}^2 \times \mathbb{R}^J_+$.
- $\sum_{j \in J} \frac{s_j}{\alpha_j} \geq 1$ is valid
- How much better can we make C'?

How we derive cuts from two row systems?



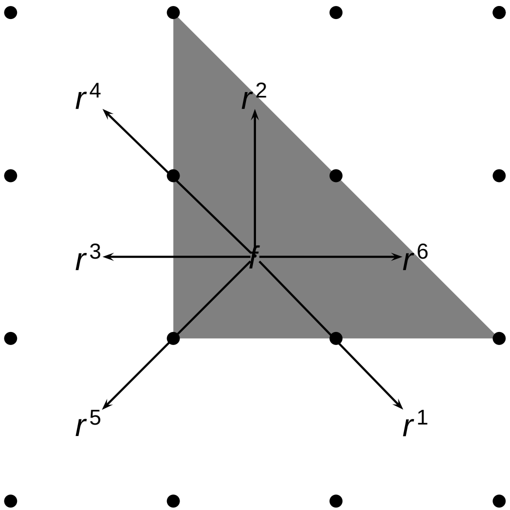
- We consider $x, s \in \mathbb{Z}^2 \times \mathbb{R}^J_+$.
- $\sum_{j\in J} \frac{s_j}{\alpha_j} \geq 1$ is valid
- C' should be a maximal convex, lattice free set containing f

How we derive cuts from two row systems?



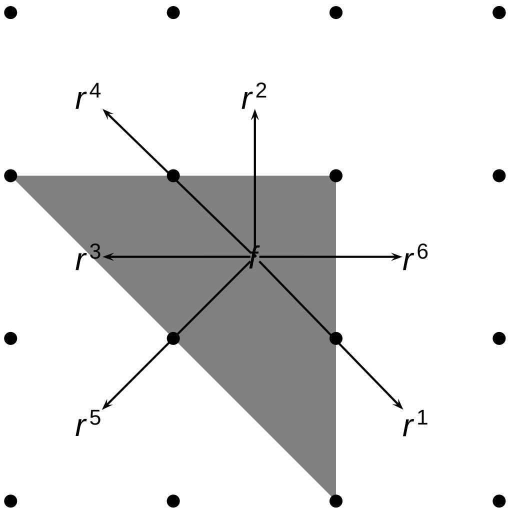
- We consider $x, s \in \mathbb{Z}^2 \times \mathbb{R}^J_+$.
- $\sum_{j \in J} \frac{s_j}{\alpha_j} \geq 1$ is valid
- C' should be a maximal convex, lattice free set containing f
- A quadrilateral or Gomory set
- completely symmetric

How we derive cuts from two row systems?



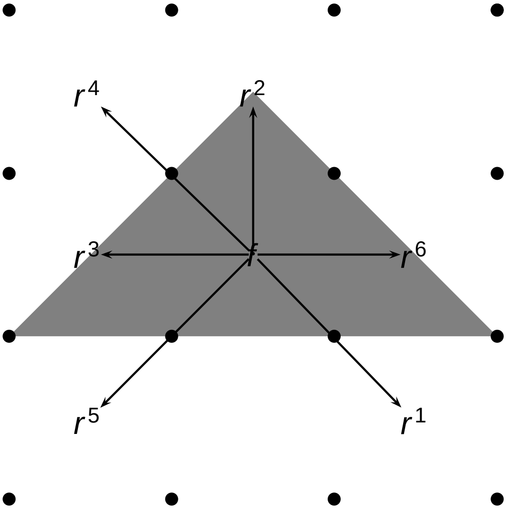
- We consider $x, s \in \mathbb{Z}^2 \times \mathbb{R}^J_+$.
- $\sum_{j \in J} \frac{s_j}{\alpha_j} \geq 1$ is valid
- C' should be a maximal convex, lattice free set containing f
- A Type 1 triangle

How we derive cuts from two row systems?



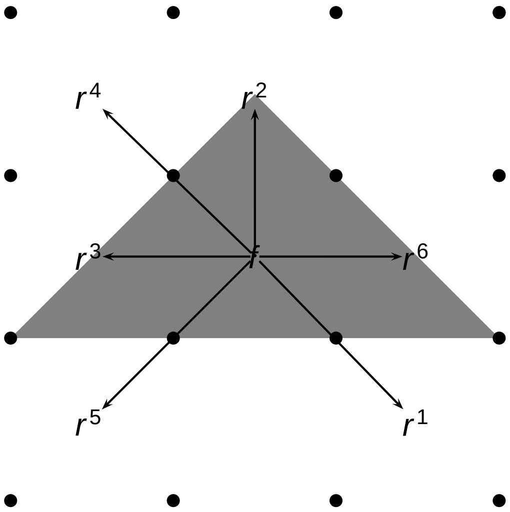
- We consider $x, s \in \mathbb{Z}^2 \times \mathbb{R}^J_+$.
- $\sum_{j \in J} \frac{s_j}{\alpha_j} \geq 1$ is valid
- C' should be a maximal convex, lattice free set containing f
- A Type 1 triangle
- 2^n possible configurations ($n =$ number of rows)

How we derive cuts from two row systems?



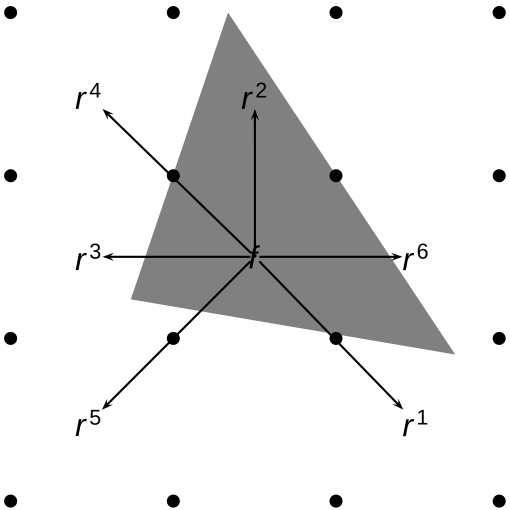
- We consider $x, s \in \mathbb{Z}^2 \times \mathbb{R}^J_+$.
- $\sum_{j \in J} \frac{s_j}{\alpha_j} \geq 1$ is valid
- C' should be a maximal convex, lattice free set containing f
- A Type 2 triangle
- $n!2^n$ possible configurations

How we derive cuts from two row systems?



