A Hybrid Approach to Beam Angle Optimization in Intensity-Modulated Radiation Therapy

Dimitris Bertsimas and Omid Nohadani

Operations Research Center, MIT (Cambridge USA)

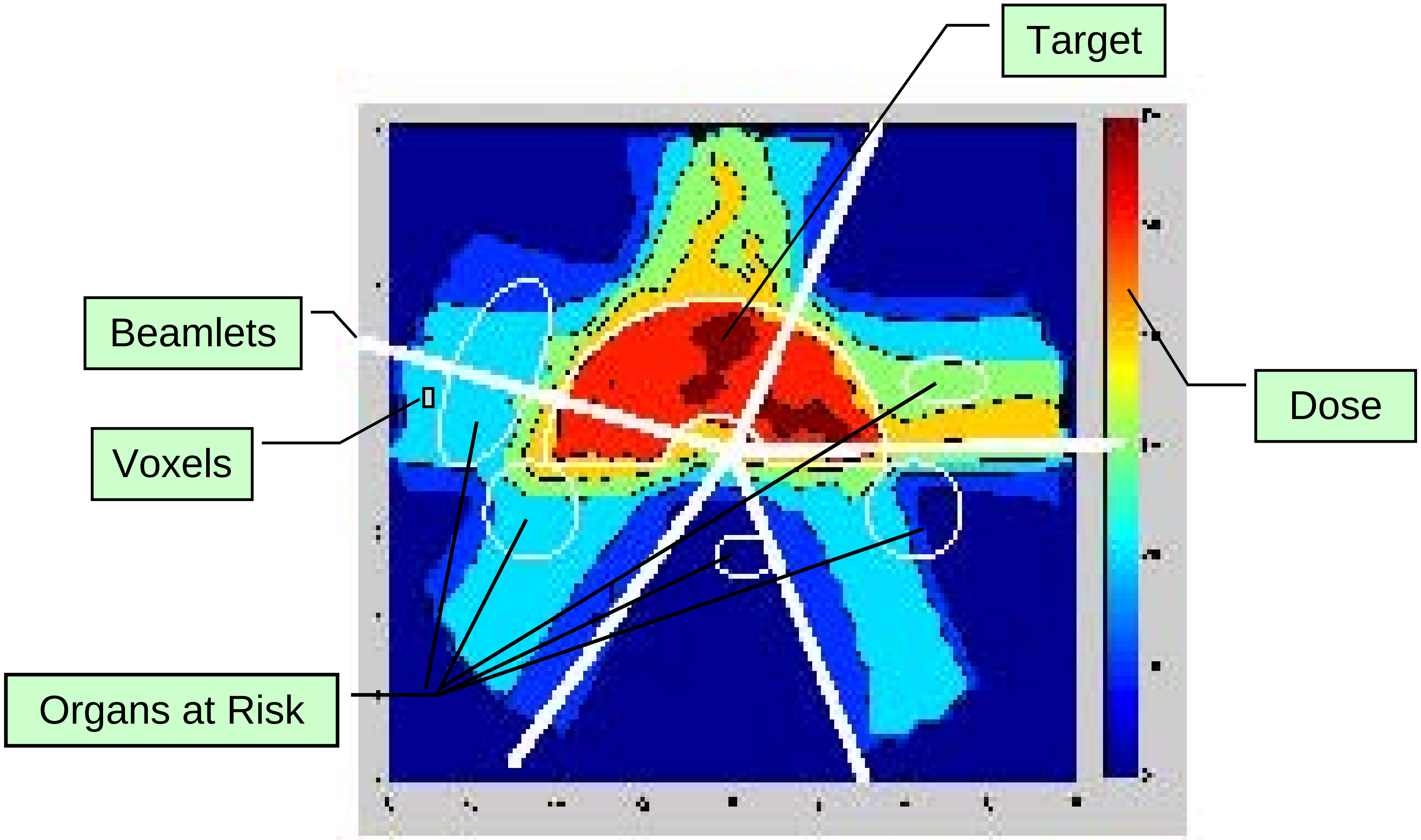
Thomas Bortfeld and David Craft

Department of Radiation Oncology, MGH and Harvard Medical School (Boston USA)

