Cooperation of Zuse Institute Berlin and ILOG

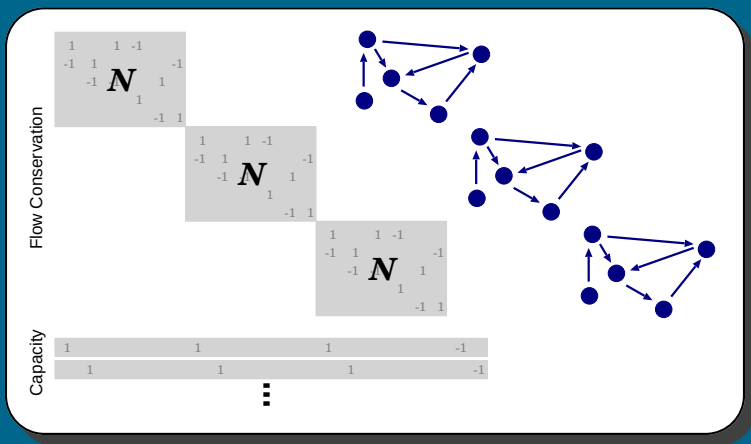
Capacitated Multi-Commodity-Flow Cuts

Part I: MCF Network Detection

Tobias Achterberg Christian Raack

Abstract: We try to extract and exploit multi-commodity flow (MCF) structures within general MIP formulations. This part describes how to detect MCF networks, that is, nodes with flow conservation, arcs with capacities, and commodities coupled by capacity constraints.

- **Literature:** Network detection for single commodity flows in the context of the network simplex: Brown & Wright [84]; Bixby & Fourer [88]; Gupinar et al. [98, 04]; Gutin & Zverovitsch [04] and others
- **Methods:** Row/column-scanning addition/deletion, signed graphs, IP formulation



Network Design

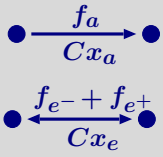
$$N f^k \begin{matrix} \leqq \\ = \\ \geqq \end{matrix} d^k \quad \text{Flow conservation for all commodities } k$$

$$\sum_{k \in K} f_a^k \leq C x_a \quad \text{Capacity constraint for all arcs } a$$

- N network matrix, f flow variables, x capacity variables
- Coupling of commodities by arc capacity constraints

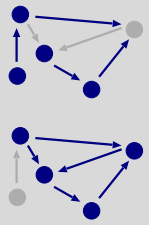
Variations:

- Model: undirected, directed / bidirected
- Flow: splittable, unsplittable, integer
- Capacity: single, modular, binary, uncapacitated, GUBs



Challenges

- **User preprocessing:** omitting one flow row per commodity, no inflow into source nodes
- **Solver preprocessing:** deleting loosely connected nodes (fixing, substituting flow variables)
- different model formulations
- additional side constraints



Algorithm

Flow Detection:

- Identify and sort flow conservation row candidates
- Row-scanning addition using flow candidates
- Identify commodities, scale and invert rows if necessary
- Throw away small components (few flow rows)
- Flow sub-matrix defines commodities and flow variables

⇒ Graph structure with one component per commodity

Capacity and Arc Detection:

- Identify and sort capacity row candidates
- Identify model type (directed, undirected)
- For all flow columns choose capacity row, assign arc-id to capacity row and corresponding flow variables

Node Detection:

- Assign node-id to flow rows (in different commodities) with similar incidence pattern w.r.t. arc-ids.

⇒ Coupling rows help to solve graph isomorphism problem.

Output:

- A set of MCF networks (usually one only).
- **flow rows ↔ nodes, capacity rows ↔ arcs**

Preliminary Results – arithmetic means

problem	original			detection – no presolve			detection – presolve		
	nodes	arcs	coms	nodes	arcs	coms	nodes	arcs	coms
SNDlib (ZIB): 52 instances				±0.3%	±0.5%	±0%	±6%	±0.1%	±0%
germany50-B-M	50	176	47	✓	✓	✓	+26%	✓	✓
cost266-U-M	37	57	36	✓	✓	✓	✓	✓	✓
ta2-B-E	65	216	42	✓	✓	✓	+9.2%	-0.9%	✓
zib54-U-E	54	81	32	✓	✓	✓	-1.9%	-1.2%	✓
avub (Atamtuerk [01]): 60 instances				±25%	±8%	±0%	±56%	±36%	±0%
nexp.50.20.8.4	50	245	1	-20%	-4.1%	✓	-56%	-38%	✓
nexp.100.20.8.4	100	990	1	-19%	-2.8%	✓	-59%	-36%	✓
nexp.150.20.8.4	150	2235	1	-18%	-3.2%	✓	-58%	-32.3%	✓
ufcn (OrtegaWolsey [03]): 84 instances				±2.5%	±3.8%	±0%	±8.8%	±9%	±0%
p500x2988	500	2988	1	✓	✓	✓	+2.6%	✓	✓
sp90x180	90	180	1	-18.9%	-28.3%	✓	-58.9%	-48.9%	✓
g200x740b	200	740	1	✓	✓	✓	+6.5%	✓	✓
p80x400	80	400	1	✓	✓	✓	-2.5%	-1%	✓
arc.set (Atamtuerk [03]): 35 instances				±0%	±0%	±0%	±20%	±13%	±0%
nu4-pr3	29	122	70	✓	✓	✓	-10.3%	-4.9%	✓
nu4-pr4	18	58	58	✓	✓	✓	-22.2%	-13.8%	✓
nu4-pr6	27	74	93	✓	✓	✓	-29.6%	-21.6%	✓
nu4-pr9	24	84	87	✓	✓	✓	-12.5%	-7.1%	✓

Relative detection error — selected instances and arithmetic means per test set
✓ means correct detection