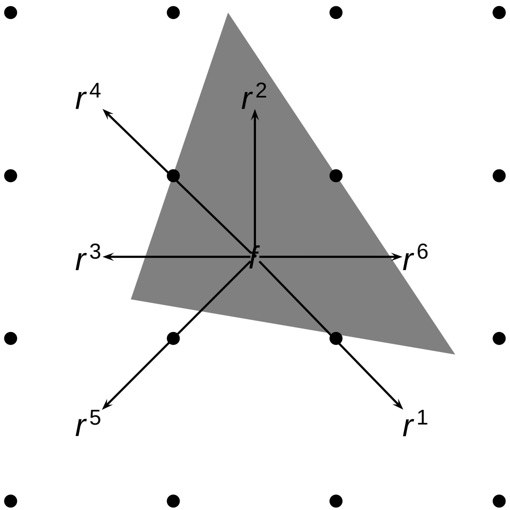
- We consider $x, s \in \mathbb{Z}^2 \times \mathbb{R}^J_+$. .
- $\sum_{j \in J} \frac{s_j}{\alpha_j} \geq 1$ is valid
- For non-dominance, every edge must contain integer point in relative interior

How we derive cuts from two row systems?



- We consider $x, s \in \mathbb{Z}^2 \times \mathbb{R}^J_+$. .
- $\sum_{j \in J} \frac{s_j}{\alpha_j} \geq 1$ is valid
- Still, far too many possible sets

How we derive cuts from two row systems?



- We consider $x, s \in \mathbb{Z}^2 \times \mathbb{R}^J_+$.
- $\sum_{j \in J} \frac{s_j}{\alpha_j} \geq 1$ is valid
- Still, far too many possible sets
- All these ideas can be extended to $x, f, r^j \in \mathbb{Q}^q$ and to $|J| = \infty$.

How we apply this?

Basic Problem:

$$\begin{aligned} \max \quad & cx \\ s.t. \quad & Ax = b \\ & x_i \in \mathbb{Z}_+ \quad \forall i \in I,\ x \in \mathbb{R}^n_+ \end{aligned} \tag{1}$$

where $I \subseteq \{1, \ldots, n\}$, $A \in \mathbb{Q}^{m \times n}$ is of full row rank, $c \in \mathbb{Q}^n$, $b \in \mathbb{Q}^m$, and $x \in \mathbb{Q}^n$.

A first relaxation:

$$\begin{aligned} x_{B'} &= f + \sum_{j \in N} r^j x_j \\ x_N &\geq 0,\ x_i \in \mathbb{Z} \forall i \in B' \end{aligned} \tag{2}$$

Where B is a basic solution, $B' = B \cap I$.

How we apply this?

Basic Problem:

$$\begin{array}{rll} \max & cx & \\ s.t. & Ax & = b \\ & x_i & \in \mathbb{Z}_+ \quad \forall i \in I,\ x \in \mathbb{R}^n_+ \end{array} \tag{1}$$

where $I \subseteq \{1, \ldots, n\}$, $A \in \mathbb{Q}^{m \times n}$ is of full row rank, $c \in \mathbb{Q}^n$, $b \in \mathbb{Q}^m$, and $x \in \mathbb{Q}^n$.

A first relaxation:

$$\begin{array}{rcl} x_{B'} & = & f + \sum_{j \in N} r^j x_j \\ x_N & \geq & 0,\ x_i \in \mathbb{Z} \forall i \in B' \end{array} \tag{2}$$

Where B is a basic solution, $B' = B \cap I$.

The math behind it

Gomory-Johnson Infinite group relaxation:

$$R_f : \quad \begin{array}{rcl} x & = & f + \sum_{finite} r s_r \\ x \in \mathbb{Z}^q & & s \in \mathbb{R}_+^{\mathbb{Q}^q} \end{array} \tag{3}$$

Theorem (Minimal valid inequalities for R_f (CB-2007))

If $f \notin \mathbb{Z}^q$, any minimal valid inequality that cuts off $(f, 0)$:

1. *Is of the form $\sum_{finite} \psi(r) s_r \geq 1$.*
2. *If $B_\psi = \{x \in \mathbb{Q}^q : \psi(x - f) \leq 1\}$, then B_ψ is convex, with no integral point in its interior. Furthermore $f \in B_\psi$.*
3. *If ψ is finite, then ψ is a continuous nonnegative homogeneous convex piecewise linear function with at most 2^q pieces.*
4. *If ψ is finite, then f is in the interior of B_ψ and B_ψ is a polyhedron of at most 2^q facets, and each of its facets contains an integral point in its relative interior.*

The math behind it

Gomory-Johnson Infinite group relaxation:

$$R_f : \begin{array}{rcl} x & = & f + \sum_{finite} r s_r \\ x \in \mathbb{Z}^q & & s \in \mathbb{R}_+^{\mathbb{Q}^q} \end{array} \qquad (3)$$

Theorem (Minimal valid inequalities for R_f (CB-2007))

If $f \notin \mathbb{Z}^q$, any minimal valid inequality that cuts off $(f, 0)$:

1. *Is of the form $\sum_{finite} \psi(r) s_r \geq 1$.*
2. *If $B_\psi = \{x \in \mathbb{Q}^p : \psi(x - f) \leq 1\}$, then B_ψ is convex, with no integral point in its interior. Furthermore $f \in B_\psi$.*
3. *If ψ is finite, then ψ is a continuous nonegative homogeneous convex piecewise linear function with at most 2^q pieces.*
4. *If ψ is finite, then f is in the interior of B_ψ and B_ψ is a polyhedron of at most 2^q facets, and each of its facets contains an integral point in its relative interior.*

The math behind it

Gomory-Johnson Infinite group relaxation:

$$R_f : \begin{array}{rcl} x & = & f + \sum_{finite} r s_r \\ x \in \mathbb{Z}^q & & s \in \mathbb{R}_+^{\mathbb{Q}^q} \end{array} \quad (3)$$

Theorem (Minimal valid inequalities for R_f (CB-2007))

If $f \notin \mathbb{Z}^q$, any minimal valid inequality that cuts off $(f, 0)$:

1. *Is of the form $\sum_{finite} \psi(r) s_r \geq 1$.*
2. *If $B_\psi = \{x \in \mathbb{Q}^p : \psi(x - f) \leq 1\}$, then B_ψ is convex, with no integral point in its interior. Furthermore $f \in B_\psi$.*
3. *If ψ is finite, then ψ is a continuous nonegative homogeneous convex piecewise linear function with at most 2^q pieces.*
4. *If ψ is finite, then f is in the interior of B_ψ and B_ψ is a polyhedron of at most 2^q facets, and each of its facets contains an integral point in its relative interior.*

The math behind it

Gomory-Johnson Infinite group relaxation:

$$R_f : \begin{array}{rcl} x & = & f + \sum_{finite} r s_r \\ x \in \mathbb{Z}^q & & s \in \mathbb{R}_+^{\mathbb{Q}^q} \end{array} \quad (3)$$

Theorem (Minimal valid inequalities for R_f (CB-2007))

If $f \notin \mathbb{Z}^q$, any minimal valid inequality that cuts off $(f, 0)$:

1. *Is of the form $\sum_{finite} \psi(r) s_r \geq 1$.*
2. *If $B_\psi = \{x \in \mathbb{Q}^p : \psi(x - f) \leq 1\}$, then B_ψ is convex, with no integral point in its interior. Furthermore $f \in B_\psi$.*
3. *If ψ is finite, then ψ is a continuous nonegative homogeneous convex piecewise linear function with at most 2^q pieces.*
4. *If ψ is finite, then f is in the interior of B_ψ and B_ψ is a polyhedron of at most 2^q facets, and each of its facets contains an integral point in its relative interior.*

The Problem:

The math behind it

Gomory-Johnson Infinite group relaxation:

$$R_f : \begin{array}{rcl} x & = & f + \sum_{finite} r s_r \\ x \in \mathbb{Z}^q & & s \in \mathbb{R}_+^{\mathbb{Q}^q} \end{array} \quad (3)$$

Theorem (Minimal valid inequalities for R_f (CB-2007))

If $f \notin \mathbb{Z}^q$, any minimal valid inequality that cuts off $(f, 0)$:

1. *Is of the form $\sum_{finite} \psi(r) s_r \geq 1$.*
2. *If $B_\psi = \{x \in \mathbb{Q}^p : \psi(x - f) \leq 1\}$, then B_ψ is convex, with no integral point in its interior. Furthermore $f \in B_\psi$.*
3. *If ψ is finite, then ψ is a continuous nonegative homogeneous convex piecewise linear function with at most 2^q pieces.*
4. *If ψ is finite, then f is in the interior of B_ψ and B_ψ is a polyhedron of at most 2^q facets, and each of its facets contains an integral point in its relative interior.*

The math behind it

Gomory-Johnson Infinite group relaxation:

$$R_f : \begin{array}{rcl} x & = & f + \sum_{finite} r s_r \\ x \in \mathbb{Z}^q & & s \in \mathbb{R}_+^{\mathbb{Q}^q} \end{array} \quad (3)$$

Theorem (Minimal valid inequalities for R_f (CB-2007))

If $f \notin \mathbb{Z}^q$, any minimal valid inequality that cuts off $(f, 0)$:

1. *Is of the form $\sum_{finite} \psi(r)s_r \geq 1$.*
2. *If $B_\psi = \{x \in \mathbb{Q}^p : \psi(x - f) \leq 1\}$, then B_ψ is convex, with no integral point in its interior. Furthermore $f \in B_\psi$.*
3. *If ψ is finite, then ψ is a continuous nonegative homogeneous convex piecewise linear function with at most 2^q pieces.*
4. *If ψ is finite, then f is in the interior of B_ψ and B_ψ is a polyhedron of at most 2^q facets, and each of its facets contains an integral point in its relative interior.*

Selecting a set B_ψ

- Three kind of maximal convex lattice free sets:
  - Type 1 *n*-dimensional simplex:
    - $T1_n := \{x \in \mathbb{R}^n : x_i \geq 0, \sum_{i=1}^n x_i \leq n\}$
    - Each facet has exactly one point in its relative interior.
    - 2^n possible *orientations*.
  - The *n*-dimensional hyper cube:
  - Type-2 *n*-dimensional simplex:
  - $\text{vol}(T1_n) = \text{vol}(G_n) = \text{vol}(T2_n)\Theta(2^{-\frac{n(n-1)}{2}})$.

Selecting a set B_ψ

- Three kind of maximal convex lattice free sets:
  - Type 1 n-dimensional simplex:
    - $T1_n := \{x \in \mathbb{R}^n : x_i \geq 0, \sum_{i=1}^{n} x_i \leq n\}$
    - Each facet has exactly one point in its relative interior.
    - 2^n possible *orientations*.
  - The n-dimensional hyper cube:
  - Type-2 n-dimensional simplex:
  - $\mathrm{vol}(T1_n) = \mathrm{vol}(G_n) = \mathrm{vol}(T2_n)\Theta(2^{-\frac{n(n-1)}{2}})$.

Selecting a set B_ψ

- Three kind of maximal convex lattice free sets:
  - Type 1 n-dimensional simplex:
    - $T1_n := \{x \in \mathbb{R}^n : x_i \geq 0, \sum_{i=1}^n x_i \leq n\}$
    - Each facet has exactly one point in its relative interior.
    - 2^n possible *orientations*.
  - The n-dimensional hyper cube:
  - Type-2 n-dimensional simplex:
  - $\mathrm{vol}(T1_n) = \mathrm{vol}(G_n) = \mathrm{vol}(T2_n)\Theta(2^{-\frac{n(n-1)}{2}})$.

Selecting a set B_ψ

- Three kind of maximal convex lattice free sets:
  - Type 1 n-dimensional simplex:
    - $T1_n := \{x \in \mathbb{R}^n : x_i \geq 0, \sum_{i=1}^n x_i \leq n\}$
    - Each facet has exactly one point in its relative interior.
    - 2^n possible *orientations*.
  - The n-dimensional hyper cube:
    - $G_n := e\frac{1}{2} + \{x \in \mathbb{R}^n : \sum_{i=1}^n s_i x_i \leq \frac{n}{2}\ \forall s \in \{-1,1\}^n\}$
    - Each facet has exactly one point in its relative interior.
    - Completely symmetric.
  - Type-2 n-dimensional simplex:
  - $\mathrm{vol}(T1_n) = \mathrm{vol}(G_n) = \mathrm{vol}(T2_n)\Theta(2^{-\frac{n(n-1)}{2}})$.