Valentina Cacchiani

University of Bologna (Italy)

Example



Beam Angle Optimization

Consider the problem of optimizing simultaneously:

- the intensities of the beamlets
- the choice of the angles from which the beams are delivered

Aim is to determine:

- a set of beam angles
- the corresponding beamlets intensities

so that:

- the prescribed dose in the tumor is achieved
- the organs at risk are spared

Mathematical Formulation

Non-Linear Programming Model

$$\min f(x,\theta) = \sum_{i\in V} \sum_{k=1}^{n} \sum_{j\in B_k} w_i D_{ij}(\theta_k) x_j,$$

$$\sum_{k=1}^{n} \sum_{j\in B_k} D_{ij}(\theta_k) x_j \geq m_i, \quad i \in T,$$

$$\sum_{k=1}^{n} \sum_{j\in B_k} D_{ij}(\theta_k) x_j \leq M_i, \quad i \in V,$$

$$x_j \geq 0, \quad j \in B,$$

$$\theta_k \geq 0, \quad k = 1,..,n$$

Dose Influence Matrix

Intensity of beamlet *j*

Prescribed Dose

Direction of beam *k*

Tumor Voxels

Max Dose

Set of Voxels

Set of Beamlets

Set of Angles

For a fixed θ we can rewrite the model as:

$$f(\theta) = \min_{x \geq 0} g(d)$$

$$s.t. \qquad d = D(\theta)x,$$

$$d \in H$$

Linear and convex function

voxel doses

d between a min and a max dose

Experimental tests show that $f(\theta)$ is

- highly non-convex
- with many local minima

For a fixed θ we obtain a Linear Programming Problem
(beamlets' intensities optimization)

Heuristic Hybrid Method

Start with *n* angles chosen randomly and almost equispaced

We alternate iterations of

- Gradient Descent
- Simulated Annealing

Helps to find a local minimum fast

Allows to escape from local minima

Gradient Computation

Solve the LP problem (with fixed set of angles)

Let x^* and d^* be the optimal beamlet solution and dose distribution

Let p^* be the optimal dual variables associated with the constraints $d = D(\theta)x$

Consider a particular voxel *v* and beamlet *b* and assume that this beamlet belongs to the first beam

Consider a small change δ in the dose influence value $D^1_{vb}(\theta_1) \rightarrow D^1_{vb}(\theta_1) + \delta$

Then to first order $g(d^*(\delta)) = g(d^*) - \delta x^*_b p^*_v$ $\Longrightarrow$ $\dfrac{\partial f}{\partial D^1_{vb}(\theta_1)} = -x^*_b p^*_v$

Now consider the slope $S^1_{vb}(\theta_1)$

$$\frac{\partial f}{\partial \theta_1} = \frac{\partial f}{\partial D^1}\frac{\partial D^1}{\partial \theta_1} = -\sum_v \sum_b x^*_b p^*_v S^1_{vb}(\theta_1)$$

Then the gradient becomes: $\nabla_\theta f(\theta) = -[p^* S^1(\theta_1) x^{1*}, \ldots, p^* S^n(\theta_n) x^{n*}]$

Simulated Annealing Phase

Generate a new set of angles

$$\theta^{new} = \theta^{old} + r * \alpha$$

random number with Gaussian distribution

step size

Move to the new set of angles according to probability

$$e^{(-(f_{new} - f_{old})/t_l)}$$

where

$$t_l = T_0 * e^{(-c * l^{1/d})}$$

initial temperature

dimension of the problem

Experimental Results

2D pancreatic phantom case

5 beams, 1257 voxels and 80 beamlets

IMRT beamlet calculation is performed by the pencil beam method