Selecting a set B_ψ

- Three kind of maximal convex lattice free sets:
  - Type 1 n-dimensional simplex:
    - $T1_n := \{x \in \mathbb{R}^n : x_i \geq 0, \sum_{i=1}^n x_i \leq n\}$
    - Each facet has exactly one point in its relative interior.
    - 2^n possible *orientations*.
  - The n-dimensional hyper cube:
    - $G_n := e\frac{1}{2} + \{x \in \mathbb{R}^n : \sum_{i=1}^n s_i x_i \leq \frac{n}{2}, \forall s \in \{-1,1\}^n\}$
    - Each facet has exactly one point in its relative interior.
    - Completely symmetric.
  - Type-2 n-dimensional simplex:
  - $\mathrm{vol}(T1_n) = \mathrm{vol}(G_n) = \mathrm{vol}(T2_n)\Theta(2^{-\frac{n(n-1)}{2}})$.

Selecting a set B_ψ

- Three kind of maximal convex lattice free sets:
  - Type 1 n-dimensional simplex:
    - $T1_n := \{x \in \mathbb{R}^n : x_i \geq 0, \sum_{i=1}^n x_i \leq n\}$
    - Each facet has exactly one point in its relative interior.
    - 2^n possible *orientations*.
  - The n-dimensional hyper cube:
    - $G_n := e\frac{1}{2} + \{x \in \mathbb{R}^n : \sum_{i=1}^n s_i x_i \leq \frac{n}{2}, \forall s \in \{-1,1\}^n\}$
    - Each facet has exactly one point in its relative interior.
    - Completely symmetric.
  - Type-2 n-dimensional simplex:
- $\mathrm{vol}(T1_n) = \mathrm{vol}(G_n) = \mathrm{vol}(T2_n)\Theta(2^{-\frac{n(n-1)}{2}})$.

Selecting a set B_ψ

- Three kind of maximal convex lattice free sets:
  - Type 1 n-dimensional simplex:
    - $T1_n := \{x \in \mathbb{R}^n : x_i \geq 0, \sum_{i=1}^n x_i \leq n\}$
    - Each facet has exactly one point in its relative interior.
    - 2^n possible *orientations*.
  - The n-dimensional hyper cube:
    - $G_n := e\frac{1}{2} + \{x \in \mathbb{R}^n : \sum_{i=1}^n s_i x_i \leq \frac{n}{2}, \forall s \in \{-1,1\}^n\}$
    - Each facet has exactly one point in its relative interior.
    - Completely symmetric.
  - Type-2 n-dimensional simplex:

  - $\mathrm{vol}(T1_n) = \mathrm{vol}(G_n) = \mathrm{vol}(T2_n)\Theta(2^{-\frac{n(n-1)}{2}})$.

Selecting a set B_ψ

- Three kind of maximal convex lattice free sets:
  - Type 1 n-dimensional simplex:
    - $T1_n := \{x \in \mathbb{R}^n : x_i \geq 0, \sum_{i=1}^n x_i \leq n\}$
    - Each facet has exactly one point in its relative interior.
    - 2^n possible *orientations*.
  - The n-dimensional hyper cube:
    - $G_n := e\frac{1}{2} + \{x \in \mathbb{R}^n : \sum_{i=1}^n s_i x_i \leq \frac{n}{2}, \forall s \in \{-1,1\}^n\}$
    - Each facet has exactly one point in its relative interior.
    - Completely symmetric.
  - Type-2 n-dimensional simplex:
  - $\mathrm{vol}(T1_n) = \mathrm{vol}(G_n) = \mathrm{vol}(T2_n)\Theta(2^{-\frac{n(n-1)}{2}})$.

Selecting a set B_ψ

- Three kind of maximal convex lattice free sets:
  - Type 1 n-dimensional simplex:
    - $T1_n := \left\{x \in \mathbb{R}^n : x_i \geq 0, \sum_{i=1}^n x_i \leq n\right\}$
    - Each facet has exactly one point in its relative interior.
    - 2^n possible *orientations*.
  - The n-dimensional hyper cube:
    - $G_n := e\frac{1}{2} + \left\{x \in \mathbb{R}^n : \sum_{i=1}^n s_i x_i \leq \frac{n}{2}, \forall s \in \{-1,1\}^n\right\}$
    - Each facet has exactly one point in its relative interior.
    - Completely symmetric.
  - Type-2 n-dimensional simplex:
    - $T2_n := \left\{x \in \mathbb{R}^n : (R_i) \sum_{j=1}^{i-1} x_j - x_i \leq i-1, \forall i \in N+1\right\}$
- $\mathrm{vol}(T1_n) = \mathrm{vol}(G_n) = \mathrm{vol}(T2_n)\Theta(2^{-\frac{n(n-1)}{2}})$.

Selecting a set B_ψ

- Three kind of maximal convex lattice free sets:
  - Type 1 n-dimensional simplex:
    - $T1_n := \{x \in \mathbb{R}^n : x_i \geq 0, \sum_{i=1}^n x_i \leq n\}$
    - Each facet has exactly one point in its relative interior.
    - 2^n possible *orientations*.
  - The n-dimensional hyper cube:
    - $G_n := e\frac{1}{2} + \{x \in \mathbb{R}^n : \sum_{i=1}^n s_i x_i \leq \frac{n}{2}, \forall s \in \{-1,1\}^n\}$
    - Each facet has exactly one point in its relative interior.
    - Completely symmetric.
  - Type-2 n-dimensional simplex:
    - $T2_n := \left\{x \in \mathbb{R}^n : (R_i) \sum_{j=1}^{i-1} x_j - x_i \leq i-1, \forall i \in N+1\right\}$
    - Facet (R_i) has $\max\{2^{n-i}, 1\}$ (interior) integer points.
    - Has $2^n n!$ possible *orientations*
- $\mathrm{vol}(T1_n) = \mathrm{vol}(G_n) = \mathrm{vol}(T2_n)\Theta(2^{-\frac{n(n-1)}{2}})$.

Selecting a set B_ψ

- Three kind of maximal convex lattice free sets:
  - Type 1 n-dimensional simplex:
    - $T1_n := \{x \in \mathbb{R}^n : x_i \geq 0, \sum_{i=1}^n x_i \leq n\}$
    - Each facet has exactly one point in its relative interior.
    - 2^n possible *orientations*.
  - The n-dimensional hyper cube:
    - $G_n := e\frac{1}{2} + \{x \in \mathbb{R}^n : \sum_{i=1}^n s_i x_i \leq \frac{n}{2}, \forall s \in \{-1,1\}^n\}$
    - Each facet has exactly one point in its relative interior.
    - Completely symmetric.
  - Type-2 n-dimensional simplex:
    - $T2_n := \left\{x \in \mathbb{R}^n : (R_i) \sum_{j=1}^{i-1} x_j - x_i \leq i-1, \forall i \in N+1\right\}$
    - Facet (R_i) has $\max\{2^{n-i}, 1\}$ (interior) integer points.
    - Has $2^n n!$ possible *orientations*
- $\mathrm{vol}(T1_n) = \mathrm{vol}(G_n) = \mathrm{vol}(T2_n)\Theta(2^{-\frac{n(n-1)}{2}})$.

Selecting a set B_ψ

- Three kind of maximal convex lattice free sets:
  - Type 1 n-dimensional simplex:
    - $T1_n := \{x \in \mathbb{R}^n : x_i \geq 0, \sum_{i=1}^n x_i \leq n\}$
    - Each facet has exactly one point in its relative interior.
    - 2^n possible *orientations*.
  - The n-dimensional hyper cube:
    - $G_n := e\frac{1}{2} + \{x \in \mathbb{R}^n : \sum_{i=1}^n s_i x_i \leq \frac{n}{2}, \forall s \in \{-1,1\}^n\}$
    - Each facet has exactly one point in its relative interior.
    - Completely symmetric.
  - Type-2 n-dimensional simplex:
    - $T2_n := \left\{x \in \mathbb{R}^n : (R_i) \sum_{j=1}^{i-1} x_j - x_i \leq i-1, \forall i \in N+1\right\}$
    - Facet (R_i) has $\max\{2^{n-i}, 1\}$ (interior) integer points.
    - Has $2^n n!$ possible *orientations*
- $\mathrm{vol}(T1_n) = \mathrm{vol}(G_n) = \mathrm{vol}(T2_n)\Theta(2^{-\frac{n(n-1)}{2}})$.

Selecting a set B_ψ

- Three kind of maximal convex lattice free sets:
  - Type 1 n-dimensional simplex:
    - $T1_n := \{x \in \mathbb{R}^n : x_i \geq 0, \sum_{i=1}^n x_i \leq n\}$
    - Each facet has exactly one point in its relative interior.
    - 2^n possible *orientations*.
  - The n-dimensional hyper cube:
    - $G_n := e\frac{1}{2} + \{x \in \mathbb{R}^n : \sum_{i=1}^n s_i x_i \leq \frac{n}{2}, \forall s \in \{-1,1\}^n\}$
    - Each facet has exactly one point in its relative interior.
    - Completely symmetric.
  - Type-2 n-dimensional simplex:
    - $T2_n := \left\{x \in \mathbb{R}^n : (R_i) \sum_{j=1}^{i-1} x_j - x_i \leq i-1, \forall i \in N+1\right\}$
    - Facet (R_i) has $\max\{2^{n-i}, 1\}$ (interior) integer points.
    - Has $2^n n!$ possible *orientations*
  - $\mathrm{vol}(T1_n) = \mathrm{vol}(G_n) = \mathrm{vol}(T2_n)\Theta(2^{-\frac{n(n-1)}{2}})$.

Selecting a set B_ψ

- Three kind of maximal convex lattice free sets:
  - Type 1 n-dimensional simplex:
    - $T1_n := \{x \in \mathbb{R}^n : x_i \geq 0, \sum_{i=1}^n x_i \leq n\}$
    - Each facet has exactly one point in its relative interior.
    - 2^n possible *orientations*.
  - The n-dimensional hyper cube:
    - $G_n := e\frac{1}{2} + \{x \in \mathbb{R}^n : \sum_{i=1}^n s_i x_i \leq \frac{n}{2}, \forall s \in \{-1,1\}^n\}$
    - Each facet has exactly one point in its relative interior.
    - Completely symmetric.
  - Type-2 n-dimensional simplex:
    - $T2_n := \left\{x \in \mathbb{R}^n : (R_i) \sum_{j=1}^{i-1} x_j - x_i \leq i-1,\ \forall i \in N+1\right\}$
    - Facet (R_i) has $\max\{2^{n-i}, 1\}$ (interior) integer points.
    - Has $2^n n!$ possible *orientations*
  - $\mathrm{vol}(T1_n) = \mathrm{vol}(G_n) = \mathrm{vol}(T2_n)\Theta(2^{-\frac{n(n-1)}{2}})$.

How we separate:

- Relaxation selection:

How we separate:

- Relaxation selection:
  - Sort fractional integer variables by pseudo cost.

How we separate:

- Relaxation selection:
  - Sort fractional integer variables by pseudo cost.
  - Discard numerically ill tableau rows ($|a| \leq 10^{-6} \Rightarrow a = 0$).

How we separate:

- Relaxation selection:
  - Sort fractional integer variables by pseudo cost.
  - Discard numerically ill tableau rows ($|a| \leq 10^{-6} \Rightarrow a = 0$).
  - Store tableau rows (in the previous order) that have a ratio $\leq 2^{12}$.

How we separate:

- Relaxation selection:
  - Sort fractional integer variables by pseudo cost.
  - Discard numerically ill tableau rows ($|a| \leq 10^{-6} \Rightarrow a = 0$).
  - Store tableau rows (in the previous order) that have a ratio $\leq 2^{12}$.
  - Up to n tableau rows are found, or no more available.

How we separate:

- Relaxation selection:
  - Sort fractional integer variables by pseudo cost.
  - Discard numerically ill tableau rows ($|a| \leq 10^{-6} \Rightarrow a = 0$).
  - Store tableau rows (in the previous order) that have a ratio $\leq 2^{12}$.
  - Up to n tableau rows are found, or no more available.
- Orientation selection:

How we separate:

- Relaxation selection:
  - Sort fractional integer variables by pseudo cost.
  - Discard numerically ill tableau rows ($|a| \leq 10^{-6} \Rightarrow a = 0$).
  - Store tableau rows (in the previous order) that have a ratio $\leq 2^{12}$.
  - Up to n tableau rows are found, or no more available.
- Orientation selection: Try all!

How we separate:

- Relaxation selection:
  - Sort fractional integer variables by pseudo cost.
  - Discard numerically ill tableau rows ($|a| \leq 10^{-6} \Rightarrow a = 0$).
  - Store tableau rows (in the previous order) that have a ratio $\leq 2^{12}$.
  - Up to n tableau rows are found, or no more available.
- Orientation selection: Try all!
- Cut selection:

How we separate:

- Relaxation selection:
  - Sort fractional integer variables by pseudo cost.
  - Discard numerically ill tableau rows ($|a| \leq 10^{-6} \Rightarrow a = 0$).
  - Store tableau rows (in the previous order) that have a ratio $\leq 2^{12}$.
  - Up to n tableau rows are found, or no more available.
- Orientation selection: Try all!
- Cut selection:
  - Discard cuts with ratio $\geq 2^{15}$.

How we separate:

- Relaxation selection:
  - Sort fractional integer variables by pseudo cost.
  - Discard numerically ill tableau rows ($|a| \leq 10^{-6} \Rightarrow a = 0$).
  - Store tableau rows (in the previous order) that have a ratio $\leq 2^{12}$.
  - Up to n tableau rows are found, or no more available.
- Orientation selection: Try all!
- Cut selection:
  - Discard cuts with ratio $\geq 2^{15}$.
  - Select at most N cuts minimizing $1/max\_abs$.

How we separate:

- Relaxation selection:
  - Sort fractional integer variables by pseudo cost.
  - Discard numerically ill tableau rows ($|a| \leq 10^{-6} \Rightarrow a = 0$).
  - Store tableau rows (in the previous order) that have a ratio $\leq 2^{12}$.
  - Up to n tableau rows are found, or no more available.
- Orientation selection: Try all!
- Cut selection:
  - Discard cuts with ratio $\geq 2^{15}$.
  - Select at most N cuts minimizing $1/max\_abs$.
  - Allow multiple rounds.

What we compare:

- We implement the cut-generation procedure as a CPLEX cut-callback.
- Use it under default CPLEX 11.0 settings (including cut generation) and preprocessing.
- We compare:
  - root LP value.
  - final running time for those that finish the run.
  - final B&B bound after time limit.
- All runs with two hours time limit.
- Base results will be CPLEX defaults with pre-processing.
- Cut generation done only at root node.

What we compare:

- We implement the cut-generation procedure as a CPLEX cut-callback.
- Use it under default CPLEX 11.0 settings (including cut generation) and preprocessing.
- We compare:
  - root LP value.
  - final running time for those that finish the run.
  - final B&B bound after time limit.
- All runs with two hours time limit.
- Base results will be CPLEX defaults with pre-processing.
- Cut generation done only at root node.

What we compare:

- We implement the cut-generation procedure as a CPLEX cut-callback.
- Use it under default CPLEX 11.0 settings (including cut generation) and preprocessing.
- We compare:
  - root LP value.
  - final running time for those that finish the run.
  - final B&B bound after time limit.
- All runs with two hours time limit.
- Base results will be CPLEX defaults with pre-processing.
- Cut generation done only at root node.

What we compare:

- We implement the cut-generation procedure as a CPLEX cut-callback.
- Use it under default CPLEX 11.0 settings (including cut generation) and preprocessing.
- We compare:
  - root LP value.
  - final running time for those that finish the run.
  - final B&B bound after time limit.
- All runs with two hours time limit.
- Base results will be CPLEX defaults with pre-processing.
- Cut generation done only at root node.

What we compare:

- We implement the cut-generation procedure as a CPLEX cut-callback.
- Use it under default CPLEX 11.0 settings (including cut generation) and preprocessing.
- We compare:
  - root LP value.
  - final running time for those that finish the run.
  - final B&B bound after time limit.
- All runs with two hours time limit.
- Base results will be CPLEX defaults with pre-processing.
- Cut generation done only at root node.

What we compare:

- We implement the cut-generation procedure as a CPLEX cut-callback.
- Use it under default CPLEX 11.0 settings (including cut generation) and preprocessing.
- We compare:
    - root LP value.
    - final running time for those that finish the run.
    - final B&B bound after time limit.
- All runs with two hours time limit.
- Base results will be CPLEX defaults with pre-processing.
- Cut generation done only at root node.

What we compare:

- We implement the cut-generation procedure as a CPLEX cut-callback.
- Use it under default CPLEX 11.0 settings (including cut generation) and preprocessing.
- We compare:
  - root LP value.
  - final running time for those that finish the run.
  - final B&B bound after time limit.
- All runs with two hours time limit.
- Base results will be CPLEX defaults with pre-processing.
- Cut generation done only at root node.

What we compare:

- We implement the cut-generation procedure as a CPLEX cut-callback.
- Use it under default CPLEX 11.0 settings (including cut generation) and preprocessing.
- We compare:
  - root LP value.
  - final running time for those that finish the run.
  - final B&B bound after time limit.
- All runs with two hours time limit.
- Base results will be CPLEX defaults with pre-processing.
- Cut generation done only at root node.

What we compare:

- We implement the cut-generation procedure as a CPLEX cut-callback.
- Use it under default CPLEX 11.0 settings (including cut generation) and preprocessing.
- We compare:
  - root LP value.
  - final running time for those that finish the run.
  - final B&B bound after time limit.
- All runs with two hours time limit.
- Base results will be CPLEX defaults with pre-processing.
- Cut generation done only at root node.

Where we compare:

- Instances from MIPLIB 3.0, MIPLIB 2003, and others.
- Total of 173 problems.
- Discarded the following problems:
  - Solved by all configurations in under 5 seconds: 48.
  - LP root gap less than 0.1% in all configurations: 26.
  - Two unstable problems discarded, roll3000, l152lav.
- Differences under 5% are not considered in the averages.
- Final test set has 96 problems.

Where we compare:

- Instances from MIPLIB 3.0, MIPLIB 2003, and others.
- Total of 173 problems.
- Discarded the following problems:
  - Solved by all configurations in under 5 seconds: 48.
  - LP root gap less than 0.1% in all configurations: 26.
  - Two unstable problems discarded: roll3000, l152lav.
- Differences under 5% are not considered in the averages.
- Final test set has 96 problems.

Where we compare:

- Instances from MIPLIB 3.0, MIPLIB 2003, and others.
- Total of 173 problems.
- Discarded the following problems:
  - Solved by all configurations in under 5 seconds: 48.
  - LP root gap less than 0.1% in all configurations: 26.
  - Two unstable problems discarded, roll3000, l152av.
- Differences under 5% are not considered in the averages.
- Final test set has 96 problems.

Where we compare:

- Instances from MIPLIB 3.0, MIPLIB 2003, and others.
- Total of 173 problems.
- Discarded the following problems:
  - Solved by all configurations in under 5 seconds: 48.
  - LP root gap less than 0.1% in all configurations: 26.
  - Two unstable problems discarded, roll3000, l152av.
- Differences under 5% are not considered in the averages.
- Final test set has 96 problems.

Where we compare:

- Instances from MIPLIB 3.0, MIPLIB 2003, and others.
- Total of 173 problems.
- Discarded the following problems:
  - Solved by all configurations in under 5 seconds: 48.
  - LP root gap less than 0.1% in all configurations: 26.
  - Two unstable problems discarded, roll3000, l152av.
- Differences under 5% are not considered in the averages.
- Final test set has 96 problems.

Where we compare:

- Instances from MIPLIB 3.0, MIPLIB 2003, and others.
- Total of 173 problems.
- Discarded the following problems:
  - Solved by all configurations in under 5 seconds: 48.
  - LP root gap less than 0.1% in all configurations: 26.
  - Two unstable problems discarded, roll3000, l152av.
- Differences under 5% are not considered in the averages.
- Final test set has 96 problems.

Where we compare:

- Instances from MIPLIB 3.0, MIPLIB 2003, and others.
- Total of 173 problems.
- Discarded the following problems:
  - Solved by all configurations in under 5 seconds: 48.
  - LP root gap less than 0.1% in all configurations: 26.
  - Two unstable problems discarded, roll3000, l152av.
- Differences under 5% are not considered in the averages.
- Final test set has 96 problems.

Where we compare:

- Instances from MIPLIB 3.0, MIPLIB 2003, and others.
- Total of 173 problems.
- Discarded the following problems:
    - Solved by all configurations in under 5 seconds: 48.
    - LP root gap less than 0.1% in all configurations: 26.
    - Two unstable problems discarded, roll3000, l152av.
- Differences under 5% are not considered in the averages.
- Final test set has 96 problems.

What we compare:

Tested settings

- Will compare several configurations:
  - default: Cplex 11.0 defaults with pre-processing.
  - General naming scheme will be C-Tt-Nn:
  - C $\in$ { Gomory (G), $T1$, $T2$}, indicate the type of lattice-free set used.
  - t $\in \{2, 5, 10\}$, is an upper bound on the number of tableau rows selected at each round of cuts at the root node.
  - n $\in \{5, 20, 100\}$, is an upper bound on the number of cuts to be added at the root node.

Tested settings

- Will compare several configurations:
  - default: Cplex 11.0 defaults with pre-processing.
  - General naming scheme will be C-Tt-Nn:
  - C $\in$ { Gomory (G), $T1$, $T2$}, indicate the type of lattice-free set used.
  - t $\in \{2, 5, 10\}$, is an upper bound on the number of tableau rows selected at each round of cuts at the root node.
  - n $\in \{5, 20, 100\}$, is an upper bound on the number of cuts to be added at the root node.

Tested settings

- Will compare several configurations:
  - default: Cplex 11.0 defaults with pre-processing.
  - General naming scheme will be C-Tt-Nn:
  - C $\in$ { Gomory (G), $T1$, $T2$}, indicate the type of lattice-free set used.
  - t $\in \{2, 5, 10\}$, is an upper bound on the number of tableau rows selected at each round of cuts at the root node.
  - n $\in \{5, 20, 100\}$, is an upper bound on the number of cuts to be added at the root node.

What we compare:

Tested settings

- Will compare several configurations:
  - default: Cplex 11.0 defaults with pre-processing.
  - General naming scheme will be C-Tt-Nn:
  - C $\in$ { Gomory (G), $T1$, $T2$}, indicate the type of lattice-free set used.
  - t $\in \{2, 5, 10\}$, is an upper bound on the number of tableau rows selected at each round of cuts at the root node.
  - n $\in \{5, 20, 100\}$, is an upper bound on the number of cuts to be added at the root node.

What we compare:

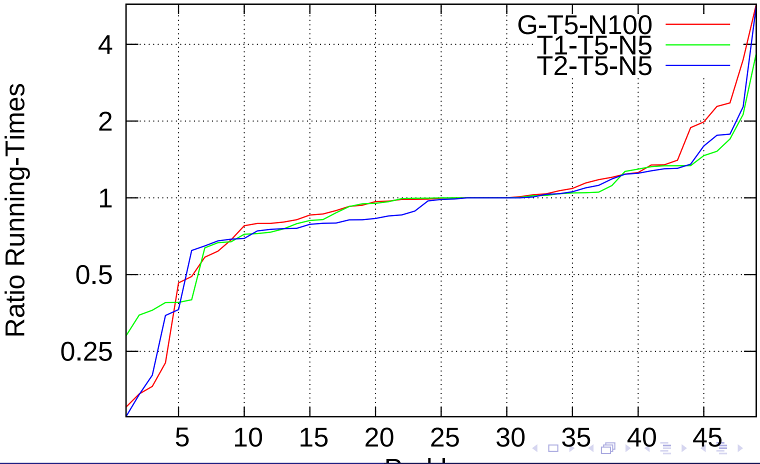
Tested settings

- Will compare several configurations:
  - default: Cplex 11.0 defaults with pre-processing.
  - General naming scheme will be C-Tt-Nn:
  - C $\in$ { Gomory (G), $T1$, $T2$}, indicate the type of lattice-free set used.
  - t $\in \{2, 5, 10\}$, is an upper bound on the number of tableau rows selected at each round of cuts at the root node.
  - n $\in \{5, 20, 100\}$, is an upper bound on the number of cuts to be added at the root node.

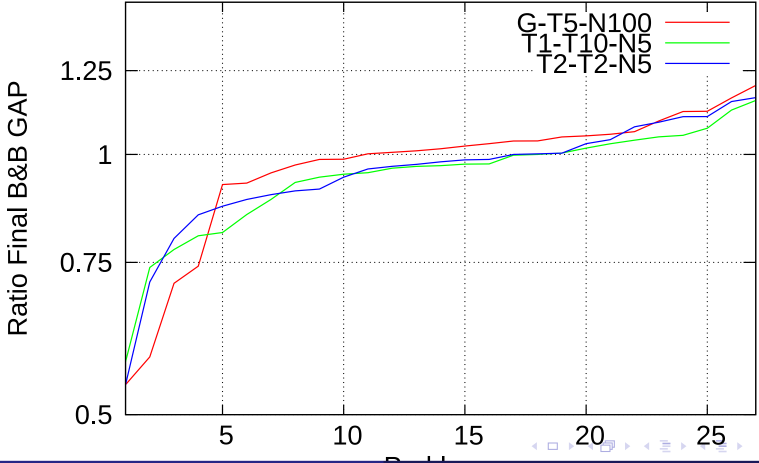
Tested settings

- Will compare several configurations:
  - default: Cplex 11.0 defaults with pre-processing.
  - General naming scheme will be C-Tt-Nn:
  - C $\in$ { Gomory (G), $T1$, $T2\}$, indicate the type of lattice-free set used.
  - t $\in \{2, 5, 10\}$, is an upper bound on the number of tableau rows selected at each round of cuts at the root node.
  - n $\in \{5, 20, 100\}$, is an upper bound on the number of cuts to be added at the root node.

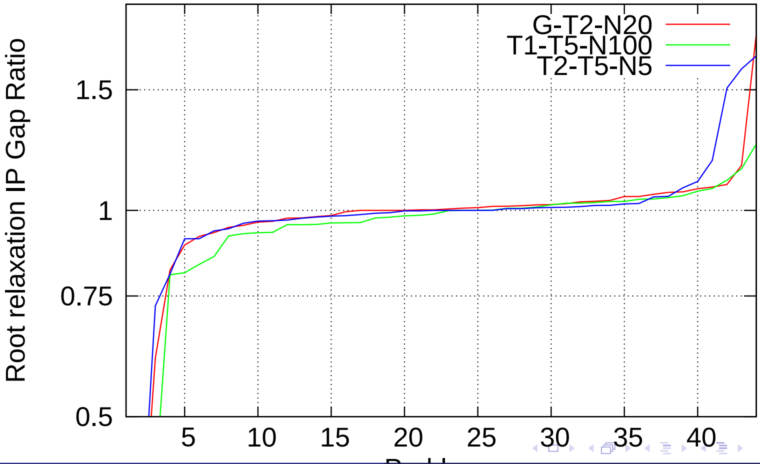
Overall speed-up (49 instances) 6.8%, 8.3%, 11.8%



Closed B&B GAP_{MIP} (27 instances) 3.7%, 6.0%, 4.5%

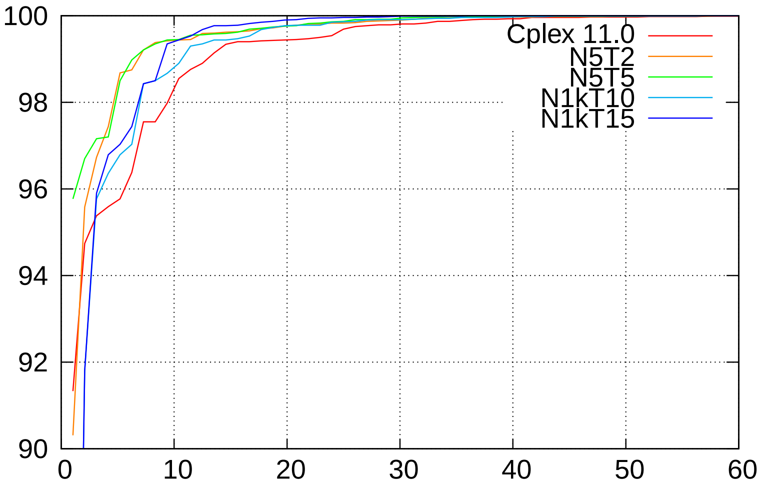


Closed Root GAP_{LP} (44 instances) 7.2%, 8.3%, 5.7%

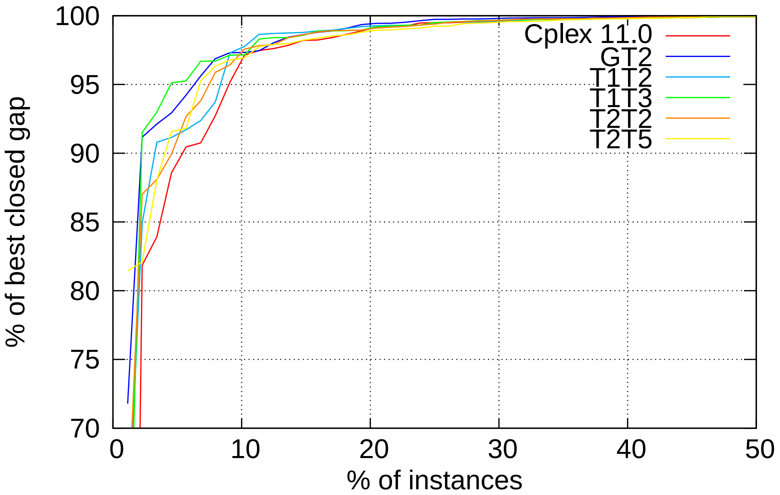


The Results

Performance Profile



Performance Profile II



Conclusions

- Of all tested configurations, only two had worst results on Root LP gap and on B&B gap, and eight had worst results on speed.
- Although the improvements are not dramatic, they still are important.
- Lots of testing of parameters.
- Numerical issues are important!
- Could we do a full separation?

Conclusions

- Of all tested configurations, only two had worst results on Root LP gap and on B&B gap, and eight had worst results on speed.
- Although the improvements are not dramatic, they still are important.
- Lots of testing of parameters.
- Numerical issues are important!
- Could we do a full separation?

Conclusions

- Of all tested configurations, only two had worst results on Root LP gap and on B&B gap, and eight had worst results on speed.
- Although the improvements are not dramatic, they still are important.
- Lots of testing of parameters.
- Numerical issues are important!
- Could we do a full separation?

Conclusions

- Of all tested configurations, only two had worst results on Root LP gap and on B&B gap, and eight had worst results on speed.
- Although the improvements are not dramatic, they still are important.
- Lots of testing of parameters.
- Numerical issues are important!
- Could we do a full separation?

Conclusions

- Of all tested configurations, only two had worst results on Root LP gap and on B&B gap, and eight had worst results on speed.
- Although the improvements are not dramatic, they still are important.
- Lots of testing of parameters.
- Numerical issues are important!
- Could we do a full separation?